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HARDWARE AND SOFTWARE FOR ENGINE FUEL SUPPLY CONTROL UNDER INCOMPLETE INFORMATION CONDITIONS

Purpose. Improving the performance of hardware and software tools for setting the phases of fuel and air supply to engine cylinders based on processing the signal of uneven crankshaft rotation.

Methodology. The research methodology is based on the theory of automatic control and numerical methods.

Findings. The results of the study are: construction of a deterministic mathematical model of an internal combustion engine; description of the mass movements of the model by a system of linear differential equations, the parameters of which are normalized using similarity theory methods; obtaining transfer functions of mechanical channels using the Mathcad software environment and the method of determinants. The coincidence of the obtained transfer functions is also established and for the first time it is proposed to represent the torque diagram of the power unit by a mechanical system with one degree of freedom; the frequency characteristics of the torque transmission channel are studied in the Matlab software environment. The effectiveness of the developed application software is established.

Originality. The scientific novelty consists in the development of the architecture of hardware and software tools for setting the phases of fuel and air supply based on the principle of feedback control based on the state of the frequency-modulated crankshaft speed signal. An information technology for processing an array of experimental data when isolating a fluctuation signal has been developed. An algorithmic support for monitoring cylinder capacities has been built by solving an overdetermined system of incompatible algebraic equations when setting a vector of cylinder weight coefficients. To find its solution, the least squares method and the Seidel iterative method with zero partial derivatives in the components of the generalized functional were used. As a result of mathematical transformations, a symmetric and positive definite Gram matrix was obtained. An uncertainty limit for the solution of the system of algebraic equations was established, upon reaching which the computational process stops.

Practical value. The practical significance lies in the development of algorithmic support for monitoring cylinder capacities with a frequency representation of the fluctuation signal using the transfer functions of mechanical channels and approximated torques. Monitoring cylinder capacities is focused on setting the parameters of a curve that passes through all points of measurement information and smoothest possible emissions due to their uncertainty. The Laplace transform under zero initial conditions was used as a mathematical apparatus for establishing information connections between cylinder torques and the fluctuation signal of the first mass of the crankshaft. A computer simulation scheme of the signal of the uneven rotation of the first mass was built in the Mathcad software environment and its parameters were identified using the method of adjusting the length of information connections.

Keywords: *internal combustion engine, hardware, algorithmic support, mathematical modeling, fluctuation signal*

Introduction. The standard setting of the fuel and air supply phases to the cylinders of internal combustion engines (ICE) affects their economic performance. To reduce the uncertainty interval around the optimal phase values, hardware developers use the method of software specification of individual fuel supply angles. A well-known method for determining the identity of the operating cycles of ICE is focused on measuring the cylinder pressure during one shaft revolution using a primary transducer of the 8QP505CS type. The analog-to-digital converter E14-140 generates the corresponding code, and the "Power Graph" software registers the indicator diagram of the individual cylinder. Comparison of several diagrams allows establishing the identity of the working cycles. In its absence, at the stage of engine tuning, fuel supply phases are adjusted. Engine modification for installation of pressure sensors and the presence of several cylinders limit the application of this method. The authors propose to use the crankshaft rotation irregularity signal for indirect measurements of the cylinder capacities of the 3RZ-FE ICE. By processing this signal, the computer system (CS) establishes the power distribution of the ICE and generates corresponding changes to the fuel supply phase settings. This will provide fuel savings of up to 5 % [1] and reduce maintenance and repair costs. Improving the performance of monitoring cylinder capacities of ICE by hardware and software means of processing indirect measurement data determines the relevance of the selected scientific and applied problem.

Problem statement and literature review. The issue of monitoring the cylinder capacities of ICE under conditions of incomplete information is given attention in the technical literature. Analysis of the force relationships of the crank mechanism showed the influence of the work-

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ing process on the deviation of the indicator pressure [2]. In work [1], the operating states of the D50 engine were simulated and their coding was given. At the same time, the generalizing functions of the experimental data were obtained by machine learning methods [3, 4].

To assess the technical condition of the engine based on the signal parameters of the uneven angular velocity of the crankshaft, work [5, 6] is devoted. In work [7], need is established to create new mathematical models to improve known diagnostic methods and use artificial intelligence systems to increase reliability.

In work [8], the use of a high-pass filter with a finite impulse response reduced the effect of interference on the signal of uneven rotation. Also, the processing of the measurement information signal was carried out in the Matlab environment. In work [9], the influence of indicators of uneven rotation of the crankshaft and the indicator torque on the change in power of four-stroke ICE was investigated.

The parameters of the movements of vibrating devices on elastic supports with an eccentric rotor, unbalances and an asynchronous motor can be established by dynamic analysis [10]. In works [11, 12], the equations of motion of the mechanism model in stable operating modes were obtained. The use of linearized differential equations [13], which approximately describe transient processes in electric motors, leads to a significant overestimation of the design torque. In work [14], a complex mathematical model of the dynamic processes of a vibrating device on elastic supports with an eccentric unbalance rotor was proposed. The working chamber of an asynchronous electric motor performs a planar parallel motion. It was found that by choosing the eccentricity, it was possible to reduce the rotor vibrations. A geometric interpretation of the unevenness of the shaft rotation during the start of the electric motor was also obtained. In work [15], a mathematical model of the unevenness of the rotation of the robot control device was developed. The model allows us to study the start-up process, steady states and spatial movements of any points of the robot. Using the ideas of the sensitivity theory, an information technology for synthesizing the parameters of the control device according to its own forms of oscillations was developed.

In work [16], the authors built an algorithmic support for monitoring the identity of cylinder capacities of ICE based on solving an overdetermined system of incompatible algebraic equations. The column vector of its right part constitutes the frequency representation of the signal of uneven rotation of the crankshaft. Known numerical methods for solving overdetermined systems of algebraic equations use matrices of a certain structure. In particular, the method of radial basis functions and some of its modifications when solving similar problems operate with completely filled matrices, which significantly limits the performance of monitoring cylinder capacities.

The disadvantage of known methods for processing frequency-modulated rotational speed signals for controlling fuel supply to the cylinders of ICE is the unsatisfactory dispersion of measurements and the performance of hardware for setting cylinder capacities, as well as the lack of algorithmic and application software for processing the input signal.

Unsolved aspects of the problem. Unresolved issues of the general problem include improving the method of non-disassembly diagnostics of ICE using a frequency-modulated crankshaft speed signal as input information, developing information technology for monitoring cylinder capacities under conditions of incomplete information, and improving methods for adjusting fuel and air supply phases in order to improve their economic performance.

The purpose of the article is to increase the performance of hardware and software fuel supply control based on processing a frequency-modulated shaft speed signal.

To achieve the goal, it is necessary to solve the following tasks:

- to develop the architecture of fuel supply control hardware;
- to build a deterministic mathematical model of the 3RZ-FE engine torque diagram and identify its parameters;
- to establish information connections between cylinder torques and the measurement information signal;
- to investigate transfer functions and adjust the frequency characteristics of hardware using neural network technology methods;
- to build a scheme for computer modeling of the measurement information signal;
- to develop information technology for monitoring cylinder capacities.

Description of the research methodology. The research methodology is based on the use of systems analysis methods, the theory of automatic control under conditions of incomplete information, the theory of similarity, and numerical methods for solving incompatible systems.

Presentation of the main material and obtained scientific results. ***Hardware architecture development.*** The authors used feedback on the state of the crankshaft unevenness signal for software control of the fuel supply of the 3RZ-FE ICE. The signal from the top dead center (TDC) sensor of the first cylinder synchronizes the operation of the hardware and software with the phase of shaft rotation. Piezoelectrically controlled injectors are used as the actuators for setting the fuel supply phases. The hardware and software architecture (Fig. 1) has the following components: PCUR – primary converter of the signal of uneven rotation, STDC – sensor of the TDC, MTFS – measuring transducer of fluctuation signal, microcomputer and A1–A4 – actuators.

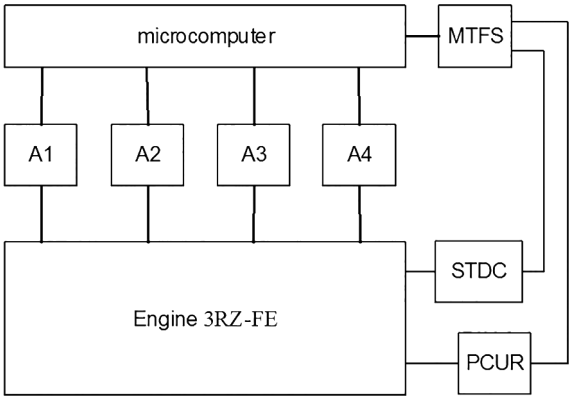

Fig. 1. CS of software control of fuel supply

Processing a set of research data involves the following computational actions:

- we set the mathematical expectation of the periods of the input signal of the PCUR block;
- the difference between the mathematical expectation and the current value of the period is the fluctuation signal, which we present in the form of an array of experimental data;
- we perform averaging of experimental data within one revolution of the crankshaft;
- we choose a method for solving the monitoring problem and present the data in the appropriate form.

Mathematical modeling of the torque diagram of the power unit. A mechanical system with four degrees of freedom was chosen as a mathematical model of the kinematic scheme of a four-stroke ICE 3RZ-FE in-line type. The mechanical parameters of the model are as follows: friction $\beta = 1.44$ Nms; malleability of relations between the masses $e = 1.32 \cdot 10^{-7}$ $(\text{Nm})^{-1}$; the moments of inertia of the cylinder masses are $J_1 = 0.246$ Nm^2. As a result of analyzing the structural diagram of the ICE, it was proposed to install the primary converter near the first cylinder. The maximum power of the 3RZ-FE ICE is 80 kW. Let us move on to developing a deterministic mathematical model of the torque diagram of the 3RZ-FE ICE.

A linear system of second-order differential equations describes the dynamics of rotation of the cylindrical masses of the proposed mechanical system [17]

$$J_i\varphi_i''(t)+\beta\varphi_i'(t)-e^{-1}\left[\varphi_{i+1}(t)-\varphi_i(t)\right]+ \\ +e^{-1}\left[\varphi_i(t)-\varphi_{i-1}(t)\right]=M_i(t), \quad (1)$$

where $i = 1, 2, ..., 4$; $\varphi_i(t)$ is an angle of oscillation of the corresponding mass; $M_i(t)$ – torque.

In order to generalize the results of the research using the theorems of similarity theory, we will reduce the system of differential equations (1) to a dimensionless form. For this purpose, the following basic quantities were chosen: J_b, β_b, e_b, M_b, φ_b, t_b. Let us introduce the following notation [16]

$$J^*=J/J_b;\quad \beta^*=\beta/\beta_b;\quad e^*=e/e_b; \\ M^*=M/M_b;\quad \varphi^*=\varphi/\varphi_b;\quad t^*=t/t_b.$$

In these expressions, the quantities with the superscript are dimensionless. We multiply each parameter of the system of differential equations (1) by the ratio of the basic quantities, since this does not change its output signal. After multiplication and taking into account the introduced notations, we obtain the following form of the system

$$\frac{J_i^*J_b\varphi_b}{M_bt_b^2}\varphi_i''+\frac{\beta^*\beta_b\varphi_b}{M_bt_b}\varphi_i'-\frac{\varphi_b}{e^*e_bM_b}(\varphi_{i+1}-\varphi_i)+ \\ +\frac{\varphi_b}{e^*e_bM_b}(\varphi_i-\varphi_{i-1})=M_i^*. \quad (2)$$

Simplification of the system of equations (2) is ensured by fulfilling the conditions

$$\frac{J_b\varphi_b}{M_bt_b^2}=1;\quad \frac{\beta_b\varphi_b}{M_bt_b}=1;\quad \frac{\varphi_b}{e_bM_b}=1. \quad (3)$$

By choosing the following values of the basic quantities $J_b = 0.246$, $\beta_b = 1{,}44$, $e_b = 1.32 \cdot 10^{-7}$, $\varphi_b = 1$ deg , based on conditions (3), we establish the numerical values of the basic coefficients of torque and time $M_b = 7.58 \cdot 10^6$, $t_b = 1.8 \cdot 10^{-4}$. Thus, by appropriate selection of the basic values of the parameters of the system of differential equations (2), we have a complete coincidence of the dimensional and dimensionless forms of its writing. As a mathematical apparatus for solving the system of equations, the authors used the Laplace transform [18]. As a result of transformations under zero initial conditions, the system of differential equations (2) takes on the following form

$$(J_ip^2+p+2)\varphi_i(p)-\varphi_{i+1}(p)-\varphi_{i-1}(p)=M_i(p). \quad (4)$$

In order to simplify further transformations of he system of algebraic equations (4), we introduce the following notation

$$a=(p^2+\mathrm{p}+1)^{-1};\quad b=(p^2+p+2)^{-1}. \quad (5)$$

Taking into account the notations, the system of algebraic equations (4) takes the following form

$$\begin{cases}\varphi_1-a\varphi_2=aM_1\\ \varphi_2-b\varphi_3-b\varphi_1=bM_2\\ \varphi_3-b\varphi_4-b\varphi_2=bM_3\\ \varphi_4-a\varphi_3=aM_4\end{cases}. \quad (6)$$

When establishing information relations between the fluctuation signal and torques, the method of determinants was used. Using the Mathcad software environment, the authors established the determinants of the system of algebraic equations (6) in the form shown in Fig. 2.

After establishing the expressions of the determinants, the system of algebraic equations (6) takes the following form

$$\varphi_1(p)=\sum_{j=1}^{4}\frac{\Delta_j}{\Delta}M_j(p)=\sum_{j=1}^{4}W_jM_j(p), \quad (7)$$

$$\left|\begin{pmatrix}1&-a&0&0\\-b&1&-b&0\\0&-b&1&-b\\0&0&-a&1\end{pmatrix}\right|\rightarrow\frac{p^6+3\cdot p^5+7\cdot p^4+9\cdot p^3+6\cdot p^2+2\cdot p}{p^6+3\cdot p^5+7\cdot p^4+9\cdot p^3+9\cdot p^2+5\cdot p+2}$$

$$\left|\begin{pmatrix}a&-a&0&0\\b&1&-b&0\\b&-b&1&-b\\a&0&-a&1\end{pmatrix}\right|\rightarrow\frac{p^4+2\cdot p^3+5\cdot p^2+4\cdot p+2}{p^6+3\cdot p^5+7\cdot p^4+9\cdot p^3+9\cdot p^2+5\cdot p+2}$$

$$\left|\begin{pmatrix}1&a&0&0\\-b&b&-b&0\\0&b&1&-b\\0&a&-a&1\end{pmatrix}\right|\rightarrow\frac{p^4+2\cdot p^3+5\cdot p^2+4\cdot p+2}{p^6+3\cdot p^5+7\cdot p^4+9\cdot p^3+9\cdot p^2+5\cdot p+2}$$

$$\left|\begin{pmatrix}1&-a&a&0\\-b&1&b&0\\0&-b&b&-b\\0&0&a&1\end{pmatrix}\right|\rightarrow\frac{p^4+2\cdot p^3+5\cdot p^2+4\cdot p+2}{p^6+3\cdot p^5+7\cdot p^4+9\cdot p^3+9\cdot p^2+5\cdot p+2}$$

$$\left|\begin{pmatrix}1&-a&0&a\\-b&1&-b&b\\0&-b&1&b\\0&0&-a&a\end{pmatrix}\right|\rightarrow\frac{p^4+2\cdot p^3+5\cdot p^2+4\cdot p+2}{p^6+3\cdot p^5+7\cdot p^4+9\cdot p^3+9\cdot p^2+5\cdot p+2}$$

Fig. 2. Scripts of determinants of the system of algebraic equations (6)

where $\varphi_1(p)$ is Laplace transform of the measurement information signal; W_j – transfer functions.

The results of analyzing the determinant expressions allow us to draw the following conclusion: the transfer functions of the mechanical channels are the same, respectively, the kinematic scheme of the four-stroke ICE 3RZ-FE of the in-line type is represented by a mechanical system with one degree of freedom. This statement significantly simplifies further transformations. Based on the analysis of Fig. 2, the transfer function expression takes the following form

$$W(p)=\frac{p^4+2p^3+5p^2+4p+2}{p(p^5+3p^4+7p^3+9p^2+6p+2)}.$$

The system of differential equations (2) assumes that the signal of measurement information is the oscillation of the twist angles of the first mass of the crankshaft of the four-stroke ICE 3RZ-FE. The appearance of the obtained transfer function of the cylinder torques allows us to establish the presence of a circuit with integration; accordingly, in this case the signal of the measuring information is the fluctuations of the rotation speed of the first mass. The information connection between these two signals establishes the following equation

$$\omega(t)=\frac{d\varphi(t)}{dt}. \quad (8)$$

Under such conditions, the expression of the transfer function of the mechanical channel has the form

$$W(p)=\frac{p^4+2p^3+5p^2+4p+2}{p^5+3p^4+7p^3+9p^2+6p+2}. \quad (9)$$

For further research of torque transmission paths, the Matlab software environment was used [19]. The information technology of the transfer function research consists of the following computational actions:

- the command line for specifying expression (9) looks like $W = tf([1\ 2\ 5\ 4\ 2], [1\ 3\ 7\ 9\ 6\ 2])$;
- the logarithmic amplitude-frequency characteristics (LAFC) of the torque transmission path are obtained using the bode(W) command. The calculation results are shown in Fig. 3;
- the Nyquist hodograph (Fig. 4) is constructed using the Nyquist (W) command;
- the zero (W) and pole (W) commands allow you to set special points of the transfer function, which represents them in the form of a series connection of elementary circuits (the calculation results are given in Table 1).

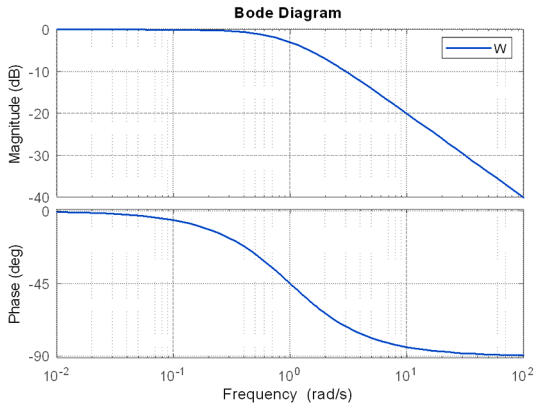

Fig. 3. LAFC of torque transmission paths

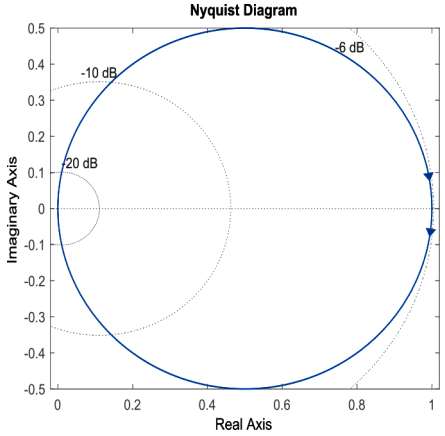

Fig. 4. Nyquist hodograph of the torque transfer function

Table 1

Special points of transfer functions

Zero (W)	Pole (W)
–0.5000 + 1.7788i	–0.5000 + 1.7788i
–0.5000 – 1.7788i	–0.5000 – 1.7788i
–0.5000 + 0.5795i	–1.0000 + 0.0000i
–0.5000 – 0.5795i	–0.5000 + 0.5795i
–	–0.5000 – 0.5795i

Canceling the roots of the numerator and denominator that have fallen off, discarding unstable and second-order roots of smallness simplifies the expressions of the transfer functions. Based on the obtained roots, the transfer functions take the following form

$$W(p)=\frac{1}{p+1}. \quad (10)$$

For further research, we will use the frequency representation of the input and measurement information signals. The transition to the frequency domain gives the transfer function of the mechanical channel the following form

$$W(j\omega)=\frac{1}{j\omega+1}=\frac{1}{\sqrt{1+\omega^2}}e^{-j\omega^{-1}}. \quad (11)$$

After establishing the transfer function, we will proceed to the approximation of the torques of the 3RZ-FE ICE.

Approximation of input signals. The indicator and compression diagrams were used as the initial information. Table 2 presents the dimensionless harmonic components of the torque from the action of gas forces. The pressure measurement of the first cylinder was carried out by discretization by the engine rotation angle with a step of six degrees, respectively, the iteration step of the experimental data is 0.0333π.

Accordingly, the torque from the action of gas forces of the 3RZ-FE ICE is mathematically represented by the following expression

$$M_1(t)=\sum_{n=1}^{6}(A_n\sin n\Omega t+B_n\cos n\Omega t). \quad (12)$$

Table 2

Discrete frequency spectrum of torque

A_n	B_n
$4.33 \cdot 10^{-5}$	$1.46 \cdot 10^{-5}$
$3.88 \cdot 10^{-5}$	$-0.11 \cdot 10^{-5}$
$2.52 \cdot 10^{-5}$	$-0.3 \cdot 10^{-5}$
$1.66 \cdot 10^{-5}$	$-0.37 \cdot 10^{-5}$
$1.09 \cdot 10^{-5}$	$-0.48 \cdot 10^{-5}$
$0.73 \cdot 10^{-5}$	$-0.39 \cdot 10^{-5}$

The torques of the other cylinders were obtained based on the known order of their operation, taking into account the displacement in the angle of rotation of the crankshaft in this form

$$M_2(t) = M_1(t)e^{-j0.5\pi}; \quad M_3(t) = M_1(t)e^{-j1.5\pi}; \quad M_4(t) = M_1(t)e^{-j\pi}. \tag{13}$$

Let us introduce the concept of differential torque, which ensures the rotation of the crankshaft of the ICE 3RZ-FE with a given angular velocity. This torque is determined using the following expression

$$M_{i,p}(t) = M_{i,1}(t) - M_{i,2}(t), \tag{14}$$

where $M_{i,1}(t)$, $M_{i,2}(t)$ are torques obtained when processing experimental data, respectively, indicator and compression diagrams.

The concept of cylinder weight coefficients is used $D_i = 0, ..., 1$ to assess the identity of their work. Deviations from the standard setting ($D_i \neq 1$) require adjustment of the fuel supply phase to the corresponding cylinder. This allowed us to perform algorithmization of the problem of monitoring the cylinder capacities of the 3RZ-FE ICE. Taking this into account, expression (14) takes the following form

$$D_i M_{1,p}(t) = M_{i,1}(t) - M_{i,2}(t). \tag{15}$$

The Fourier transform was used as the mathematical tool for further research. Under these conditions, expression (7) takes the following form

$$\varphi_1(j\omega) = \sum_{i=1}^{4} W_i D_i M_{i,p}(j\omega) + \sum_{i=1}^{4} W_i M_{i,2}(j\omega). \tag{16}$$

Let us move on to building a scheme for computer simulation of the fluctuation signal. For this, it is convenient to use the Mathcad software environment.

Development of a computer modeling scheme. It is compiled taking into account the capabilities of the Mathcad environment based on expressions (11–13). The output signal is the fluctuations in the rotation speed of the first mass. Torque models take into account normalization by amplitude. For this purpose, amplifiers are used in the circuit, the conversion coefficients of the input signal of which are equal to $M_b^{-1} = 1.32 \cdot 10^{-7}$. The learning model method [20] was used to identify the parameters of a mechanical system with one degree of freedom. For comparison, the fluctuation signal obtained by processing experimental data was used. When setting up the computer simulation scheme (Fig. 5), the weighting coefficients of the cylinders were used μ_i, which are changed by amplifiers.

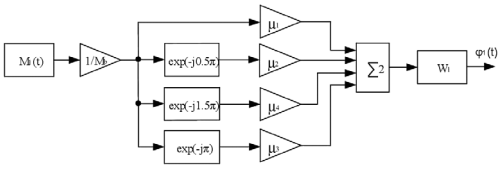

Fig. 5. Computer simulation scheme

Selected for comparison $\mu_1 = 1$, $\mu_2 = 1$, $\mu_3 = 1$, $\mu_4 = 1$ and $\mu_1 = 1.5$, $\mu_2 = 1.0$, $\mu_3 = 0.75$, $\mu_4 = 0.5$. The graphs of fluctuation signals during one revolution of the crankshaft (T) at these values of the cylinder weight coefficients are shown in Fig. 6. It was established that their change corrects the output signal. Computer modeling established the signal of fluctuations from the action of torques in the absence of fuel supply to the cylinders.

Algorithmic support for hardware. Determination of weighting factors D_i cylinders is performed by solving an overdetermined system of incompatible algebraic equations of the form (17), which is based on the expression

$$B(j\omega)D = \Omega_1(j\omega) - \sum_{3=1}^{4} W(j\omega)M_{i,2}(j\omega), \tag{17}$$

where $B(j\omega) = W(j\omega)\sum_{i=1}^{4} M_{i,1}(j\omega)$ is a matrix of size $n \times i$, whose coefficients are determined using the expressions (12, 13); $\sum_{i=1}^{4} W(j\omega)M_{i,2}(j\omega)$ – a column vector of fluctuations in the absence of fuel supply, its coefficients are set in advance; D – a column vector of system solutions; $\Omega_1(j\omega)$ – a column vector of measurement information.

After transformations, expression (17) took the following form (18)

$$C(j\omega)D = \frac{\Omega_1(j\omega)}{W(j\omega)} - \sum_{i=1}^{4} M_{i,2}(j\omega), \tag{18}$$

where $C(j\omega) = \sum_{i=1}^{4} M_{i,1}(j\omega)$.

The computational procedure for monitoring the cylinder capacities of the 3RZ-FE ICE involves setting the parameters of a curve that passes through all points

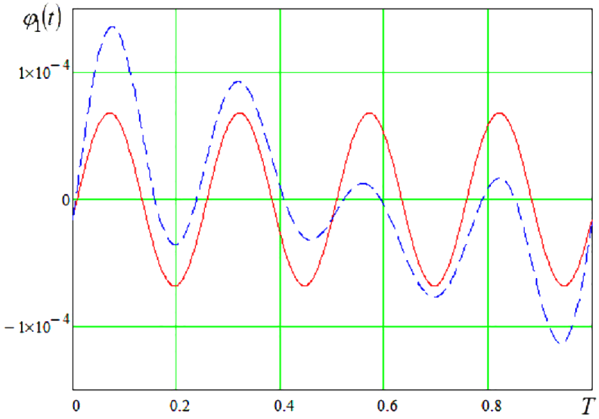

Fig. 6. Results of computer simulation:

$\mu_1 = 1$, $\mu_2 = 1$, $\mu_3 = 1$, $\mu_4 = 1$ – *solid;* $\mu_1 = 1.5$, $\mu_2 = 1.0$, $\mu_3 = 0.75$, $\mu_4 = 0.5$ – *dash*

of measurement information and smoothest possible emissions due to their uncertainty. The coefficients u_0, u_1 of a direct will be established by the results $k + 1$ experiments conducted with a step 0.0333π. For each equation of system (18), we assume a non-binding of the following form

$$r_k = u_1 x_k + u_0 - f_k, \tag{19}$$

where $k = 0,0.0333\pi, 0.0666\pi, ...; f_k = f(x_k)$ – the calculated value of the fluctuation signal, which is set at the point x_k using the expression $\frac{\Omega_1(j\omega)}{W(j\omega)} - \sum_{i=1}^{4} M_{i,2}(j\omega)$.

Since the non-binding takes both positive and negative values, accordingly, in order to ignore the signs, the mathematical operation of squaring is used. A functional equal to the sum of the squares of the non-binding is introduced for consideration.

$$\Phi(u_1,u_0) = \sum_{k=0}^{n} r_k^2 = \sum_{k=0}^{n} (u_1 x_k + u_0 - f_k)^2. \tag{20}$$

The following values are taken as the generalized solution of the system of equations (18) u_0, u_1, which provide the minimum value of the functional $\Phi(u_1, u_0)$. Setting of weighting coefficients D_i provided that the sum of squares is minimum, the non-binding is carried out using a system of two equations of the following form

$$\frac{d\Phi}{du_0} = 0; \quad \frac{d\Phi}{du_1} = 0. \tag{21}$$

The non-binding is defined as

$$r_k = \sum_{m=0}^{3} D_m \psi_m(x_k) - f_k, \tag{22}$$

where $\psi_m(x_k)$ – some functions that form the basis. In this case, trigonometric functions are used, so we have $\psi_m(x_k) = \cos(mx)$. Therefore, for further research, the real part of the system of equations (18) was used. The expression $\sum_{m=0}^{3} D_m \psi_m(x_k)$ is a generalized polynomial, which, based on this choice of basis function, has become algebraic. Therefore, the system of equations (18) and the generalized functional take the form

$$\begin{cases} D_0\psi_0(x_0) + D_1\psi_1(x_0) + D_2\psi_2(x_0) + D_3\psi_3(x_0) = f_0 \\ D_0\psi_0(x_1) + D_1\psi_1(x_1) + D_2\psi_2(x_1) + D_3\psi_3(x_1) = f_1 \\ \vdots \\ D_0\psi_0(x_k) + D_1\psi_1(x_k) + D_2\psi_2(x_k) + D_3\psi_3(x_k) = f_k \end{cases}; \tag{23}$$

$$\Phi(D_0, D_1, D_2, D_3) = \sum_{k=0}^{n} \left(\sum_{m=0}^{3} D_m \psi_m(x_k) - f_k \right)^2.$$

The solution of the system of equations was found by the least squares method, equating all partial derivatives with respect to the components of the generalized functional to zero. As a result of these transformations, we obtained the following expression

$$\frac{d\Phi}{dD_k} = 2\sum_{k=0}^{n} \psi_m(x_k) \left[\sum_{m=0}^{3} D_m \psi_m(x_k) - f_k \right] = 0. \tag{24}$$

By changing the order of summation, we obtained a system of equations of the form

$$\sum_{m=0}^{3} \left[\sum_{k=0}^{n} \psi_m(x_k)\psi_k(x_k) \right] D_m = \sum_{k=0}^{n} f_k \psi_k(x_k). \tag{25}$$

Let us rewrite the last relation (25) in the form of a system of equations

$$\begin{cases} D_0(\psi_0,\psi_0) + D_1(\psi_0,\psi_1) + D_2(\psi_0,\psi_2) + \\ + D_3(\psi_0,\psi_3) = (\psi_0, f) \\ D_0(\psi_1,\psi_0) + D_1(\psi_1,\psi_1) + D_2(\psi_1,\psi_2) + \\ + D_3(\psi_1,\psi_3) = (\psi_1, f) \\ \vdots \\ D_0(\psi_k,\psi_0) + D_1(\psi_k,\psi_1) + D_2(\psi_k,\psi_2) + \\ + D_3(\psi_k,\psi_3) = (\psi_k, f) \end{cases}. \tag{26}$$

The indefinite algebraic system of the least squares method has the form $Bu = f$. Matrix elements B are scalar multiplications $(\psi_i,\psi_j) = \sum_{i=0}^{k} \psi_j(x)\psi_k(x_i)$. As a result of the transformations, the Gram matrix is obtained

$$\begin{vmatrix} (\psi_0,\psi_0)(\psi_0,\psi_1)(\psi_0,\psi_2)(\psi_0,\psi_3) \\ (\psi_1,\psi_0)(\psi_1,\psi_1)(\psi_1,\psi_2)(\psi_1,\psi_3) \\ \vdots \\ (\psi_k,\psi_0)(\psi_k,\psi_1)(\psi_k,\psi_2)(\psi_k,\psi_3) \end{vmatrix}. \tag{27}$$

On the right side of the system are the projections of the free component of the initial problem onto the space of the basis function $(\psi, f) = \sum_{i=0}^{k} \psi_k(x_i) f_i$. The matrix is symmetric and positive definite, so the inconsistent system has a solution and it is one. This solution is obtained thanks to the Seidel iterative method, the main idea of which is to use in the calculation $k + 1$ approximation of unknowns D_0, D_1, D_2, ..., D_{i-1}. To apply this method, the inconsistent system of equations is reduced to the form

$$\begin{cases} D_0^{(1)} = a_{01}D_1^{(0)} + a_{02}D_2^{(0)} + a_{03}D_3^{(0)} + f_0 \\ D_1^{(1)} = a_{10}D_0^{(1)} + a_{12}D_2^{(0)} + a_{13}D_3^{(0)} + f_1 \\ D_2^{(1)} = a_{20}D_0^{(1)} + a_{21}D_2^{(1)} + a_{23}D_3^{(0)} + f_2 \\ \vdots \\ D_k^{(1)} = a_{k0}D_0^{(1)} + a_{k1}D_1^{(1)} + a_{k2}D_2^{(1)} + a_{k3}D_3^{(1)} + f_k \end{cases}, \tag{28}$$

where the diagonal coefficients of the system are maximal in modulus and the conditions for convergence of the equations are checked. In the case of lack of convergence, it is necessary to perform simple mathematical transformations. The column vector of the free terms of the system of equations (23) is used as the zero approximation. The peculiarity of the Seidel method is that the approximation obtained in the first equation is used in the second, and the approximations obtained in the first and second equations are used in the third, etc. The convergence theorem for the simple iteration method is also valid for the Seidel method.

The uncertainty of the solution of the system of equations (28), upon reaching which the computational process stops, satisfies the conditions

$$\left|D_i^{k+1} - D_i^k\right| \le \varepsilon, \quad \text{at} \quad i = 1,2,3,\ldots. \qquad (29)$$

The size ε is accepted in advance and this condition is fulfilled for all equations. The developed software for monitoring the cylinder capacities of the ICE 3RZ-FE was tested using the example of processing the signal of a computer simulation scheme with the following values of weighting factors $\mu_1 = 1.5$, $\mu_2 = 1.0$, $\mu_3 = 0.75$, $\mu_4 = 0.5$. The following column vector of the solution is obtained $D = \{1.501, 0.998, 0.747, 0.504\}^T$. The calculation results prove the effectiveness of the developed application software.

Conclusions.

1. Hardware and software tools for controlling the supply of the fuel-air mixture with feedback on the state of the signal of uneven rotation of the crankshaft of the ICE 3RZ-FE are proposed. Information technology for isolating a discrete fluctuation signal is developed and its frequency spectrum is established. Algorithmic support for converting the input signal is developed, which meets the performance requirements.

2. For the first time, a mechanical system with four degrees of freedom was used as a mathematical model of the 3RZ-FE ICE. Under the conditions of taking into account friction, the motion of its masses is described by a system of second-order differential equations. The parameters of the system were reduced to a dimensionless form using theorems of similarity theory and the Laplace transform was used to solve it under zero initial conditions. A system of algebraic equations was obtained, the solution of which significantly simplifies monitoring the identity of the operating cycles of the 3RZ-FE ICE.

3. The method of determinants was used to establish the transfer functions of mechanical torque transformations. The coincidence of the expressions of the transfer functions was established, accordingly, the mathematical model of the ICE 3RZ-FE was first presented as a mechanical system with one degree of freedom.

4. The Mathcad software environment was used and a scheme for computer simulation of the movements of the first mass of the 3RZ-FE ICE was built with the signal of uneven crankshaft rotation. The model parameters were identified by comparison with experimental data. The method for regulating the length of connections of neural network technologies was applied to adjust the parameters of the scheme for computer simulation.

5. The frequency representation of the signal of uneven rotation was used and an information technology for monitoring the cylinder capacities of the ICE 3RZ-FE was developed. The algorithmic support for solving the overdetermined system of incompatible algebraic equations implements the least squares and Seidel methods. The uncertainty of the solution of the system of equations was established, upon reaching which the computational process stops. According to the calculation results, the hardware and software tools set the amplitude coefficients of the cylinders, on the basis of which the settings of the fuel-air mixture supply phases of the ICE 3RZ-FE are changed.

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Апаратно-програмні засоби керування подачею палива двигуна за умов неповної інформації

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Мета. Покращення продуктивності апаратно-програмних засобів задання фаз подачі палива й повітря в циліндри двигуна на основі обробки сигналу нерівномірності обертання колінчастого валу.

Методика. Дослідження базуються на теорії автоматичного керування й чисельних методах.

Результати. Результатами досліджень є: побудова детермінованої математичної моделі двигуна внутрішнього згоряння; опис рухів мас моделі системою лінійних диференціальних рівнянь, параметри якої нормалізовані за допомогою методів теорії подібності; отримання передавальних функцій механічних каналів за допомогою програмного середовища Mathcad і методу визначників. Також встановлено збіг отриманих передавальних функцій і вперше запропоновано подати крутну схему силового агрегату механічною системою з одним ступенем свободи; у програмному середовищі Matlab досліджені частотні характеристики каналу передачі крутних моментів. Встановлена ефективність розробленого прикладного програмного забезпечення.

Наукова новизна. Полягає у розробці архітектури апаратно-програмних засобів задання фаз подачі палива на основі використання принципу управління зі зворотним зв'язком за станом частотно-модульованого сигналу швидкості обертання колінчастого валу. Розроблена інформаційна технологія обробки масиву дослідних даних з метою визначення сигналу флуктуацій. Створене алгоритмічне забезпечення моніторингу циліндрових потужностей на основі розв'язання перевизначеної системи несумісних алгебраїчних рівнянь для встановлення параметрів вектору вагових коефіцієнтів циліндрів. Для пошуку її рішення використані метод найменших квадратів та ітераційний метод Зейделя за умови нульових часткових похідних компонентів узагальненого функціоналу. Як наслідок математичних перетворень, отримана симетрична й позитивно визначена матриця Грама. Встановлена межа невизначеності розв'язання системи алгебраїчних рівнянь, при досягненні якої обчислювальний процес зупиняється.

Практична значимість. Полягає у розробці алгоритмічного забезпечення моніторингу циліндрових потужностей за частотного подання сигналу флуктуацій із використанням передавальних функцій механічних каналів і апроксимованих крутних моментів. Моніторинг циліндрових потужностей орієнтований на встановлення параметрів кривої, що проходить через усі точки вимірювальної інформації та згладжує можливі викиди за рахунок їхньої невизначеності. Використане перетворення Лапласа за нульових початкових умов в якості математичного апарату встановлення інформаційних зв'язків між крутними моментами циліндрів і сигналом флуктуацій першої маси колінчастого валу. У програмному середовищі Mathcad створена схема комп'ютерного моделювання сигналу нерівномірності обертання першої маси й, на основі використання методу регулювання довжини інформаційних зв'язків, проведена ідентифікація її параметрів.

Ключові слова: *двигун внутрішнього згоряння, апаратні засоби, алгоритмічне забезпечення, математичне моделювання, сигнал флуктуацій*

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