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JUSTIFICATION OF COAL MINES CONVEYOR LINE PARAMETERS

Purpose. Justification of conveyor line parameters of a coal mine using initial data on the actual distribution of random freight traffic and taking into account the influence of the speed of transportation of minerals.

Methodology. Conducting a comprehensive study. Searching for a theoretical model of freight traffic distribution:
- description of the distribution of the actual freight traffic by means of various indicative algebraic functions;
- analysis of freight traffic distribution models by various criteria.

Obtaining an equation of friction of flexible bodies, taking into account the influence of the speed of movement of the flexible body:
- solution of the classical problem of sliding of a flexible body on a block by the method proposed by Euler, but taking into account new concepts of friction of bodies and conservation of mechanical energy;
- experimental study of the influence of the conveyor belt speed on the traction capacity of the conveyor.

Findings. A theoretical model of random freight flow distribution for mine conveyor transport has been constructed in the form of an algebraic exponential function with separately defined parameters of the ascending and descending branches. A new revision of the equation of friction of flexible bodies is based on the method proposed by Euler, but with the use of new ideas about friction of bodies and conservation of mechanical energy, which became known after his conclusions, and also taking into account the centrifugal forces of a flexible body. According to the study, the traction force of a belt conveyor coincides with that predicted in accordance with the new edition of the equation of friction of flexible bodies, and the friction force of flexible bodies predicted according to the Euler equation has a deviation of 25 % or more. The effect of the conveyor belt speed on the reduction of the traction capacity of a belt conveyor is significantly weaker (up to 3 times) compared to what is assumed by the Euler equation of friction of flexible bodies.

Originality. For the first time, a theoretical model of random freight flow distribution for mine conveyor transport has been constructed in the form of an exponential algebraic function, growing and falling, the branches of which are described by the normal law of distribution of a random variable with separately defined parameters. A new version of the equation of friction of flexible bodies has been substantiated, overcoming the existing discrepancies between the Euler law of friction of flexible bodies and the general laws of classical mechanics, and also takes into account the influence of centrifugal forces of a flexible body.

Practical value. The initial data of random freight flow, substantiated by the application of the proposed theoretical model of its distribution, and a new version of the equation of friction of flexible bodies have a positive effect on the calculation and selection of rational parameters of the conveyor transport line, contribute to energy savings, and also increase the efficiency and reliability of the mine conveyor-transport system.

Keywords: *mine, conveyor transport line, distribution of cargo flow, friction of flexible body*

Introduction. In response to the global energy crisis, increasing coal production in Ukraine necessitates the critical optimization of coal transportation within mining enterprises. Coal extraction often involves complex processes akin to drilling operations, where substantial material movement is required to access resources [1, 2]. A key component of this system is the conveyor transport network, which, similar to the extensive reach of a drilling borehole [3], spans significant distances to enable the efficient movement of extracted coal.

Literature review. Mine conveyor systems offer distinct advantages, including high throughput capacity, operational reliability, low maintenance costs, and potential for automation. However, analysis of high-performance conveyor systems reveals challenges such as extensive branching and limited throughput of transport lines, reminiscent of the complexities involved in managing material flow in long underground tunnels [4]. The efficiency and reliability of these systems are often compromised due to uneven material flow and suboptimal conveyor parameters, much like how irregular material extraction can disrupt resource mining operations.

Current approaches to calculating conveyor line parameters rely on simplified models that assume symmetric load flow distribution, disregarding the critical influence of coal transportation speed [5]. This omission is analogous to ignoring the dynamics of material movement in extraction processes. Additionally, many Ukrainian coal mines operate in gas-bearing seams, which impose strict safety and operational constraints on equipment such as belt conveyors, especially in high-risk mining environments [6, 7].

These factors frequently lead to the use of non-optimal conveyor parameters, thereby undermining the overall performance and reliability of the transportation system. To address this issue, mines must implement precisely calculated conveyor line parameters that reflect the actual distribution of stochastic material flow and account for coal transportation speed. Such an approach will enhance the performance of conveyor systems, ensuring a more reliable and efficient coal transportation process.

The mine's conveyor transport system relies on a series of belt conveyors to move coal, reflecting the significant material displacement observed during mineral extraction operations [8]. Key parameters of the conveyor transport line include throughput, type, power, length and number of belt conveyors, conveyor belt type, coal transportation speed, and the corresponding belt pre-tensioning force [9].

Modern methods for calculating these parameters use input data on stochastic coal load flow to select the type of belt conveyors and conveyor belts, determine the traction capacity of the line, and calculate the maximum tension and pre-tensioning force of the conveyor belt. These calculations are based on the general Euler friction law for flexible bodies and assume equilibrium in the mechanical system; however, they frequently neglect the impact of coal transportation speed. Furthermore, these methods assume that the stochastic load flow follows a normal distribution, despite empirical data indicating that Euler's friction law does not align with practical observations [10].

Oversimplifications in these calculations further complicate the situation. For instance, belt tension – an essential parameter – is typically calculated solely based on moment equilibrium within the mechanical system [11, 12], ignoring the combined influence of the external torque from the drive motor and the pre-tensioning force. This oversight compromises the accurate distribution of belt tension, violating classical mechanics principles and disrupting the balance of forces and moments within the system.

Unsolved aspects of the problem. Modern coal mines operate under complex production and technological conditions, which demand enhanced efficiency, reliability, and energy efficiency from conveyor transport systems. One of the key factors affecting the performance of such systems is the irregular and stochastic nature of the material flow, which is often inadequately addressed in classical engineering calculations. Traditional models, including Euler's friction equation for flexible bodies, fail to account for this stochastic load behavior and do not fully reflect current understandings of contact mechanics within conveyor systems [13].

The relevance of this study is driven by the need to develop new theoretical approaches that consider the actual operating conditions of mine transport systems. Specifically, the aim is to construct a mathematical model of the actual load flow distribution that incorporates its random nature, and to adapt the friction equation for flexible bodies in accordance with modern concepts of mechanical interaction, including the influence of centrifugal forces and energy conservation.

Despite the results obtained, several issues remain unresolved. In particular, the generalizability of the proposed load distribution model to other types of mines or transported materials requires further investigation. The influence of external factors (such as temperature, humidity, and emergency conditions) on the behavior of the transport system also remains insufficiently explored. Additionally, further experimental validation of the revised friction equation is necessary across various operating modes and equipment types. Moreover, the practical implementation of the proposed models in automated design systems for conveyor lines currently lacks sufficient methodological support.

The core challenge of this research lies in improving calculation and design methods for mine conveyor transport systems based on the consideration of complex stochastic characteristics of the material flow and the integration of advanced understandings of friction in flexible bodies. This opens new opportunities for enhancing the overall efficiency and performance of mine transport systems.

Purpose. The objective of this study is to provide a scientific rationale for the determination of optimal conveyor line parameters for coal transportation under mining conditions. The research is based on the analysis of empirical data reflecting the actual distribution of the random material flow characteristic of real-world operations. Special attention is given to evaluating the impact of mineral transportation speed on the technical and economic performance of the conveyor system – specifically, its productivity, energy efficiency, and reliability. The results of the study aim to identify optimal design and operational parameters of the conveyor line to ensure its stable and efficient operation under variable loading conditions and in compliance with production and technological requirements.

Results and discussion. Consider the operational model of a belt conveyor at *Dnipro University of Technology (Dnipro, Ukraine)* as shown in Fig. 1.

For the proper functioning of any horizontal belt conveyor, it is essential to account for the primary components and forces acting on the belt during motion. These include: P – the pre-tensioning force of the conveyor belt, which ensures adequate contact between the belt and the drive pulley, enabling stable torque transmission without slippage. This force is applied when the system is at rest or during startup to create initial belt tension; N_x – the normal reaction component that arises between the conveyor belt and the drive unit or drive pulley. It acts perpendicular to the contact surface and determines the frictional force required for effective drive operation; v – the belt speed, indicating the direc-

Fig. 1. Operational Model of a Horizontal Belt Conveyor ($\beta = 0$)

tion and rate at which the load is transported along the conveyor; *1, 2, 3, 4* – characteristic points on the conveyor used to describe the position of individual components or belt segments in the diagram. These points may represent locations where forces are applied, changes in belt direction, or other critical positions.

According to modern methodologies, the reaction force on the drive pulley – or the sum of the belt tensions at characteristic points – does not equal the pre-tensioning force at the tensioning station, which violates the mechanical equilibrium of the system [14]. Moreover, during operation, the traction forces around the drive pulley cause a symmetric (inverse) redistribution of belt tension compared to the idle state, reflecting equilibrium conditions. However, the calculated total tension at these points often deviates from observed values [15, 16].

To ensure high efficiency and operational reliability of conveyor systems in mining environments, it is crucial to select optimal conveyor line parameters. These parameters significantly affect the system's stability and service life. Their selection should be based not only on standard design methodologies but also on the actual distribution of the stochastic material flow specific to the conditions of mine transport.

Particular attention should be given to the use of precise mathematical models, such as the friction equation for flexible bodies, which allow for an accurate representation of the interaction between the conveyor belt, the pulleys, and the transported load. This, in turn, enables more precise modeling of belt tension variation along its length, which is critically important for ensuring proper belt – pulley adhesion and preventing slippage. During system design, it is also necessary to maintain mechanical equilibrium across all system elements, including the belt, idlers, pulleys, and other structural components. This requires analyzing the equilibrium of forces acting on each element, considering their interconnections and influence on the system's overall dynamics.

The influence of the load transportation speed – particularly coal – must also be carefully considered, as it directly affects system productivity, belt wear rate, energy consumption, and the overall characteristics of the transport process. Optimization of this parameter must take into account the properties of the material, feed intensity, and the operational environment.

In summary, applying a comprehensive, scientifically justified approach to determining conveyor line parameters – which includes accounting for material flow dynamics, employing accurate physical and mathematical models, and maintaining system mechanical equilibrium – substantially enhances the efficiency, reliability, and durability of the conveyor transport system.

Coal transportation logistics depend on the stochastic nature of the material flow, a critical element of the system. Analysis of the actual distribution and irregularity of this flow enables the informed selection of transport system parameters. Contemporary methods typically select parameters based on the coefficient of variation of the random material flow, assuming it follows a Gaussian (normal) distribution [17]. However, the actual material flow in a conveyor line is often asymmetric, with the mode of the distribution frequently deviating from the expected value predicted by the normal distribution model [18].

For instance, Fig. 2 presents a representative histogram of the actual material flow distribution in a conveyor transport line.

The histogram illustrates a pronounced asymmetry in the distribution of random load flows, with a noticeable deviation from the central grouping. This distribution clearly diverges from the expected value assumed by the standard normal distribution model. The grouping center of the load flow is critically important for determining key parameters such as nonuniformity, probable maximum value, and the coefficient of variation. Relying on inaccurate data regarding load flow and its irregularity during the design and calculation of conveyor transport lines often leads to suboptimal parameter selection, which undermines the efficiency of the transport system.

This discrepancy highlights the inadequacy of the symmetric Gaussian (normal) distribution model commonly used in practice [11]. To address this issue, the research considered alternative theoretical models for the distribution of random load flows in conveyor transport lines. Various distribution laws were evaluated, including the normal distribution, gamma distribution, and a specially constructed indicative algebraic function with distinct parameters for the increasing and decreasing segments of the distribution curve [19].

As a result of the analysis, an optimal theoretical model for load flow distribution was substantiated. This model employs branches of an exponential-algebraic function rooted in the normal distribution law but with independently defined parameters for its rising and falling segments

$$y = a \cdot e^{-\frac{(x-x_0)^2}{2 \cdot \sigma_k^2}}, \qquad (1)$$

where x is the independent variable; a is the maximum value; x_0 is the mathematical expectation (mean); σ_k^2 is the variance of the deviation of the random variable (σ_1^2 – variance for $x \le x_0$; σ_2^2 – variance for $x \ge x_0$).

Since σ_k^2 constitute a unified model, a necessary condition is the consistency of their values at the transition point $x = x_0$. Mathematically, this requires that the equality $\sigma_1^2(x_0) = \sigma_2^2(x_0)$ is satisfied. This condition ensures the continuity of the variance function at x_0, which is a critical prerequisite for the stability and correctness of the model, particularly in the context of statistical analysis and stochastic process modeling.

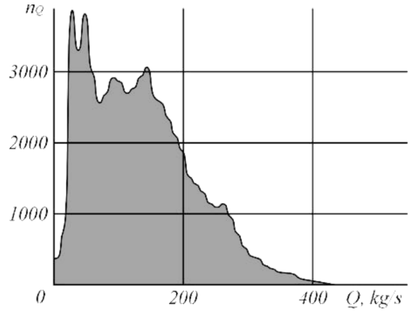

Fig. 2. Histogram of the Load Flow Distribution on the Eastern Main Route of the "Dovzhanska-Kapitalna" Mine (DTEK "Sverdlovsk Anthracite" LLC):

Q – load flow, kg/s; n_Q – the number of instantaneous load flow values

Properties of the proposed theoretical model of cargo flow distribution are as follows: the domain of the model is from $-\infty$ to $+\infty$; the model has a single extremum; the model can be either symmetric or asymmetric relative to the mean (grouping center or the most probable/factual cargo flow value); the rates of increase and decrease of the model's branches are defined by the specified variances of deviation $(\sigma_1^2$ and $\sigma_2^2)$ from the mean x_0.

The development of an effective and adequate theoretical model for the distribution of random cargo flows in conveyor transport lines plays a key role in optimizing the coal and mineral transportation processes at mining enterprises. Under conditions of increasing extraction volumes and stringent requirements for production continuity, the operational efficiency of the transport system directly affects overall mine productivity, economic indicators, and energy efficiency.

This issue becomes particularly relevant in modern high-intensity mining operations, where cargo flow dynamics are characterized by instability, stochasticity, and high variability. The loads on transport infrastructure components – such as belt conveyors, transfer nodes, and storage bins – often approach critical limits, increasing the risks of overloading, emergency situations, and equipment wear. Consequently, there is an objective need for mathematical models that can not only assess the current state of cargo flows but also predict their behavior in the presence of random disturbances.

The model proposed in this study is universal, adaptive, and highly sensitive to input parameters, making it suitable for a wide range of operational scenarios – both in stable conditions and under irregular or intermittent material inflows. Unlike traditional approaches based on averaged load values or assumptions of uniform distributions, this model accounts for the stochastic nature of the process, enabling accurate descriptions of random fluctuations, impulse-like material surges, and temporary accumulations or downtimes.

A distinctive advantage of the model is its ability to adequately reflect both symmetric (normal) and asymmetric, skewed, or multimodal distributions of cargo flow, which is especially important in cases where the system operates under periodic fluctuations or is influenced by external factors – such as changes in mining equipment operation, directional shifts in transport, or uneven feed rates. These scenarios are typical in modern mining operations and necessitate corresponding adaptations in mathematical modeling to avoid inaccurate load assessments and associated operational risks.

The model's high flexibility allows for not only real-time analysis of the current operational state but also the development of predictive control systems and the optimization of internal logistics chains. In particular, it can be integrated into automated control systems (SCADA), utilizing sensor data to adjust conveyor operation parameters in real time – thus significantly improving the overall efficiency of the transport infrastructure.

The proposed model of random cargo flow distribution holds not only significant theoretical value but also opens new practical opportunities for enhancing the reliability, safety, and economic efficiency of transportation processes in the mining industry. Its further development could serve as a foundation for the creation of integrated digital solutions aimed at intelligent management of production flows under conditions of elevated dynamics and uncertainty.

Particular attention should be given to the model's ability to adapt to the complex dynamics of material flow, which is characteristic of variable extraction volumes resulting from natural, technological, or organizational factors. In practical terms, this means that the model can effectively predict and simulate cargo flows in real time – even under conditions of unstable mine productivity, fluctuations in loading speed, or temporary equipment malfunctions. This ensures that theoretical calculations remain aligned with actual operational conditions, thereby significantly enhancing the efficiency of conveyor system management.

Theoretical and empirical studies conducted confirm the superiority of this model over existing alternatives, particularly in terms of accurately capturing the actual distribution of cargo flow at various stages of transportation – from loading to unloading. To validate the reliability of the model, key analytical criteria were applied, including the position of the flow grouping center, load amplitude, rates of increase and decrease in mass flow, the level of fluctuations, and deviations from the mean. The analysis based on these indicators has demonstrated that the model not only reveals general trends but also precisely replicates local anomalies often overlooked by simplified approaches. The results obtained indicate a high degree of correspondence between the model and real operating conditions of the mining transport infrastructure. As such, it provides a reliable analytical foundation for making technically and economically sound decisions regarding the design, modernization, and optimization of conveyor system parameters. Specifically, its implementation can reduce the likelihood of line overloading, minimize energy losses, and prevent uneven mechanical wear caused by unstable material flows. Furthermore, the accuracy of the model is critically important for eliminating systemic inefficiencies that often arise from erroneous or overly generalized assumptions about the behavior of cargo flows. Owing to its advanced mathematical structure and the incorporation of key statistical parameters, the proposed model significantly improves the reliability of production process planning and ensures stable operation of transport lines even under variable mining and geological conditions.

The model itself is formulated as an indicative algebraic function, with its ascending and descending branches defined by the normal distribution law but governed by separately specified parameters. This structure allows for a nuanced representation of cargo flow, accounting for its inherent variability. Parameter calculations are based on probability theory and mathematical statistics, ensuring precision while adhering to the principles of normal distribution for each branch [20]. Such an approach enables the accurate determination of critical transport line parameters, allowing the system to effectively manage the uneven and unpredictable nature of coal movement.

The impact of coal transport speed on the performance of the conveyor line is a key operational factor. This speed directly influences the traction capacity of the belt conveyor, which is modulated by centrifugal forces acting on the conveyor belt. These forces – analogous to mechanical stresses encountered in resource extraction processes – must be carefully managed to maintain operational efficiency [10].

The classical computational scheme for a flexible body sliding over a stationary block provides the fundamental basis for analyzing this dynamic, as illustrated in Fig. 3. By integrating velocity considerations into the model, the system can be optimized to balance throughput capacity and mechanical stability, thereby ensuring the reliable transportation of coal.

Leonhard Euler theoretically derived the equation for friction in flexible bodies based on the prevailing principles of classical mechanics and then-established understanding of natural laws.

$$\ln\frac{S_1}{S_2} = f \cdot \varphi, \tag{2}$$

where f is the coefficient of friction between the flexible body and the pulley.

Take into account the centrifugal forces acting on the flexible body [17]

$$\ln\frac{S_1 - q \cdot v^2}{S_2 - q \cdot v^2} = f \cdot \varphi, \tag{3}$$

where q is the linear mass of the flexible body.

At the same time, the tension in the flexible body along the line of contact with the pulley is described by an exponential function

$$S(\alpha) = S_2 \cdot e^f, \tag{4}$$

where $S(\alpha)$ is the tension in the flexible body at a given cross-section; α is the angle of the cross-section of the flexible body.

According to these equations, the flexible body, under the action of forces applied at its ends, slides along the pulley in the direction of the greater force and exceeds the lesser force by the total amount of frictional force generated between the bodies

$$\begin{gathered} F = S_1 - S_2 = S_2 \cdot (e^f - 1); \\ F = S_1 - S_2 = (S_2 - q \cdot v^2) \cdot (e^f \cdot - 1), \end{gathered} \tag{5}$$

where F is the frictional force between the bodies.

Experimental investigations into the frictional properties of flexible bodies, particularly the influence of their velocity, have consistently revealed inaccuracies in Euler's classical equations for flexible body friction [8, 19]. Despite this, widely used methods for calculating conveyor line parameters continue to rely on Euler's friction equation, which does not account for the velocity of the flexible body – and, consequently, the coal transportation speed. This limitation persists even though global scientific efforts spanning nearly two centuries have aimed at improving Euler's equation, which, despite its shortcomings, remains the most widely accepted model [20, 21]. Ignoring the experimentally validated influence of movement velocity in such calculations is fundamentally flawed and undermines the accuracy of conveyor system design.

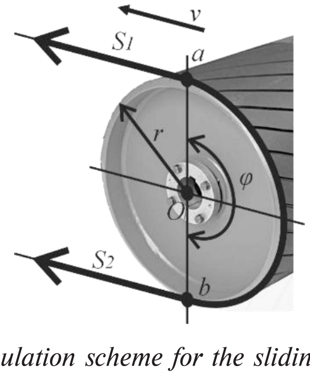

Fig. 3. Calculation scheme for the sliding of a flexible body over a pulley:

S_1, S_2 – forces applied at the ends of the flexible body; r – radius of the pulley; φ – angle of wrap of the flexible body around the pulley; v – direction and velocity of the flexible body's movement

Since 2007, researchers of Dnipro University of Technology have been working on improving Euler's friction model for flexible bodies by developing a new solution. This work has led to a revised equation for flexible body friction that incorporates the centrifugal forces acting on the body [22]. The new model integrates contemporary understandings of friction and mechanical energy conservation – concepts that were introduced after Euler's time. By combining Euler's foundational equations with the contributions of the Dnipro University of Technology team, a new system of differential equations has been proposed. This system provides a more comprehensive basis for modeling the equilibrium of a flexible body and significantly enhances the accuracy of conveyor transport line parameter calculations

$$\begin{aligned} &\{dN = S \cdot da; \leftarrow (dN = (S - q \cdot v^2) \cdot da); \\ &\{dS = dF; \\ &\left\{dF = \frac{F_C}{\varphi} \cdot da + \mathrm{tg}\beta \cdot dN,\right. \end{aligned} \tag{6}$$

where $S(f, a)$ is the tension in the flexible body as a function of the friction coefficient and the angle of contact; F_c is the frictional force between the bodies when the normal reaction is zero (the first parameter of Coulomb friction); $\mathrm{tg}\,\beta$ is the tangent of the slope of the friction force dependence on the normal reaction between the bodies (the second parameter of Coulomb friction).

The revised system of differential equations describing the equilibrium of a flexible body during sliding consists of four distinct equations, extending Euler's classical system, which originally comprised only three equations. The original Eulerian system has been enhanced through the incorporation of Coulomb's law of friction, replacing the outdated Amonton's law. A fourth equation has been introduced, based on the formulation of the law of conservation of mechanical energy in a closed system. This updated law accounts for both the kinetic and potential energy of the flexible body, thus going beyond the earlier formulation of Leibniz's law of mechanical energy conservation. Consequently, this new friction equation for flexible bodies represents a solution to an improved theoretical model of mechanical system equilibrium. The updated equation integrates both the friction coefficient and Coulomb friction parameters, offering a more comprehensive approach to modeling the dynamics of conveyor transport systems

$$\begin{aligned} F &= f \cdot \frac{(S_1 + S_2) \cdot \varphi}{2} = f \cdot N = \\ &= F_c + \mathrm{tg}\beta \cdot N = \left(\frac{F_c}{N} + \mathrm{tg}\beta\right) \cdot N, \end{aligned} \tag{7}$$

where N denotes the normal reaction between the bodies (the normal compressive force between the bodies); $f(N)$ represents the coefficient of friction between the flexible body and the pulley as a function of the normal reaction.

Taking into account the centrifugal forces acting on the flexible body

$$F = f \cdot \frac{(S_1 + S_2 - 2 \cdot q \cdot v^2) \cdot \varphi}{2} = f \cdot N = \\ = F_c + \mathrm{tg}\beta \cdot N = \left(\frac{F_c}{N} + \mathrm{tg}\beta \right) \cdot N. \quad (8)$$

Specifically, the coefficient of friction depends on the normal reaction between the bodies and is expressed as follows

$$f = \left(\frac{F_c}{N} + \mathrm{tg}\beta \right) = f(N). \quad (9)$$

The tension of the flexible body along the contact line with the pulley, according to the new version of the equation, is described by a linear function

$$S(\alpha) = \frac{S_1 - S_2}{\varphi} \cdot \alpha + S_2. \quad (10)$$

The debate regarding the validity and completeness of Euler's equation describing friction in flexible bodies has persisted for nearly two centuries. Its enduring relevance is not only due to the fundamental nature of the issue – which lies at the intersection of classical mechanics, applied mathematics, and engineering – but also because of the ever-increasing need for precise modeling of frictional processes in flexible structures. Such models find applications across a wide range of fields, from biomechanics and materials science to robotics, microsystems engineering, and aerospace engineering.

One of the principal sources of this scientific controversy is that Euler's original formulation has often been criticized for excessive idealization and subjective interpretation of boundary conditions and interacting forces. Specifically, Euler's model relies on simplifying assumptions about the nature of frictional forces – often treating them as purely geometrical reactions without considering the complex mechanics of contact. Furthermore, the absence of direct experimental validation in real-world conditions has significantly impeded efforts to expand or adapt the classical model to more complex systems. While the initial assumptions embedded in Euler's equation were reasonable at the time, they are now considered too restrictive for describing the full scope of physical interactions in flexible bodies under modern scrutiny.

To address these contradictions and improve the current theoretical framework, this study introduces a revised approach supported by both theoretical and analytical evidence, establishing the scientific integrity and practical significance of the updated friction equation for flexible bodies. The new model enables a clear and mathematically rigorous separation of physical influences – such as geometric deformation, contact mechanics, distribution of normal pressure, and spatial variation in the friction coefficient contingent upon local loading. As a result, the revised equation is more universal and adaptable across diverse mechanical interaction scenarios than its classical counterpart.

A notable focus of the revised model is its distinction from both Euler's original equation and other contemporary methods of modeling friction in flexible systems. One key innovation is the incorporation of the interaction between frictional forces and normal reaction – allowing for more accurate descriptions of behavior under conditions of sliding, microscopic adhesion, and transitional friction. In classical approaches, these factors are usually either ignored or approximated indirectly using simplified analytical expressions or assumed coefficients.

Unlike Euler's original methodology, which often introduced friction apriori – based on assumptions concerning contact geometry or problem symmetry – the revised equation is derived in accordance with the principles of contact mechanics. It retains the empirical linear relationship between frictional force and normal reaction, consistent with Amontons-Coulomb laws, while extending beyond Euler's conception to accommodate modern insights in contact mechanics.

The updated equation preserves continuity with classical theories but significantly broadens their applicability. It can account not only for ideal sliding conditions but also for static friction and transitional regimes – factors that are crucial in many engineering and natural processes. A key feature of the model is the introduction of the concept of a partial friction coefficient, defined locally as the ratio of frictional force to normal reaction at each point of contact. This enables compatibility with existing empirical laws for rigid bodies while adapting the model to flexible systems that manifest complex, nonlinear deformation behaviors.

The presented revision of the friction equation for flexible bodies opens new prospects for more precise mathematical modeling, provides a foundation for the development of efficient numerical algorithms, and expands the range of engineering problems that can be solved by accounting for the actual mechanical properties of flexible bodies. The proposed approach not only complements the existing theoretical framework but also creates prerequisites for further interdisciplinary research in the fields of deformable systems mechanics, contact interaction physics, and applied mathematics.

Moreover, the revised equation aligns with the modern law of conservation of mechanical energy in a closed system, wherein the potential energy of the flexible body remains constant under specified test conditions regardless of the friction coefficient [23]. It also satisfies the equilibrium conditions for forces and moments in the mechanical system depicted in Fig. 4 [24, 25]. In particular, for a wrapping angle of the flexible body around the pulley of 180 degrees, the calculated reaction force equals the sum of forces applied at the ends of the body, and the computed friction moment corresponds to the moment of forces acting at the pulley ends. Furthermore, the equation is corroborated by empirical data, in contrast to Euler's model [26, 27].

Unlike the classical Euler equation and many other existing formulations, the proposed new version of the friction equation demonstrates full consistency with contemporary mathematical analysis methods. This not only endows the model with theoretical rigor but also confirms its suitability for the precise solution of flexible body mechanics problems involving frictional forces. Due to this conformity, the new equation provides a mathematically justified resolution to the classical Euler problem under conditions where frictional effects cannot be neglected – an issue previously complicated by the approximate nature of the original model.

To substantiate this claim, it is worth noting that the differential of the function in the new friction equation formulation – that is, the derivative of friction force along the curve length of the flexible body – fully complies with Coulomb's law for an elemental segment in contact with the support surface. This implies that, at every infinitesimally small section of the body, the friction force is proportional to the normal reaction, with the proportionality coefficient remaining constant or variable according to local contact conditions – fully consistent with empirical observations described in classical works by Coulomb.

Additionally, the validity of this approach is confirmed by the new system of equilibrium equations derived within the revised model. This system accounts not only for traditional forces – normal and tangential – but also their distribution along the length of the deformable body, including reactions caused by internal stresses, deformations, contact interactions, and sliding conditions. Such an approach enables the formulation of equilibrium equations in differential form, suitable for accurate analytical or numerical solutions, as well as for the setup of variational problems in the context of deformable media mechanics.

Furthermore, the new model opens the possibility for consistent generalization, particularly to incorporate nonlinear dependencies between friction and normal pressure, as well as to model complex boundary conditions arising during friction regime transitions (e.g., from static to dynamic sliding). Thus, the revised formulation of the equation is not only mathematically consistent with the fundamental laws of mechanics but also better suited for modeling real physical processes occurring in flexible structures under contact forces.

In other words

$$dF(\alpha) \equiv \frac{F_C}{\varphi} \cdot d\alpha + \text{tg}\beta \cdot dN, \tag{11}$$

where $F(\alpha)$ is the friction force between the bodies as a function of the wrap angle; α is the wrap angle.

It is understood that the first parameter of Coulomb friction, F_c, characterizes the molecular component of the friction force between the bodies (dependent on the contact area), while the coefficient of friction f depends on the normal force N.

Thus, for a planar problem

$$\begin{gathered} dF(a) = F'(a) \cdot da = (f \cdot N)' \cdot da = \\ = (F_c(a) + tq\beta \cdot N)' \cdot da = \\ = \left(\left(\frac{F_c}{r \cdot \phi} \cdot r \cdot a \right)' + (\text{tg}\beta \cdot N)' \right) \cdot da = \\ = \frac{F_C}{\varphi} \cdot da + \text{tg}\beta \cdot N' \cdot da = \frac{F_C}{\varphi} \cdot da + \text{tg}\beta \cdot dN. \end{gathered} \tag{12}$$

Thus, the correctness of the solution to Euler's classical problem of a flexible body sliding over a pulley has been confirmed.

Experimental studies have provided definitive evidence supporting ongoing debates regarding the accuracy of Euler's classical friction problem for flexible bodies, complementing previously presented arguments. The research confirms that the traction force resulting from the friction between the conveyor belt and the drum precisely corresponds to the friction force calculated using the revised equation for flexible body friction. In contrast, calculations based on Euler's original equation demonstrate errors of 25 % or more, highlighting its limitations.

Moreover, the influence of conveyor belt speed on the traction capacity of the belt conveyor was identified. This effect is accurately described by the new friction equation, as confirmed by experimental data. Notably, the actual impact of belt speed on traction capacity is significantly lower (on average up to three times less, and in some cases even more) than what is predicted by Euler's original equation for flexible body friction. These findings are consistent with practical data, thereby confirming the advantages of the revised friction equation [28, 29].

Fig. 4 illustrates the diagrams showing the dependence of the maximum traction force of a belt conveyor on belt speed under a given pretension force.

The revised friction equation for flexible bodies is more refined than Euler's formulation and should be widely adopted in education, scientific research, engineering, mechanical design, and industrial applications. This new formulation provides a more accurate representation of friction dynamics, especially in systems such as belt conveyors, where precise calculations are critical. Recommendations were developed for computing conveyor transport line parameters in mining operations, incorporating findings on the impact of mineral transportation speed on conveyor performance and the enforcement of mechanical equilibrium conditions within the system [8, 23]. These recommendations enhance the design and operation of conveyor systems by aligning theoretical models with practical outcomes.

Conclusions. Within the scope of this research, a robust and coherent theoretical model was established to describe the distribution of stochastic cargo flow within a mine's conveyor transport line. The development of this model is based on an indicative algebraic function that adequately characterizes the probabilistic behavior of material movement along the conveyor chain. The function features ascending and descending branches,

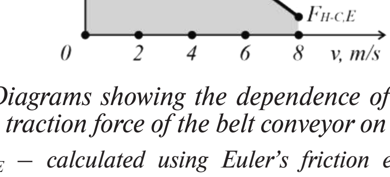

Fig. 4. Diagrams showing the dependence of the maximum traction force of the belt conveyor on belt speed:

$F_{H\text{-}C,E}$ – calculated using Euler's friction equation for flexible bodies; $F_{H\text{-}C}$ – calculated using the revised friction equation for flexible bodies

each modeled according to a normal (Gaussian) distribution law but with distinct parameters for mean and variance. This approach enables the modeling of asymmetry and inhomogeneity in the flow, reflecting the real operating conditions of mine transport systems.

In addition to the cargo flow model, the study produced a revised friction equation for flexible bodies that incorporates a crucial factor: the velocity of translational and deformation movement. This updated equation builds on Euler's classical methodology, supplemented by modern understanding of friction, as well as principles of mechanical energy conservation in non-ideal systems. The friction model employed in the equation is consistent with empirical laws such as Coulomb's and Amonton's, alongside more recent developments in tribology, allowing for a more accurate depiction of flexible-body behavior under complex contact conditions.

Unlike the classical Euler equation, which includes simplifications that can lead to discrepancies between theoretical predictions and observed phenomena, the proposed revision resolves these inconsistencies by ensuring compatibility with classical mechanics and conservation laws. Notably, the model enables accurate representation of both dynamic and static interaction modes between the flexible body, the support surface, or the transported object.

The integrated application of the new cargo flow distribution model together with the revised friction equation enabled more precise and reliable calculations of key parameters in conveyor transport lines – such as belt tension force, load factor, maximum throughput capacity, and loading dynamics along the route. The adoption of these models resulted in a significant improvement in the overall efficiency, stability, and operational reliability of the mine's transportation system, offering considerable practical value for boosting mining enterprise productivity and reducing energy consumption.

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Обґрунтування параметрів конвеєрної лінії вугільних шахт

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Мета. Обґрунтування параметрів конвеєрної лінії вугільної шахти із використанням вихідних даних фактичного розподілу випадкового вантажопотоку та з урахуванням впливу швидкості транспортування корисних копалин.

Методика. Проведення комплексного дослідження. Пошук теоретичної моделі розподілу вантажопотоків:

- опис розподілу фактичного вантажопотоку за допомогою різних індикативних алгебраїчних функцій;
- аналіз моделей розподілу вантажопотоків за різними критеріями.

Отримання рівняння тертя гнучких тіл, що враховує вплив швидкості руху гнучкого тіла:

- розв'язання класичної задачі ковзання гнучкого тіла по блоку методом, запропонованим Ейлером, проте з урахуванням нових концепцій тертя тіл і збереження механічної енергії;
- експериментальне дослідження впливу швидкості конвеєрної стрічки на тягову здатність конвеєра.

Результати. Побудована теоретична модель розподілу випадкового вантажного потоку для конвеєрного транспорту шахти у вигляді алгебраїчної експоненціальної функції з окремо визначеними параметрами зростаючої й спадаючої гілок. Новий перегляд рівняння тертя гнучких тіл обґрунтовано методом, запропонованим Ейлером, проте із застосуванням нових ідей про тертя тіл і збереження механічної енергії, що стали відомими після його висновків, а також з урахуванням відцентрових сил гнучкого тіла. Згідно із дослідженням, сила тяги стрічкового конвеєра збігається із прогнозованою відповідно до нової редакції рівняння тертя гнучких тіл, а прогнозована, згідно із рівнянням Ейлера, сила тертя гнучких тіл має відхилення від 25 % і більше. Вплив швидкості конвеєрної стрічки на зменшення тягової здатності стрічкового конвеєра значно слабший (до 3 разів) порівняно із тим, що передбачає рівняння Ейлера щодо тертя гнучких тіл.

Наукова новизна. Уперше побудована теоретична модель розподілу випадкового вантажопотоку для конвеєрного транспорту шахти у вигляді алгебраїчної експоненціальної функції, зростаючої й спадаючої, гілки якої описуються нормальним законом розподілу випадкової величини з окремо визначеними параметрами. Обґрунтовано новий варіант рівняння тертя гнучких тіл, що долає існуючі невідповідності між законом тертя Ейлера щодо гнучких тіл і загальними законами класичної механіки, а також ураховує вплив відцентрових сил гнучкого тіла.

Практична значимість. Вихідні дані випадкового вантажопотоку, обґрунтовані застосуванням запропонованої теоретичної моделі його розподілу, та нова версія рівняння тертя гнучких тіл позитивно впливають на розрахунок і вибір раціональних параметрів конвеєрної транспортної лінії, сприяють економії електроенергії, а також підвищують ефективність і надійність роботи конвеєрно-транспортної системи шахти.

Ключові слова: *шахта, конвеєрна транспортна лінія, розподіл вантажопотоку, тертя гнучкого тіла*

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