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MATHEMATICAL MODEL FOR HEAT TRANSFER DURING UNDERGROUND COAL GASIFICATION PROCESS

Purpose. To develop a mathematical model of the "coal-gas" medium heat transfer during underground coal gasification to predict the combustion face advance velocity and the duration of gasification column mining.

Methodology. To detect the temperature fields in the coal seam and gas, depending on the displacement length of the phase transition boundary, a boundary-value problem of mathematical physics has been developed. To solve this boundary-value problem, Boltzmann transformations, as well as methods for solving ordinary differential equations, are used. Newton-Raphson method, which has quadratic convergence, is used to find the roots of the transcendental equation.

Findings. Tendencies in the formation of mathematical models when studying temperature fields around an underground gasifier have been analyzed, with highlighting their disadvantages. A mathematical model of heat transfer during underground gasification has been developed, taking into account the phase transition boundaries of the "coal-gas" medium. A computational experiment was conducted to determine the temperature of the phase transition boundary based on the length of the gasification column and the duration of the process.

Originality. A mathematical model for heat transfer during coal gasification in the form of a boundary-value problem of mathematical physics, which consists of parabolic heat-transfer equations, the Stefan condition at the phase transition boundary, and the Dirichlet boundary conditions, has been constructed. As a result of solving the boundary-value problem, a self-similar solution has been obtained for the distribution of the coal seam and gas temperature fields, as well as the position of the phase transition boundary depending on the gasification duration and on the medium density, thermal conductivity coefficients, specific heat capacity of gas and coal, specific calorific value and temperature of coal combustion, initial coal temperature and constant temperature of the gasification process. The conducted analysis of numerical calculations provides a deeper understanding of the dynamics of underground coal gasification process and makes necessary corrections to achieve the maximum process efficiency.

Practical value. A methodology for determining the displacement length of the phase transition boundary of the "coal-gas" medium has been developed, taking into account the change in the combustion face temperature along the gasification zone length depending on the duration of this process. Application of the methodology makes it possible to predict the time of mining the gasified coal column for drawing up a calendar plan for mining operations.

Keywords: *underground gasification, mathematical model, coal, gas, phase transition, Stefan conditions*

Introduction. Underground coal gasification (UCG) is a complex heterogeneous process of converting coal into a gaseous product (producer gas) in underground conditions. Today, this technology is classified as a low-carbon technology, which reduces the amount of CO_2 emissions compared to traditional coal combustion methods [1]. Underground coal gasification has a history of more than 130 years, during which a number of field and laboratory studies have been conducted [2]. Significant steps forward were made in the *20th* century with a large number of experiments in different countries, including the United States of America, China, Australia and others [3, 4]. Such experience has laid a solid foundation for the industrialization of this technology [5].

Modern underground coal gasification projects are implemented on several continents, providing a stable supply of energy and chemical products. One of the key advantages of underground gasification is the ability to use low-quality coal, which is not suitable for conventional combustion, thereby expanding the resource base of the power industry.

In addition, the introduction of underground coal gasification technology reduces the technogenic impact on the environment by reducing the volumes of accumulated waste rocks and coal beneficiation waste during coal mining by underground or open-pit methods [6, 7]. The implementation of this technology also reduces the risks associated with coal mining, in particular, not having workers directly in underground conditions increases their safety in the coal mining industry.

In the long run, underground coal gasification can become a key technology in the transition to a cleaner and more sustainable energy system, providing a stable energy supply with minimal environmental impact.

Literature review. Underground coal gasification process includes a complex of preparatory works (drilling wells, forming a reaction channel and igniting a coal seam) and main works (coal gasification, control of underground gasifier operating modes) [8, 9]. Knowing the duration of each process provides an effective plan for mining the reserves. This plan determines the directions of development of mining operations in compliance with the mining system parameters stipulated by the field development projects, the volumes of minerals to be mined, the volumes of geological exploration, stripping, reclamation, mining capital and mining preparatory works, the volumes of mineral raw material recycling, as well as other works provided for in the technical projects for mining the fields, while ensuring adequate and safe labor conditions [10].

To date, a significant amount of thorough research in the field of underground coal gasification has focused on the development of numerical models characterizing the correlation between analytical and laboratory research results [11, 12]. However, the real underground gasification process conditions are extremely complex and involve many factors that affect the efficiency of this process. One of the key aspects influencing gas generation in the combustion face is the change in the temperature field both in the combustion face plane and around the underground gasifier [13].

In real conditions of the coal seam occurrence, the thermal conductivity coefficient and specific heat capacity of the rock mass change under the influence of temperature, pressure and moisture content [14, 15]. These parameters may vary significantly depending on the mining-geological conditions of coal occurrence and its composition. For example, the presence of moisture in a coal seam, roof and bottom rocks leads to a decrease in temperature and the formation of local zones of low temperature, which influences the efficiency of heat transfer through the reaction zones of the combustion face [16]. Spe-

cific heat capacity, which determines the ability of roof and bottom rocks to accumulate heat, also changes with temperature. It influences coal heating rate throughout the thickness of the seam and the length of the gasification column [17]. Therefore, when developing numerical models, it is necessary to consider the temperature field distribution parameters to obtain accurate predictions of the gasification process efficiency and its dynamics. Also, knowledge about the temperature field distribution parameters is important to assess the possibility of intensifying gasification process as one of the main methods.

To accurately model the underground coal gasification process, it is necessary to use complex numerical models capable of taking into account the variability of parameters and real process conditions. Such models adequately predict the behavior of the studied phenomenon or system, as well as allow determining the optimal parameters of heat transfer, diffusion, etc. [18]. This approach allows for more accurate predictions and optimization of gasification processes, increasing their efficiency and reducing the negative impact on the environment.

To date, a number of analytical studies have been conducted in the field of constructing numerical models characterizing both chemical and kinetic processes arising in the combustion face plane and around the underground gasifier [19, 20]. In particular, models have been developed to study the coal combustion thermodynamic parameters, the behavior of gas mixtures influenced by high temperatures, as well as the kinetics of the formation and destruction of chemical compounds [21, 22]. One such example is coal combustion numerical modeling, which allows predicting heat release and combustion product formation [23]. Another example is models for optimizing gasification processes in underground conditions, taking into account heat and mass transfer processes, as well as chemical reactions between rock and gas constituents [24].

Stationary models of the underground gasification process consisting of interrelated differential equations are described in [25]. These models allow analytically solving equations and obtaining expressions of parameters influencing the coal gasification process. But the gasification process is characterized by dynamic changes in the temperature field depending on the chemical reaction zone, both in the combustion face and behind the gasification column. Therefore, the use of this model leads to inaccurate predictions and limits the ability to use the model for real-time process control.

In [26], a two-dimensional model of thermal fields was developed, based on the Fourier thermal conductivity equation, taking into account heat transfer between solid and liquid phases. But to determine the gasification process efficiency, it is essential to substantiate the heat transfer parameters at the coal-gas interphase boundary.

The authors of [27] developed an extended mathematical model describing the combustion face of an underground gasifier, which is presented as a cylindrical channel. This made it possible to obtain a mathematical equation for non-stationary thermal conductivity in a moving coordinate system that describes the radial extent of the gasification channel and its expansion. This model takes into account the influence of the moving combustion front expanding with time on the temperature field distribution, which is important in determining the process efficiency, since different coal seam areas can be at different stages of combustion and gasification. Similar studies on the temperature field distribution parameters were also conducted in [28]. Unfortunately, this approach is quite logical when studying the coal gasification process, which is implemented through vertical wells using the "well-gasifier" technology. The above-mentioned models represent the combustion face as a cylindrical channel, which does not always correspond to real conditions. Coal seams can have complex geometries, which can lead to inaccuracies in predicting temperature distribution.

The authors in [29] mathematically describe the identification of the coal seam temperature field $T(x, t)$ and the displacement length of the phase transition boundary $S(t)$, which consists of integrating the differential thermal conductivity equation based on solving the transcendental equation by the Newton-Raphson method. It should be noted that the development of this model involves only solving a single-phase problem.

Unsolved aspects of the problem. There are significant inherent uncertainties in the existing models. They do not take into account all the complexities and variability of the real conditions of coal seam occurrence, in particular, changes in the thermophysical properties of coal and surrounding rocks along the length of the gasification column under the influence of high temperatures. This results in numerical modelling data providing inaccurate predictions of temperature distribution, reaction rates and combustion face front displacement. This, in turn, influences the ability to effectively control the gasification process, which is critical to achieving stable and safe underground gasifier operation.

It is therefore important to develop more accurate and reliable models to effectively monitor and control temperature conditions during the gasification process. These models should take into account not only the main thermodynamic underground gasifier parameters, but also their variability in the "coal-gas" medium, which can significantly influence heat transfer and chemical reactions in the combustion face, and predict the duration of gasification column mining.

The development of such models requires the use of advanced mathematical modeling and numerical analysis methods, as well as detailed experimental studies on model validation. It is important to integrate data from laboratory experiments and field studies to obtain more realistic and accurate predictions. Thus, the development of more accurate and reliable underground coal gasification models is a critical step to improve the efficiency, safety and environmental sustainability of this technology, opening up new opportunities for the use of coal resources in the modern energy sector.

Therefore, the purpose of this research is to develop a mathematical model of the "coal-gas" medium heat transfer during underground coal gasification to predict the combustion face advance velocity and the duration of gasification column mining, which is an urgent scientific-applied task.

To solve the purpose set, this paper analyzes mathematical models when studying underground coal gasification technology; characterizes the "coal-gas" medium interphase boundaries; develops a mathematical model for heat transfer of the underground coal gasification in the form of a two-phase Stefan problem; substantiates the parameters for temperature change and gasification duration at the phase transition boundary along the length of the gasification column.

Methods. When conducting numerical studies on underground coal gasification, it is assumed that there are no heterogeneity in the coal seam, and the heat- and mass-transfer conditions along the upper and lower surfaces of the seam do not change. This simplification allows for a more controllable and predictable model to be created that enables the study of the main gasification process parameters. Several key aspects are important in such models. Firstly, these are thermodynamic conditions, including temperature, pressure, and gas phase composition. Determination of these parameters allows modeling of reaction processes occurring in a coal seam, such as combustion, gasification, and slag formation. Secondly, it is important to take into account the kinetic parameters of reactions, which determine the rate of coal conversion into the gas phase. Modeling of the heat transfer process is accompanied by a change in the aggregate state of the underground gasifier medium as a result of coal gasification. Therefore, a special feature of this problem is the changing size of the area in which the temperature field is studied. Note that the physical properties of the medium when passing through the phase transition boundary (in our case, it is thermal conductivity) change abruptly.

The main characteristic of phase transformations during coal gasification is the temperature at which the phases are in a

state of thermodynamic equilibrium (phase transition point). For phase transitions of the first kind, it is characteristic that at the phase transition point there is a heat emission or absorption and a change in volume. In general view, the moving boundary of the gas-coal phase transition $S(t)$ is shown in Fig. 1.

According to the given image (Fig. 1), assume that coal occupies half-space $x > 0$ and let the coal temperature at $t < 0$ is the same and equal to $T_0 < T_g$, where T_g is the coal gasification temperature.

From $t = 0$ at $x = 0$, a constant temperature $T_p \geq T_g$ is maintained. In this case, at $t > 0$ and $x \approx 0$, a gassed-out space appears, the size of which $S(t)$ increases over time (Fig. 1). The phase transition boundary $x = S(t)$, at any moment in time, separates coal from combustion products, moving with a certain velocity $v = dS/dt$ in the direction of the coal seam. By the terms of the problem, $S(0) = 0$.

Assuming that the properties of the medium change abruptly during the phase transition, the thermal conductivity equations for two phases take the form

$$\frac{\partial T_1(x,t)}{\partial t} = a_1 \frac{\partial^2 T_1(x,t)}{\partial x^2}, \quad t > 0, \quad 0 < x < S(t); \tag{1}$$

$$\frac{\partial T_2(x,t)}{\partial t} = a_2 \frac{\partial^2 T_2(x,t)}{\partial x^2}, \quad t > 0, \quad S(t) < x < \infty, \tag{2}$$

where $T_1(x, t)$, $T_2(x, t)$ are gas and coal temperatures, respectively; $a_i = \frac{\lambda}{c\gamma}$ – the temperature conductivity coefficient; γ_i – medium density; λ_i – the thermal conductivity coefficient, c_i – specific heat capacity; $i = 1$ – values related to gas, and $i = 2$ – values related to coal.

Given that only coal exists at the initial moment of time, the initial condition will be written as follows

$$T_2(x, 0) = T_0. \tag{3}$$

The boundary conditions of the problem are formulated as follows

$$T_1(0, t) = T_P; \tag{4}$$

$$T_2(\infty, t) = T_0. \tag{5}$$

At the phase transition boundary, the Stefan conditions are fulfilled (γ_2)

$$T_1(x,t)\big|_{x=S-0} = T_2(x,t)\big|_{x=S+0} = T_g; \tag{6}$$

$$\lambda_2 \frac{\partial T_2(x,t)}{\partial x}\bigg|_{x=S+0} - \lambda_1 \frac{\partial T_1(x,t)}{\partial x}\bigg|_{x=S-0} = \gamma \cdot Q \frac{dS}{dt}, \tag{7}$$

where Q is specific calorific value.

Thus, the task of identifying the coal seam temperature field $T_2(x, t)$, gas temperature $T_1(x, t)$, the length $S(t)$ of the phase transition boundary displacement consists in integrating the differential thermal conductivity equations (1–2) for initial condition (3), for boundary conditions (4–5) for Stefan conditions at the phase transition boundary (6–7). This allows for the exploration of the dynamics of thermal processes within a coal seam during underground gasification and enables the prediction of the phase transition boundary behavior depending on time and spatial coordinates. By applying Stefan conditions, the velocity of the phase transition boundary movement is linked to the heat flux passing through this boundary. It is formulated based on the principle of energy conservation, which states that the heat released or absorbed at the phase transition boundary must equal the amount of heat required for the material's phase transition at this boundary.

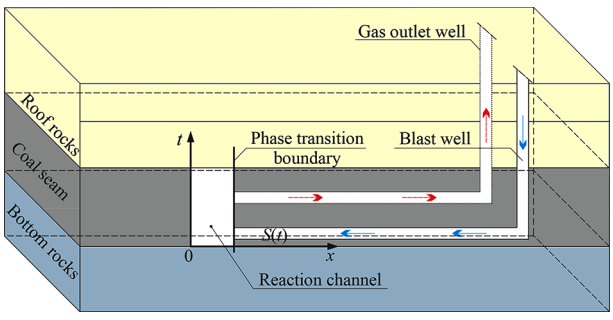

Fig. 1. Image of the "coal-gas" medium interphase boundary S(t)

Results. When conducting numerical studies using Boltzmann transformation $\vartheta = \frac{x}{\sqrt{t}}$, equations (1–2) are reduced to ordinary differential equations for the function $T_1(\vartheta)$, $T_2(\vartheta)$

$$a_i \frac{d^2 T_i(x,t)}{d\vartheta^2} + \frac{1}{2}\vartheta \frac{dT_i}{d\vartheta} = 0, \quad i = 1,2. \tag{8}$$

Further, let us make a substitution for $\theta_i = \frac{dT_i}{d\vartheta}$, $i = 1, 2$, then equation (8) takes the form

$$\frac{1}{\theta_i}\frac{d\theta_i}{d\vartheta} = -\frac{1}{2a_i}\vartheta, \quad i = 1,2. \tag{9}$$

The solution to ordinary differential equations (9) is

$$\theta_i(\vartheta) = B_i \exp\left(-\frac{\vartheta^2}{4a_i}\right), \tag{10}$$

where B_i is unknown constant.

By integrating equation (10), the general solutions to equation (8) can be found

$$T_i(\vartheta) = A_i + B_i \int_0^{\vartheta} \exp\left(-\frac{\varsigma^2}{4a_i}\right) d\varsigma = A_i + B_i erf\left(\frac{\vartheta}{2\sqrt{a_i}}\right), \tag{11}$$

where A_i is unknown constant; $erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-\varsigma^2) d\varsigma$ – an error function.

In particular, the error function satisfies the conditions

$$erf(0) = 0, \quad erf(\infty) = 1. \tag{12}$$

Returning to the variables x, t, the solution to equations (1–2) can be written as follows

$$T_1(x,t) = A_1 + B_1 \cdot erf\left(\frac{x}{2\alpha_1\sqrt{t}}\right); \tag{13}$$

$$T_2(x,t) = A_2 + B_2 \cdot erf\left(\frac{x}{2\alpha_2\sqrt{t}}\right), \tag{14}$$

where $\alpha_1 = \sqrt{a_1}$.

From the boundary conditions (4–5), given (12), we have

$$A_1 = T_p; \tag{15}$$

$$A_1 + B_2 = T_0. \tag{16}$$

From the condition at the phase transition boundary (6) we have

$$A_1 + B_1 \cdot erf\left(\frac{S(t)}{2\alpha_1\sqrt{t}}\right) = T_g; \tag{17}$$

$$A_2 + B_2 \cdot erf\left(\frac{S(t)}{2\alpha_2\sqrt{t}}\right) = T_g. \tag{18}$$

Each of these conditions (17, 18) can be satisfied for any $t > 0$ only if the function $erf(x)$ argument in these equations is not time-dependent. But this is possible if $S(t) = \beta \cdot \sqrt{t}$, where β is an unknown constant.

Substituting the above-mentioned relationship into conditions (17 and 18) allows for obtaining a new solution that de-

pends on the parameter β. Next, to determine the value of this constant β, it is necessary to consider the initial and boundary conditions of the problem, as well as the physical characteristics of the medium, such as thermal conductivity, density, and specific heat capacity. Analyzing these conditions will help to find a specific value of β that satisfies the problem's conditions, which in turn will allow determining the temperature distribution within the coal seam and the nature of the phase transition boundary movement during underground coal gasification.

This approach is widely used in heat conduction problems with moving boundaries, particularly in Stefan problems, where it is important to consider the influence of time on phase transition processes and heat propagation.

Thus, the law of motion $S(t)$ of the phase transition boundary is determined with accuracy to a certain constant β

$$S(t) = \beta \cdot \sqrt{t}. \tag{19}$$

Substituting (19) into the ratio (17–18), we have

$$A_1 + B_1 \cdot erf\left(\frac{\beta}{2\alpha_1}\right) = T_g; \tag{20}$$

$$A_2 + B_2 \cdot erf\left(\frac{\beta}{2\alpha_2}\right) = T_g. \tag{21}$$

Now, from equations (15, 16), and (20, 21), four constants A_i, B_i, $i = 1, 2$ can be found

$$A_1 = T_p; \tag{22}$$

$$B_1 = \frac{T_? - T_\partial}{erf\left(\frac{\beta}{2\alpha_1}\right)}; \tag{23}$$

$$A_2 = \frac{T_0 erf\left(\frac{\beta}{2\alpha_2}\right) - T_g}{erf\left(\frac{\beta}{2\alpha_2}\right) - 1}; \tag{24}$$

$$B_2 = \frac{T_g - T_0}{erf\left(\frac{\beta}{2\alpha_2}\right) - 1}. \tag{25}$$

To determine the constant β, it is necessary to use the condition at the phase transition boundary (7) and, taking into account the known ratio

$$\frac{derf(z)}{dz} = \frac{2}{\sqrt{\pi}}\exp(-z^2), \tag{26}$$

obtain a transcendental equation to determine β

$$F(\beta) = \frac{\exp\left[-\left(\frac{\beta}{2\alpha_1}\right)^2\right]\lambda_1(T_g - T_p)}{erf\left(\frac{\beta}{2\alpha_1}\right)\cdot\alpha_1} - \frac{\exp\left[-\left(\frac{\beta}{2\alpha_2}\right)^2\right]\lambda_2(T_g - T_0)}{\left(erf\left(\frac{\beta}{2\alpha_2}\right) - 1\right)\cdot\alpha_2} - \sqrt{\pi}\gamma_2\frac{Q\beta}{2} = 0. \tag{27}$$

Consequently, the temperatures $T_1(x, t)$, $T_2(x, t)$ of gas and coal, respectively, will be calculated using the formulas

$$T_1(x,t) = T_p + \frac{T_g - T_p}{erf\left(\frac{\beta}{2\alpha_1}\right)}\cdot erf\left(\frac{x}{2\alpha_1\sqrt{t}}\right), \tag{28}$$

$$0 < x < S(t);$$

$$T_2(x,t) = \frac{1}{erf\left(\frac{\beta}{2\alpha_2}\right) - 1}\left[\begin{array}{l}\left(T_0 erf\left(\frac{\beta}{2\alpha_2}\right) - T_?\right) + \\ (T_g - T_0)erf\left(\frac{x}{2\alpha_2\sqrt{t}}\right)\end{array}\right], \tag{29}$$

$$x > S(t),$$

where A_i, B_i, $i = 1, 2$ are calculated by formulas (22–25); β is determined from solving the transcendental equation (27).

Transcendental equation (27) roots β can be found by the Newton-Raphson method, which has quadratic convergence. Successive approximations by the Newton-Raphson method are calculated by the formula

$$\beta_{n+1} = \beta_n - \frac{F(\beta_n)}{\frac{dF(\beta_n)}{d\beta}}, \tag{30}$$

where $F(\beta)$ is calculated by formula (27).

The Newton-Raphson method is an efficient and widely used technique for finding the roots of nonlinear equations, especially when the function $f(\beta)$ is sufficiently smooth and its derivative can be easily computed. However, it is important to note that ensuring the convergence of the method requires choosing an appropriate initial guess. If the initial value is too far from the actual root, the method may converge slowly or even diverge, meaning it could move away from the desired root. This is particularly crucial when dealing with functions that have a complex form or multiple roots, as an incorrect initial guess could lead to convergence to the wrong root or to divergent behavior.

It should also be considered that the Newton-Raphson method requires the calculation of the derivative of the function $f'(\beta)$ at each step, which can be computationally challenging or even infeasible in cases of complex functions. In such scenarios, modifications of this method or other numerical methods, such as the secant method, may be used. The secant method does not require the calculation of the derivative, but it only has linear convergence.

Despite these limitations, the Newton-Raphson method remains one of the most popular numerical methods due to its efficiency and accuracy, particularly when the initial guess is chosen well. It is widely used in engineering, physical, and mathematical problems where precise and fast root finding is critically important.

Explore an example of conducting a numerical study at the specified values, given in Table 1.

Table 1

Input data for the computational experiment

Parameters	Gas	Coal
Temperature	T_1, °C	T_2, °C
	1,400	800
Coal initial temperature	T_0, °C	
	38	
Medium density	γ_1, kg/m^3	γ_2, kg/m^3
	0.66	1,340
Thermal conductivity coefficient	λ_1, J/(m · s · deg.)	λ_2, J/(m · s · deg.)
	0.035	0.1
Specific heat capacity	c_1, J/kg · K	c_2, J/kg · K
	2,260	1,100.00
Specific calorific value	Q, kJ/kg	
	27,000	
Gasification duration	t, hours	
	0–100	

From the solution of the transcendental equation (30), with the selected numerical parameters, it has been found that $\beta = 1.04 \cdot 10^{-4}$. Fig. 2 shows the dependence of the change in the "coal-gas" medium temperature field. Duration of the gasification process $t = 0 - 100$ hours. In this case, the phase transition boundary $S(100) = 0.0624$.

Analysis of the dependence shown in Fig. 2 indicates that the temperature field in the "coal-gas" medium changes linearly with one slope angle before the phase transition boundary and with another slope angle after it, which corresponds to Stefan conditions (6, 7). In this case, the "coal-gas" medium phase boundary is described by the parabola branch depending on the gasification duration (Fig. 3).

The parabolic nature of this dependence highlights the importance of variables such as temperature conditions, chemical reaction rates and physical-chemical properties of coal. The presence of a high temperature in the combustion face and high chemical reaction rate contribute to a faster displacement of the phase transition boundary, which leads to an acceleration of the coal gasification process. In particular, high temperatures in the combustion face provide efficient heat transfer, which allows the conditions required for coal gasification to be achieved more quickly. This, in turn, increases the rate of chemical reactions, such as oxidation and reduction, which are key in the synthesis gas formation process. The physical-chemical properties of coal, including its thermal conductivity, heat capacity and composition, are also important factors. These properties determine how efficiently coal can absorb and conduct heat, as well as how quickly chemical reactions may occur.

Further research envisages optimizing the model parameters when varying the blast mixtures supplied to the underground gasifier and developing adaptive algorithms for actual prediction and control of the gasification process.

Conclusions. The proposed mathematical model of heat transfer is the basis for the development of an engineering methodology for determining the displacement length of the phase transition boundary of the "coal-gas" medium, characterizing the changes in the temperature of combustion face along the gasification column length depending on the given process duration. It forms the basis for creating a software product aimed at determining the efficiency of conducting underground coal gasification process.

It has been determined that the dependence of the temperature change at the phase transition boundary along the length of the gasification zone corresponds to Stefan conditions, which take into account the phase transition occurring during coal gasification and provide a more accurate reflection of the actual heat transfer processes in the "coal-gas" medium. Analysis of this dependence indicates a stable regime, where heat transfer is determined mainly by the temperature gradient and coal seam properties such as heat capacity and thermal conductivity.

The determined dependence of the change in the phase transition boundary on the gasification duration allows predicting the combustion face front advance velocity of the underground gasifier. These data are important for scheduling mining operations and predicting the parameters of the underground coal gasification process.

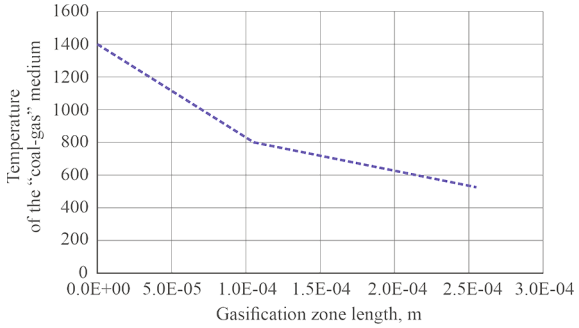

Fig. 2. Dependence of the change in the "coal-gas" medium temperature on the gasification zone length

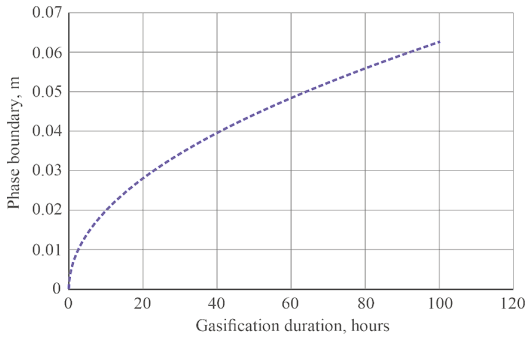

Fig. 3. Dependence of the change in the phase transition boundary on the gasification duration

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Математична модель теплообміну процесу підземної газифікації вугілля

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Мета. Розробка математичної моделі теплообміну середовища «вугілля–газ» при підземній газифікації вугілля задля прогнозування швидкості посування вогневого вибою та тривалості відпрацювання стовпів газифікації.

Методика. Для знаходження температурних полів у вугільному пласті та газі в залежності від довжини переміщення межі фазового переходу була побудована крайова задача математичної фізики. Для рішення цієї крайової задачі було застосовано перетворення Больцмана, а також методи розв'язання звичайних диференційних рівнянь, а для знаходження коренів трансцендентного рівняння використано метод Ньютона–Рафсона, що має квадратичну збіжність.

Результати. Проаналізовані тенденції щодо формування математичних моделей при дослідженні температурних полів навколо підземного газогенератора з виділенням їх недоліків. Розроблена математична модель теплообміну при підземній газифікації, що враховує межі фазового переходу середовища «вугілля–газ». Проведено обчислювальний експеримент зі встановлення температури межі фазового переходу за довжиною стовпа газифікації та тривалості процесу.

Наукова новизна. Побудована математична модель теплообміну при газифікації вугілля у вигляді крайової задачі математичної фізики, що складається із параболічних рівнянь теплопровідності, умови Стефана на межі фазового переходу та граничних умов Діріхле. У результаті розв'язання крайової задачі було отримане автомодельне рішення розподілу температурних полів вугільного пласта й газу, а також положення межі фазового переходу в залежності від тривалості газифікації та від щільності середовища, коефіцієнтів теплопровідності, питомої теплоємності газу та вугілля, питомої теплоти згоряння вугілля, температури горіння вугілля, початкової температури вугілля та постійної температури процесу газифікації. Проведений аналіз чисельних розрахунків дозволяє глибше зрозуміти динаміку процесу підземної газифікації вугілля та вносити необхідні корективи для досягнення максимальної ефективності процесу.

Практична значимість. Розроблена методика з визначення довжини переміщення межі фазового переходу середовища «вугілля–газ» з урахуванням зміни температури вогневого вибою за довжиною області газифікації від тривалості даного процесу. Застосування методики дозволяє прогнозувати час відпрацювання стовпа вугілля, що буде газифікуватися, для складання календарного плану ведення гірничих робіт.

Ключові слова: *підземна газифікація, математична модель, вугілля, газ, фазовий перехід, умови Стефана*

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