https://doi.org/10.33271/nvngu/2026-4/068

O. Bazhinov[1],
orcid.org/0000-0002-5755-8553,
M. Arpabekov[*2],
orcid.org/0000-0002-7998-2507,
I. Taran[3],
orcid.org/0000-0002-3679-2519,
T. Bazhynova[1],
orcid.org/0000-0003-3003-4028,
E. Kykla[1],
orcid.org/0009-0006-8547-2120

1 – Kharkiv National Automobile and Highway University, Kharkiv, Ukraine
2 – L. N. Gumilyov Eurasian National University, Astana, Kazakhstan
3 – Dnipro University of Technology, Dnipro, Ukraine
* Corresponding author e-mail: arpabekov_mi@enu.kz

INTELLIGENT CONTROL OF IN-WHEEL MOTORS IN A VEHICLE WITH ANALYTICAL DETERMINATION OF TORQUE DISTRIBUTION

Purpose. To derive a closed-form analytical solution for the optimal distribution of drive torque between the left and right sides of an all-wheel-drive vehicle equipped with in-wheel motors. The proposed solution maximizes the instantaneous efficiency of the wheel propulsion system while accounting for load asymmetry and driving conditions.

Methodology. The study employs analytical methods based on a reduced energy model of the wheel propulsion system, quadratic optimization techniques, and simulation modelling in the MATLAB/Simulink environment to verify the performance of the proposed torque distribution law.

Findings. A closed-form analytical expression was obtained for determining the optimal torque distribution coefficient between the left and right sides of the vehicle. Its application ensures the maximization of the instantaneous efficiency of the wheel propulsion system. It was demonstrated that the optimal torque distribution depends on the normal tyre-road reactions, vehicle geometric parameters, and driving conditions. Simulation results confirmed the effectiveness of the proposed control law under both traction and curvilinear driving conditions.

Originality. For the first time, the problem of torque distribution between the left and right sides of an all-wheel-drive vehicle with independently driven in-wheel motors has been formulated as the problem of maximizing the instantaneous efficiency of the wheel propulsion system. This made it possible to derive a control law suitable for real-time implementation without the need for numerical optimization.

Practical value. The proposed analytical torque distribution law can be integrated into the upper-level control system of all-wheel-drive vehicles equipped with independently driven in-wheel motors. Its implementation improves vehicle energy efficiency without increasing the complexity of the control algorithms or the associated computational burden.

Keywords: *in-wheel motor, all-wheel-drive vehicle, vehicle speed, torque, regenerative braking*

Received: 25 February 2026
Accepted: 13 May 2026
Available online: 27 August 2026

Introduction. Intensification of transport electrification processes and stricter requirements regarding the energy efficiency and environmental friendliness of vehicles necessitate the development of new power architectures. One promising area is the use of in-wheel motors, in which the traction electric motor is integrated directly into the wheel, thereby eliminating the need for traditional mechanical transmission components.

The use of in-wheel motors enables independent control of torque on each wheel, opening up additional potential for improving handling, manoeuvrability and traction efficiency of the vehicle. At the same time, such architecture requires solving the problem of optimal torque distribution between the sides or axles, taking into account the conditions of driving, load and environmental resistance.

Despite a large amount of research in the field of torque vectoring and regenerative braking, the issue of analytical rationale for energy-efficient torque distribution in all-wheel-drive vehicles with in-wheel motors remains insufficiently explored.

© Bazhinov O., Arpabekov M., Taran I., Bazhynova T., Kykla E., 2026

The purpose of the research is to develop an analytical method for determining the optimal torque distribution coefficient between the sides of an all-wheel-drive vehicle equipped with in-wheel motors, aimed at maximizing the instantaneous efficiency of the wheel propulsion system.

Literature review. The development of electric vehicles with distributed drive systems and in-wheel motors has significantly expanded the possibilities for controlling the traction and braking modes of the vehicle. The presence of an independent drive for each wheel makes it possible not only to implement traction control and stabilization functions, but also to use torque distribution as a tool for increasing energy efficiency [1]. In [1], a parameterized torque distribution strategy for an all-wheel-drive electric vehicle is proposed, focused on minimizing losses in power electric paths.

These approaches were further developed in [2, 3], where the torque distribution is treated as an optimization problem based on the efficiency maps of electric motors. It is proven that load redistribution between drives reduces total electrical losses. However, in such studies, the focus is on electromechanical losses in

drives, while losses in tire-road surface contact and rolling resistance are generally described in simplified form. Similar issues are also characteristic of research into the energy efficiency of electric rolling stock, where the optimization of modes of propulsion system operation is examined at a system level, taking operational factors into account [4].

At the same time, a significant number of studies are devoted to torque vectoring and vehicle motion stabilization. In particular, [5 and 6] prove that torque distribution can simultaneously improve handling and reduce energy consumption. In [7], it is shown that traction optimization in extreme manoeuvring conditions requires a comprehensive consideration of both stability and energy efficiency. The issue of improving the efficiency of braking modes as a tool for improving the operational performance of transport systems was also examined in studies of rail transport. The impact of improving service braking on the potential to increase train weights and the energy efficiency of the transportation process was proven [8].

Hierarchical control structures form a separate direction, where the upper level provides dynamics and safety, while the lower level implements optimal torque distribution [9]. In [10 and 11], algorithms that combine energy optimization with road-wheel adhesion constraints are discussed. However, the implementation of such approaches is often based on numerical optimization or predictive control, which complicates their use in real-time systems.

The problem of minimizing losses due to tyre slippage is actively investigated in [12 and 13], where it is shown that energy dissipation in the tyre can be comparable to, or even exceed, losses in electric motors. Thus, the optimization of torque distribution should take into account not only the electrical, but also the mechanical components of the losses. In modern transport research, regression models and machine learning methods are used to assess the behaviour of loaded components and predict their service life [8], thus emphasizing the expediency of taking into account variable dynamic loads and operational factors when forming control algorithms [14].

Studies [15, 16] examine the use of optimization algorithms (including heuristic and Model Predictive Control (MPC) approaches) to determine the torque distribution between wheels or axles. Such methods demonstrate reduced energy consumption under model conditions; however, their versatility and ease of implementation remain limited [17]. System-level modelling of transport processes and vehicle operation provides an effective means of improving transport system performance by incorporating operational conditions into the analysis and enabling a more comprehensive evaluation of energy efficiency. Another promising research direction involves automated generation and continuous updating of digital models of complex engineering systems using information obtained from sensor measurements and actuator states, thereby improving modelling accuracy and enhancing the adaptability of control algorithms [18, 19].

Review papers [20, 21] systematize current approaches to torque distribution for in-wheel motor vehicles. The generalization shows that most strategies are focused either on driving stability or on minimizing losses in the power drive. An analytical determination of the distribution that directly maximizes the instantaneous efficiency of the wheel propulsion system, taking into account actual load asymmetries, is rarely examined.

Recent studies [22, 23] have attempted to account for both electrical and mechanical losses simultaneously; however, the solutions obtained are mostly based on numerical optimization or pre-tabulated dependences. The cited sources do not provide a generalized analytical dependence, suitable for real-time implementation, for determining the optimal torque distribution between the wheels.

Thus, the analysis of current research indicates that the issue of an analytical rationale for optimal torque distribution between the wheels/axles of in-wheel motor vehicle in order to maximize the instantaneous efficiency of the wheel propulsion system remains insufficiently explored. The existing approaches are mainly based on numerical methods or optimization of individual loss components, which complicates their generalisation and practical implementation. This research is specifically devoted to solving this problem.

Purpose and objectives of research. The purpose of this research is to analytically determine the law of total torque distribution between the left and right sides of an all-wheel-drive vehicle with in-wheel motors, which provides maximum instantaneous efficiency of the wheel propulsion system.

To achieve the set purpose, the following objectives should be achieved:

- to develop an energy model of the instantaneous efficiency of the wheel propulsion system, taking into account rolling losses and elastic-deformation losses in the tyre;
- to reduce the problem to quadratic function optimization with respect to the distribution coefficient;
- to find a closed-form analytical expression and prove the existence of a global maximum;
- to verify the performance of the resulting law using simulation modelling.

In-wheel motor energy efficiency and torque distribution optimization. *Instantaneous efficiency of a single in-wheel motor*. The energy efficiency of in-wheel motor is determined by its instantaneous efficiency, which is expressed as the product of two components – the power component and the elastic-deformation component [24, 25]

$$\eta_k^{mit} = \eta_{fk}^{mit} \eta_{pr}^{mit}. \quad (1)$$

The power component is determined as follows

$$\eta_{fk}^{mit} = 1 - \frac{M_{fk}}{M_k}, \quad (2)$$

where M_k is torque to the in-wheel motor, N · m; M_{fk} is rolling resistance torque, N · m.

Rolling resistance torque is specified by the ratio

$$M_{fk} = R_{zk} f r_0, \quad (3)$$

where R_{zk} is normal road reaction, N; f is rolling resistance coefficient; r_0 is in-wheel motor dynamic radius, m.

The elastic component of the efficiency takes into account losses associated with tyre deformation

$$\eta_{pr}^{mit} = 1 - \frac{M_k}{C_{circ}}\left(1 - \frac{M_{fk}}{M_k}\right), \quad (4)$$

where C_{circ} is equivalent circular tyre stiffness.

Substituting (2 and 4) into (1), we can obtain the full expression for the instantaneous in-wheel motor efficiency

$$\eta_k^{mit} = \left(1 - \frac{M_{fk}}{M_k}\right)\left[1 - \frac{M_k}{C_{circ}}\left(1 - \frac{M_{fk}}{M_k}\right)\right]. \quad (5)$$

Thus, the efficiency is determined as the ratio of the useful torque and losses to rolling and deformation.

In-wheel motor operation mode in the vehicle transmission. The in-wheel motor is integrated directly into the wheel as a self-contained electromechanical drive unit, eliminating the need for intermediate mechanical transmission components. Such a configuration enables independent wheel torque control and provides an effective basis for energy-efficient torque distribution between the left and right sides of the vehicle.

This research is aimed at using Permanent Magnet Synchronous Motors (PMSMs) as a traction electric motor in in-wheel motors, since this type of electric motor has a high specific power factor, low rotor inertia, high torque at low speeds, and high efficiency in a wide speed range, particularly in the weakened field mode. The research focuses on improving the energy efficiency of multi-axle vehicles by rationally selecting the torque distribution coefficient between the left and right sides of the vehicle.

Achieving the set purpose involves determining the dependence for calculating the efficiency of the wheel propulsion system in an all-wheel-drive multi-axle vehicle. For a vehicle with independent control of the left and right sides, the total torque distribution between them should be taken into account.

Introduce the distribution coefficient

$$\beta_M = \frac{M_L}{M_L + M_R}, \quad \beta_M \in [0,1], \quad (6)$$

where M_L, M_R is total torques of the left and right sides, respectively.

The instantaneous efficiency of the wheel propulsion system in an all-wheel-drive vehicle, following the transformations, can be expressed as a quadratic function with respect to β_M

$$\begin{aligned}\eta_{ru}^{mit} = {} & b_1 b_2 - a_1 a_2 - \beta_M^2\left(a_3 + b_3\right) + \\ & + \beta_M\left(a_2 + a_1 a_3 - b_2 + b_1 b_3\right),\end{aligned} \quad (7)$$

where the coefficients are determined as follows

$$a_1 = \frac{M_{fl}}{M_e U_{tr} \eta_{tr}^{mit}}; \quad a_2 = 1 + \frac{M_{fl}}{C_{circ}}; \quad a_3 = \frac{M_e U_{tr} \eta_{tr}^{mit}}{C_{circ}}; \quad (8)$$

$$\begin{aligned} b_1 &= 1 - \frac{M_{fR}}{M_e U_{tr} \eta_{tr}^{mit}}; \quad b_2 = 1 + \frac{M_{fR}}{C_{circ}} - \frac{M_e U_{tr} \eta_{tr}^{mit}}{C_{circ}}; \\ b_3 &= a_3, \end{aligned} \quad (9)$$

where M_{fl}, M_{fR} are rolling resistance torques of the left and right sides; M_e is effective torque of the wheel propulsion system; U_{tr} is transmission gear ratio; η_{tr}^{mit} is instantaneous transmission efficiency.

Formulation of the optimization problem. Since the expression (7) is a quadratic function with respect to β_M, the problem of maximizing efficiency arises

$$\max_{\beta_M \in [0,1]} \eta_{ru}^{mit}\left(\beta_M\right). \quad (10)$$

The first derivative is equal to

$$\frac{d\eta_{ru}^{mit}}{d\beta_M} = \left(a_2 + a_1 a_3 - b_2 + b_1 b_3\right) - 2\left(a_3 + b_3\right)\beta_M. \quad (11)$$

Solution of the equation

$$\frac{d\eta_{ru}^{mit}}{d\beta_M} = 0, \quad (12)$$

is

$$\left(\beta_M\right)_{opt} = \frac{a_2 + a_1 a_3 - b_2 + b_1 b_3}{2(a_3 + b_3)}. \quad (13)$$

Proving the maximum. Second derivative

$$\frac{d^2\eta_{ru}^{mit}}{d\beta_M^2} = -2(a_3 + b_3). \quad (14)$$

Since

$$a_3 + b_3 = 2\frac{M_e U_{tr} \eta_{tr}^{mit}}{C_{circ}} > 0,$$

we have

$$\frac{d^2\eta_{ru}^{mit}}{d\beta_M^2} < 0. \quad (15)$$

Therefore, the function is strictly concave, and the value found corresponds to a single global maximum.

Closed-form analytical expression. (8 and 9) being substituted into (13), a compact result can be obtained

$$\left(\beta_M\right)_{opt} = 0.5\left(1 + \frac{M_{fl} - M_{fR}}{M_e U_{tr} \eta_{tr}^{mit}}\right). \quad (16)$$

At $M_{fl} = M_{fR}$, $\beta_M = 0.5$, which corresponds to a uniform distribution.

Relation to vehicle parameters. Rolling resistance torques are determined as follows

$$M_{fl} = mfr_0 g\frac{b}{L}; \quad M_{fR} = mfr_0 g\frac{a}{L}, \quad (17)$$

where m is vehicle mass; g is free fall acceleration; a, b are distances from mass centre to the sides; L is base between sides.

After substituting

$$\left(\beta_M\right)_{opt} = 0.5\left(1 + \frac{mfr_0 g(b-a)}{LM_e U_{tr} \eta_{tr}^{mit}}\right). \quad (18)$$

Form for steady-state motion. In steady-state motion mode

$$M_e U_{tr} \eta_{tr}^{mit} = \frac{N_e \eta_{tr}^{mit}}{V_a} r_0. \quad (19)$$

Hence

$$\left(\beta_M\right)_{opt} = 0.5\left(1 + \frac{mgf\left(b-a\right)V_a}{LN_e \eta_{tr}^{mit}}\right). \quad (20)$$

Considering aerodynamic drag

$$P_w = \frac{C_x}{2}\rho F V_a^2; \quad N_e \eta_{tr}^{mit} = P_w V_a. \tag{21}$$

Finally obtain

$$(\beta_M)_{opt} = 0.5\left(1 + \frac{2mgf(b-a)}{C_x \rho F L V_a}\right). \tag{22}$$

Since the optimization problem is solved in a closed-form, the law $(\beta_M)_{opt}$ can be applied in real time as a top-level algorithm for forming torque commands on the sides based on the measured/estimated variables (m, V_a, f, C_x, ρ, F) and geometry (a, b, L).

For correct physical implementation, it is necessary to provide restrictions

$$\beta_M \in [0.1],$$

that is, if the calculated value falls outside the interval, clipping is applied without altering the basic logic of the method.

The analysis of equation (22) shows that the optimal torque distribution coefficient is a function of the vehicle parameters and its driving conditions, including its mass, road conditions, geometric parameters and speed mode. As these parameters change during operation, the optimal torque distribution should be formed in an adaptive mode based on current measured or estimated values.

The use of vehicles with distributed electric drive and in-wheel motors enables controlled torque distribution between the wheels or sides without the use of mechanical differentials. This creates the possibility of increasing the energy efficiency of vehicle motion by optimizing the operation modes of electric motors in real time.

Examine the steady-state motion mode of a vehicle with independent control of electric motors of the left and right side in-wheel motors, as well as the acceleration mode. In the steady-state motion mode of a vehicle, the load of in-wheel motors will be

$$\frac{M_e U_{tr} \eta_{tr}^{mit}}{r_0} = P_{mex\Sigma} = \frac{N_e \eta_{tr}^{mit}}{V_a}, \tag{23}$$

where V_a is linear speed of the vehicle, m/s.

After substituting (23) into (22), determine

$$(\beta_M)_{opt} = 0.5\left[1 + \frac{mgf(b-a)V_a}{L N_e \eta_{tr}^{mit}}\right]. \tag{24}$$

Dependence (24) shows that implementing the proposed torque distribution law requires the use of an electronic control unit, which forms control actions on in-wheel motors based on measured or estimated vehicle motion parameters, in particular the normal reactions of the bearing surface, driving speed and the effective power of the drives.

To minimise the number of required sensors, the effective power of the in-wheel motors can be estimated from its functional relationship with the total resistance to vehicle motion. In the following analysis, power losses in the tyres are assumed to be incorporated into the overall transmission losses [26, 27]

$$N_e \eta_{tr}^{mit} = P_w \cdot V_a, \tag{25}$$

where P_w is aerodynamic drag force of a vehicle; C_x is frontal aerodynamic drag coefficient

$$P_w = \frac{C_x}{2} \cdot \rho F V_a^2, \tag{26}$$

where ρ is air density; F is front area of the vehicle. After substituting (26) into (25) and then into (24), have

$$(\beta_M)_{opt} = 0.5\left[1 + \frac{2mgf\left(b-a\right)}{C_x \rho F V_a L}\right]. \tag{27}$$

The proposed analytical torque distribution method is developed for vehicles equipped with Permanent Magnet Synchronous Motors (PMSMs) integrated into direct-drive in-wheel units. Each motor is controlled independently, allowing the traction torque to be distributed between the left and right sides of the vehicle according to the derived analytical law. The upper-level controller determines the required torque allocation, while the motor control system ensures accurate tracking of the commanded operating mode. Such a control architecture enables the implementation of the proposed torque distribution strategy in real time without the use of mechanical differentials or numerical optimization.

Depending on each individual direction and speed of the wheel, the resulting combination of all these forces creates a total force vector in any desired direction, enabling movement in any direction. The speed of each motor is calculated as follows

$$\begin{bmatrix} \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \end{bmatrix} = \frac{1}{R}\begin{bmatrix} 1 & 1 & -(l_1+l_2) \\ -1 & 1 & l_1+l_2 \\ -1 & 1 & -(l_1+l_2) \\ 1 & 1 & -l_1+l_2 \end{bmatrix} \times \begin{bmatrix} v_x(t) \\ v_y(t) \\ \Omega_z(t) \end{bmatrix},$$

where $\omega_i(t)$ is rotor speed of the i^{th} wheel ($i = 1–4$); $v_x(t)$, $v_y(t)$ are linear and angular velocities; Ω_z is vehicle turning speed left/right; l is distances between wheel contact points in the longitudinal and transverse directions; R is wheel radius.

The electric motor model is defined using the required operating and geometric parameters, including angular velocity, rotor diameter, and rotor mass. For simulation purposes, the operating cycle is divided into consecutive time intervals, each characterized by constant speed or acceleration. The force acting during each interval is then calculated as follows

$$F_i = F_{c,i} + F_{dyn,i},$$

where F_i is the resulting force at the i^{th} time interval; $F_{c,i}$ is the static force at the i^{th} time interval, taking transmission losses into account, N; $F_{dyn,i}$ is dynamic force at the i^{th} interval.

The electric motor changes its speed throughout the entire movement; consequently, the force varies according to a dynamic law. Calculate the dynamic force for each i^{th} interval using the formula

$$F_{dyn,i} = m_i a_i.$$

Static force is calculated based on the specific process performed by the mechanism.

$$F_{c,i} = \frac{G_i}{i\eta_{tr}} = \frac{m_i g}{i\eta_{tr}}.$$

As the load diagram does not account for the inertia of the electric motor, the required rated power P_{calc} is determined using a safety factor of $k_{res} = 1.1-1.3$.

$$P_{calc} = k_{res} \cdot P_e.$$

Generator mode and regenerative braking of the in-wheel motor. The in-wheel motor, integrated into the distributed electric drive system of the vehicle, functions in two main modes: traction and generator. In the generator mode, the electric motor converts the mechanical energy of the vehicle motion into electrical energy, enabling regenerative braking.

The transition to generator mode occurs when an electromagnetic torque is created, directed against the direction of wheel rotation. In this case, the electromechanical converter operates as a synchronous generator, and electrical energy is transferred through the power inverter to the traction battery.

From an energy point of view, the recuperation process consists of two successive stages:

- the conversion of kinetic energy of a vehicle into electrical energy in an electric motor;
- accumulation of received electrical energy in chemical form in a battery.

Rational implementation of generator mode involves coordinating the electromagnetic braking torque with the permissible charge modes of the battery system and the limitations of the power electronics. Recuperation efficiency is determined by driving speed, the battery charge level and road conditions.

It is important to note that at low driving speeds, the efficiency of the generator mode decreases due to insufficient level of electromotive force, which limits the amount of current generated. Therefore, the regenerative braking system must function together with the mechanical braking system to ensure the necessary driving safety level.

Thus, the integration of generator mode into the in-wheel motor control system expands the functionality of the vehicle, ensuring the energetic feasibility of the braking process without disturbing the driving stability.

Integration with the control system. While the vehicle is in motion, signals from the accelerator pedal, as well as the left and right side control bodies (joysticks) are integrated into a single control circuit. The speed of the vehicle is determined by the angular velocities of the in-wheel motor, and the control actions are formed according to the specified driving modes.

The output signals of vehicle model, including the accelerator pedal position and the brake signal, are used as the upper-level input actions. The main upper-level output is the formation of an input speed signal for the FOC-controller of the in-wheel motors, both when moving in a straight line and when controlling the left and right sides (Fig. 1).

Research results. The resulting analytical expression (22) determines the optimal total torque distribution coefficient between the left and right sides of the vehicle,

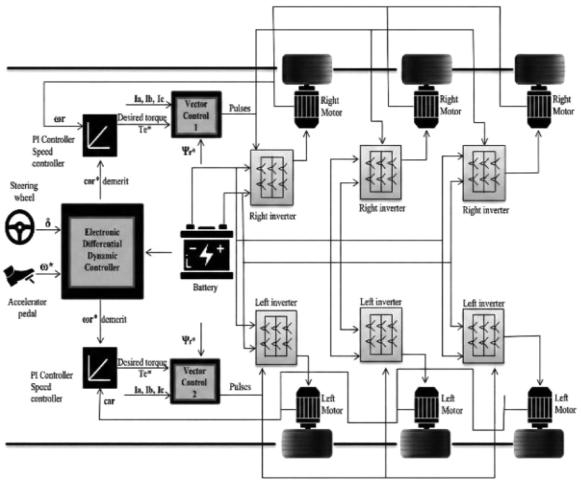

Fig. 1. Left and right side drive control system

taking into account the asymmetry of normal reactions and rolling resistance torques. This means that, in the presence of an uneven load, even with the same requirement for the total traction torque, the optimal control action must be asymmetrical. This approach minimizes rolling losses and elastic-deformation losses in the tyres within the framework of the accepted energy model.

To test the performance of the proposed torque distribution law, simulation modeling was performed in the MATLAB/Simulink environment. The results (Figs. 2–6) indicate the correct functioning of the control system in different driving modes, namely:

- coordinated formation of in-wheel motor torques and speeds in traction and recuperation modes;
- stable practice of various speed commands for the left and right sides during turning modes without loss of controllability;
- no significant static deviation between the command and actual in-wheel motor speed values in the studied modes.

Thus, the analytical determination of the torque distribution coefficient (22) can be used as the upper-level command formation law for the side drives, while the FOC (Field-Oriented Control) circuit can be used as a low-level system for processing specified speed/torque signals.

Examine cases involving a controlled vehicle in different conditions, illustrating the initial results of the Simulink model showing the initial data for in-wheel motor speed and torque, the tracked vehicle speed, and the speed commands.

First case. The first simulation scenario represents straight-line vehicle motion controlled by the accelerator pedal, including acceleration, regenerative braking, and steady-speed operation. The vehicle starts from rest under the maximum available drive torque. As illustrated in Fig. 2, the motor speed increases from zero during acceleration, followed by regenerative braking with negative torque, subsequent re-acceleration, steady-speed motion, and controlled deceleration before returning to a constant-speed regime. Throughout the simulation, the steering angle remains equal to zero ($\delta = 0$). Fig. 2 presents the commanded and actual motor speeds (rpm), Fig. 3 illustrates the phase current (A), and Fig. 4 shows the electromagnetic torque together with the output torque ($N \cdot m$).

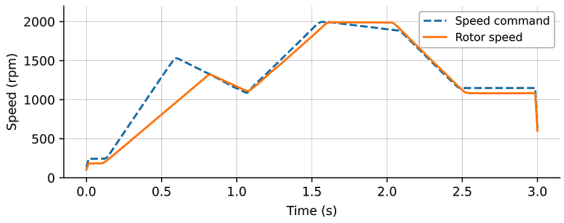

Fig. 2. Speed command and rotor speed

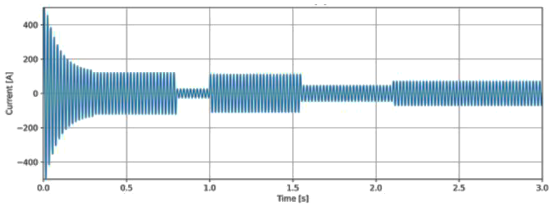

Fig. 3. Linear Current (A)

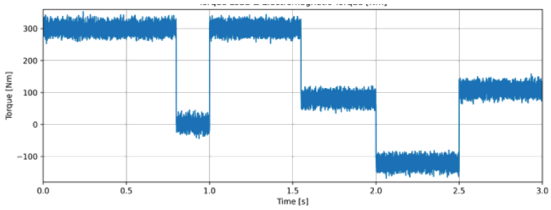

Fig. 4. Load Torque and Electromagnetic Torque (N · m)

Second case. This case shows the initial vehicle reaction, as well as the in-wheel motor commands and speeds as the vehicle moves in a straight line from a standstill, then accelerates at $(T) < 2$ and moves at a constant speed in the range of $2 < (T) < 4.5$, and then applies a steering wheel rotation angle of $\delta = 4.5$ to the left. In this case, the results show the vehicle speed (m/s) and the rotational speeds of the left and right in-wheel motors (rad/s) during steering.

Fig. 5, *a* shows the difference between the speed command signals of the two motors; Figs. 5, *b*, *c* show the difference between the control signal and the rotation speed (rad/s) of the in-wheel motor on the left and right sides, respectively; Fig. 5, *d* shows the vehicle speed.

Third case. In this case, the output reaction of the vehicle speed and in-wheel motor speed, as well as the signals, is shown. When a vehicle is moving straight on a hard sandy road, at a time $(T) > 3$ s, and then turns right at an angle of $\delta = 0$ to $\delta = -15$ from $T = 3.5$ s to $t = 5$ s, the steering wheel is at a constant speed. At $7 < T < 8$, the vehicle accelerates, while the steering wheel has an angle of $\delta = -15$, and finally at $T > 8$, the vehicle rotates at an angle of $\delta = -15$ at a higher constant speed. The results are presented in Fig. 6.

Quantitative generalization of modeling results. To summarize the results obtained from the graphs (Figs. 2–6), a comparative assessment of the main dynamic indicators of the left and right drive systems was performed in three modes: straight-line motion with acceleration/braking, controlled turning on a level road, and turning on an uneven surface. The comparative indicators obtained from the simulation results are summarized in Table.

It should be noted that the maximum speed difference $\Delta\omega$ values given in the table refer specifically to the value of divergence between the speed modes of the left and right sides, and not to their identity.

In turning mode (Figs. 5 and 6), the system forms different angular velocity values for the in-wheel motors: for example, around 26 rad/s for the outer side and around 20 rad/s for the inner side. Thus, the speed difference (6 rad/s) provides the kinematic possibility of turning the vehicle without the use of a mechanical differential.

The sameness of the values given in the table for the left and right sides means that the control system operates symmetrically and there is no distortion between the control channels, while physically the angular velocities of the wheels when turning are different.

It is therefore the controlled asymmetry of the speed in-wheel motor modes that ensures the maneuverability of the vehicle, rather than their equality.

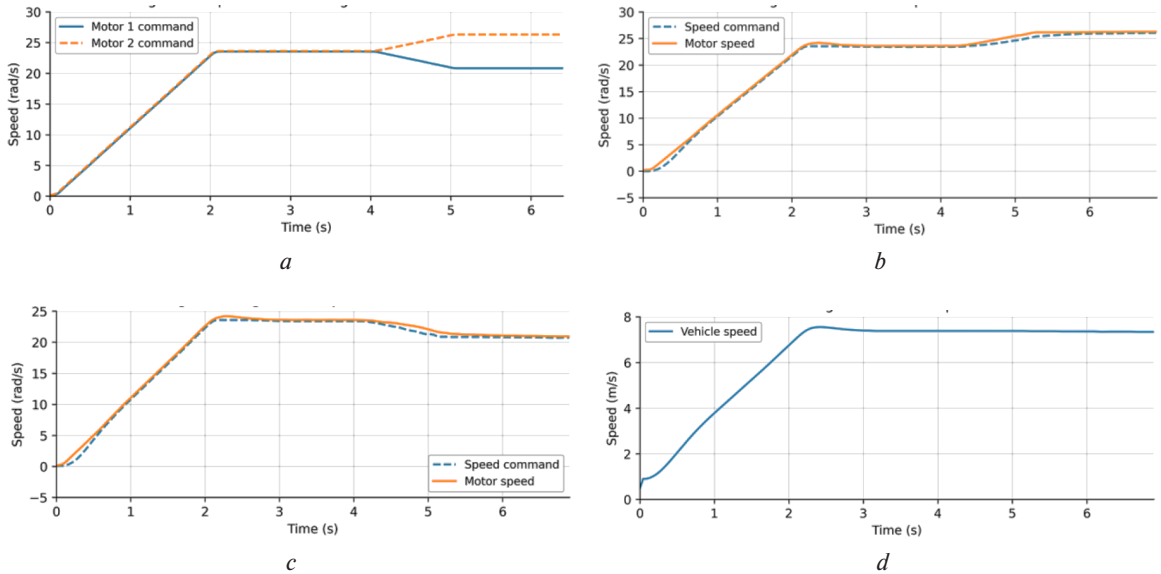

Fig. 5. Simulation results for controlled left-side turning on a level road:

a – the speed command signals of the two motors (rad/s); b – command signal and rotation speed of the left vehicle motor (rad/s); c – command signal and rotation speed of the right vehicle motor (rad/s); d – vehicle speed, (m/s)

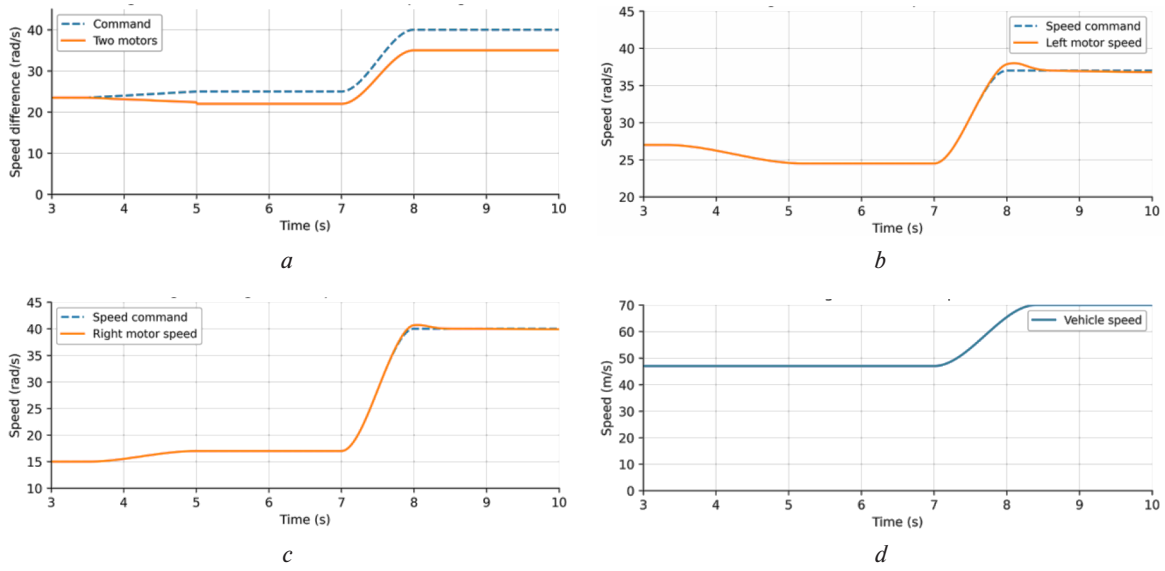

Fig. 6. Simulation results for right-side turning on an uneven road:

a – the difference between the speed command signals of the two motors (rad/s); b – command signal and output speed (rad/s) of the left-side in-wheel motor; c – command signal and output speed (rad/s) of the right-side in-wheel motor; d – vehicle speed, (m/s)

Table

Comparative indicators based on modeling results

Mode	Indicator	Left side	Right side	Note
Straight-line motion	Maximum motor speed ω_{max}, rpm	2,000	2,000	Plateau 2,000 rpm
	Time to reach a steady state mode t_{st}, s	0.3–0.5	0.3–0.5	After changing the speed command
	Maximum electromagnetic torque $M_{em,max}$, N · m	300	300	Upper moment level on the graph
	Minimum (recuperative) torque $M_{em,min}$, N · m	–120	–120	Negative torque in the braking phase
	Maximum current I_{max}, A	450	450	Starting peak, scale ±500 A
Turning on a level road	Maximum speed difference $\Delta\omega_{max}$, rad/s	6	6	Commands diverge 26 vs 20 rad/s
	Static speed deviation e_{st}, rad/s	0.3–0.5	0.3–0.5	"Command-fact" are almost identical
	Speed re-adjustment σ, %	3 %	3 %	A slight re-adjustment after acceleration/turning
	Time to set after the change δ: t_{δ}, s	0.4–0.6	0.4–0.6	After the point of command divergence (4.5 s)
	Vehicle speed V_{st}, m/s	7.3–7.6	–	Plateau after acceleration
Turning on an uneven road	Maximum speed difference $\Delta\omega_{max}$, rad/s	4–5	4–5	Based on the graph of difference/divergence between commands
	Time to set after a maneuver t_{δ}, s	0.5–0.8	0.5–0.8	After a sudden mode change (7–8 s)
	Maximum motor speed ω_{max}, rad/s	40–41	40–41	Plateau around 40 rad/s
	Sign of steadiness	steadily	steadily	No breakdown/run is visible
	Vehicle speed V_{max}, m/s	70	–	Increase from 45 to 70 m/s

Results and discussion. The obtained modeling results confirm the performance of the analytical torque distribution law (22) as part of a hierarchical control system for a vehicle with in-wheel motors. The proposed approach allows for the formation of controlled asymmetry of torques and speeds of the left and right sides depending on driving conditions without disturbing the stability of the drive system operation.

In traction mode, the system provides coordinated formation of electromagnetic torque and in-wheel motor speed characteristics with minimal static deviation between the command and actual values. In turning modes, a controlled difference in the angular velocities of the sides is achieved, which corresponds to the kinematic maneuvering conditions.

The implementation of generator mode within the control structure enables regenerative braking without changing the torque distribution logic. The conversion of kinetic energy into electrical energy occurs within the permissible operation modes of the electromechanical system and power electronics. In this case, the combined operation of the electrical and mechanical braking systems ensures the necessary level of driving safety.

The proposed analytical torque distribution strategy, combined with the lower-level FOC controller, provides

an effective framework for improving vehicle energy efficiency while maintaining a simple control architecture and eliminating the need for numerical optimization.

Conclusions.

1. An analytical model for the instantaneous efficiency of the wheel propulsion system of an all-wheel-drive vehicle with in-wheel motors has been developed, taking into account rolling losses and elastic-deformation losses in the tyres.

2. The problem of distributing the total torque between the left and right sides is reduced to optimizing a quadratic function relative to the distribution coefficient, which made it possible to obtain a closed-form analytical expression for a value that provides a global maximum of the instantaneous efficiency.

3. It has been proven that the optimal distribution coefficient depends on normal reactions, geometric parameters of the vehicle and driving conditions, which necessitates the adaptive formation of the control action.

4. The results of simulation modelling have confirmed the performance of the proposed torque distribution law in traction, turning and recuperative modes without loss of control system stability.

5. The analytical law obtained can be used as a top-level algorithm for forming torque commands in vehicle control systems with independent electric drive on each side without the use of numerical optimization.

Conflicts of interest. The authors declare no financial or personal conflicts of interest that could affect the results and conclusions presented in this research. The research was conducted solely for the purpose of objectively examining scientific data on the control of torque distribution between the sides of a vehicle.

References.

1. Dizqah, A. M., Lenzo, B., Sorniotti, A., Gruber, P., Fallah, S., & De Smet, J. (2016). A Fast and Parametric Torque Distribution Strategy for Four-Wheel-Drive Energy-Efficient Electric Vehicles. *IEEE Transactions on Industrial Electronics*, *63*(7), 4367-4376. https://doi.org/10.1109/tie.2016.2540584

2. De Filippis, G., Lenzo, B., Sorniotti, A., Gruber, P., & De Nijs, W. (2018). Energy-Efficient Torque-Vectoring Control of Electric Vehicles With Multiple Drivetrains. *IEEE Transactions on Vehicular Technology*, *67*(6), 4702-4715. https://doi.org/10.1109/tvt.2018.2808186

3. Koehler, S., Viehl, A., Bringmann, O., & Rosenstiel, W. (2017). Energy-Efficiency Optimization of Torque Vectoring Control for Battery Electric Vehicles. *IEEE Intelligent Transportation Systems Magazine*, *9*(3), 59-74. https://doi.org/10.1109/mits.2017.2709799

4. Fischer, S., Hermán, B., Sysyn, M., Kurhan, D., & Kocsis Szürke, S. (2025). Quantitative analysis and optimization of energy efficiency in electric multiple units. *Facta Universitatis, Series: Mechanical Engineering*, *23*(2), 351-375. https://doi.org/10.22190/FUME241103001F

5. Chatzikomis, C., Zanchetta, M., Gruber, P., Sorniotti, A., Modic, B., Motaln, T., Blagotinsek, L., & Gotovac, G. (2019). An energy-efficient torque-vectoring algorithm for electric vehicles with multiple motors. *Mechanical Systems and Signal Processing*, *128*, 655-673. https://doi.org/10.1016/j.ymssp.2019.03.012

6. Torinsson, J., Jonasson, M., Yang, D., & Jacobson, B. (2020). Energy reduction by power loss minimisation through wheel torque allocation in electric vehicles: a simulation-based approach. *Vehicle System Dynamics, 60*(5), 1488–1511. https://doi.org/10.1080/00423114.2020.1858121

7. Hu, X., Wang, P., Hu, Y., & Chen, H. (2020). A stability-guaranteed and energy-conserving torque distribution strategy for electric vehicles under extreme conditions. *Applied Energy*, *259*, 114162. https://doi.org/10.1016/j.apenergy.2019.114162

8. Novytskyi, O., Taran, I., & Zhanbirov, Z. (2019). Increasing mine train mass by means of improved efficiency of service braking. *E3S Web of Conferences*, *123*, 01034. https://doi.org/10.1051/e3sconf/201912301034

9. Mangia, A., Lenzo, B., & Sabbioni, E. (2021). An integrated torque-vectoring control framework for electric vehicles featuring multiple handling and energy-efficiency modes selectable by the driver. *Meccanica*, *56*(5), 991-1010. https://doi.org/10.1007/s11012-021-01317-3

10. Wu, S., Li, Y., Guan, Y., Liu, T., & Che, C. (2023). Distribution Method of Automotive Torque for Hub Motor Considering Energy Consumption Optimization. *International Journal of Automotive Technology*, *24*(3), 913-928. https://doi.org/10.1007/s12239-023-0075-9

11. Xu, T., Zhao, Y., Deng, H., Guo, S., Li, D., & Lin, F. (2023). Integrated optimal control of distributed in-wheel motor drive electric vehicle in consideration of the stability and economy. *Energy*, *282*, 128990. https://doi.org/10.1016/j.energy.2023.128990

12. Wang, J., Lv, S., Sun, N., Gao, S., Sun, W., & Zhou, Z. (2021). Torque Vectoring Control of RWID Electric Vehicle for Reducing Driving-Wheel Slippage Energy Dissipation in Cornering. *Energies*, *14*(23), 8143. https://doi.org/10.3390/en14238143

13. Liang, Y., Zhao, W., Wu, J., Xu, K., Zhou, X., Luan, Z., & Wang, C. (2025). Energy-efficient driving for distributed electric vehicles considering wheel loss energy: A distributed strategy based on multi-agent architecture. *Applied Energy*, *384*, 125462. https://doi.org/10.1016/j.apenergy.2025.125462

14. Kou, L., Sysyn, M., Liu, J., Fischer, S., Nabochenko, O., & He, W. (2023). Prediction system of rolling contact fatigue on crossing nose based on support vector regression. *Measurement*, *210*, 112579. https://doi.org/10.1016/j.measurement.2023.112579

15. Klein, D., Mandl, P., Plöchl, M., Klinger, F., & Edelmann, J. (2026). Power optimal torque distribution for overactuated electric vehicles: analysis and experimental validation. *Automotive and Engine Technology*, *11*(1). https://doi.org/10.1007/s41104-025-00164-8

16. Hua, M., Chen, G., Zhang, B., & Huang, Y. (2018). A hierarchical energy efficiency optimization control strategy for distributed drive electric vehicles. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, *233*(3), 605-621. https://doi.org/10.1177/0954407017751788

17. Chen, H., Du, P., Wang, Y., Jin, D., & Lian, X. (2020). Dynamic energy-efficient torque allocation algorithm for in-wheel motor-driven vehicle. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, *234*(7), 1815-1825. https://doi.org/10.1177/0954407019899205

18. Taran, I. A., & Klimenko, I. Yu. (2014). Transfer ratio of double-split transmissions in case of planetary gear input. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, (6), 60-66.

19. Szántó, N., Fischer, S., & Monek, G. D. (2025). A Novel Method for Simulation Model Generation of Production Systems Using PLC Sensor and Actuator State Monitoring. *Journal of Sensor and Actuator Networks*, *14*(3), 55. https://doi.org/10.3390/jsan14030055

20. He, S., Fan, X., Wang, Q., Chen, X., & Zhu, S. (2022). Review on Torque Distribution Scheme of Four-Wheel In-Wheel Motor Electric Vehicle. *Machines*, *10*(8), 619. https://doi.org/10.3390/machines10080619

21. Zhao, K., Fan, X., Huang, Z., Yu, X., Wang, L., & Peng, J. (2024). A review of drive torque distribution control for distributed drive electric vehicles. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, *239*(12), 5291-5315. https://doi.org/10.1177/09544070241286016

22. Wang, F., Feng, Y., Tan, F., Liang, J., Wang, D., Jang, S., & Li, Z. (2025). Torque allocation strategy for distributed-drive electric vehicles with synergistic optimization of energy saving and safety. *Energy*, *360*, 3, 100022. https://doi.org/10.1016/j.energ.2025.100022

23. Li, W., Xu, H., Liu, X., Wang, Y., Zhu, Y., Lin, X., Wang, Z., & Zhang, Y. (2024). Regenerative braking control strategy for pure electric vehicles based on fuzzy neural network. *Ain Shams Engineering Journal*, *15*(2), 102430. https://doi.org/10.1016/j.asej.2023.102430

24. Bazhinov, A., Bazhinova, T., Podrigalo, M., Kholodov, M., Haiek, Y., & Sierikova, I. (2022). Dynamics Hybrid Vehicle Driven with Electric Motor Driving Wheels from Batteries. *SAE Technical Paper Series*, *1*. https://doi.org/10.4271/2022-01-0667

25. Gu, J., Ouyang, M., Lu, D., Li, J., & Lu, L. (2013). Energy efficiency optimization of electric vehicle driven by in-wheel motors. *International Journal of Automotive Technology*, *14*(5), 763-772. https://doi.org/10.1007/s12239-013-0084-1

26. Puma-Benavides, D. S., Cevallos-Carvajal, A. S., Masaquiza-Yanzapanta, A. G., Quinga-Morales, M. I., Moreno-Pallares, R. R., Usca-Gomez, H. G., & Murillo, F. A. (2024). Comparative Analysis of Energy Consumption between Electric Vehicles and Combustion Engine Vehicles in High-Altitude Urban Traffic. *World Electric Vehicle Journal*, *15*(8), 355. https://doi.org/10.3390/wevj15080355

27. Monastyrskyi, Y., Taran, I., Kokayev, U., & Sistuk, V. (2025). Assessing the performance efficiency of haul trucks and diesel-trolley trucks when changing their technological states and parameters of traffic routes. *Mining of Mineral Deposits, 19*(2), 47-55. https://doi.org/10.33271/mining19.02.047

Інтелектуальне керування мотор-колесами транспортного засобу з аналітичним визначенням розподілу крутного моменту

О. Бажинов[1], *М. Арпабеков*[*2], *І. Таран*[3], *Т. Бажинова*[1], *Е. Кикла*[1]

1 – Харківський національний автомобільно-дорожній університет, м. Харків, Україна
2 – Євразійський національний університет імені Л. М. Гумільова, м. Астана, Казахстан
3 – Національний технічний університет «Дніпровська політехніка», м. Дніпро, Україна
[*] Автор-кореспондент e-mail: arpabekov_mi@enu.kz

Мета. Розроблення аналітичного методу визначення оптимального коефіцієнта розподілу крутного моменту між лівим і правим бортами повнопривідного автомобіля з мотор-колесами. Запропонований метод спрямований на максимізацію миттєвого коєфіцієнта корисної дії колісного рушія з урахуванням асиметрії навантаження й умов руху.

Методика. У роботі використані аналітичні методи, засновані на застосуванні спрощеної енергетичної моделі колісного рушія, методи оптимізації квадратичних функцій, а також імітаційне моделювання в середовищі MATLAB/Simulink для перевірки працездатності отриманого закону розподілу крутного моменту.

Результати. Отримано аналітичний вираз у замкненій формі для визначення оптимального коєфіцієнта розподілу крутного моменту між лівим і правим бортами автомобіля, використання якого забезпечує максимізацію миттєвого коєфіцієнта корисної дії колісного рушія. Показано, що оптимальний розподіл крутного моменту залежить від нормальних реакцій опорної поверхні, геометричних параметрів автомобіля й умов руху. Результати імітаційного моделювання підтвердили працездатність запропонованого закону керування як у режимі реалізації тягового зусилля, так і під час руху автомобіля по криволінійній траєкторії.

Наукова новизна. Уперше задача розподілу крутного моменту між бортами повнопривідного автомобіля з незалежним приводом мотор-коліс сформульована як задача максимізації миттєвого коєфіцієнта корисної дії колісного рушія, що дало змогу отримати придатний для реалізації в реальному часі закон керування без застосування чисельної оптимізації.

Практична значимість. Запропонований аналітичний закон розподілу крутного моменту може бути інтегрований до верхнього рівня системи керування повнопривідними автомобілями із незалежними електроприводами мотор-коліс. Його застосування дає змогу підвищити енергоефективність руху транспортного засобу без збільшення складності алгоритмів керування й обчислювальних витрат.

Ключові слова: *мотор-колесо, повнопривідний автомобіль, швидкість руху, крутний момент, рекуперація*