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ANALYSIS OF THE THIRD CLASS MECHANISM USING THE MODELING METHOD IN THE MATHCAD SOFTWARE ENVIRONMENT

Purpose. To carry out a kinematic analysis of a flat twelve-link hinged mechanism with three leading links, the basis of which is a structural group of links of the third class of the fourth order with rotational kinematic pairs, which is used in technological equipment.

Methodology. The kinematic study of a flat complex mechanism of the third class with three leading links was performed using the mathematical modeling method in the Mathcad software environment.

Findings. Using the method of mathematical modeling, the mechanism was presented in the form of free vectors built on its links, for which, taking into account the presence of three leading links, three vector contours were selected, which made it possible to draw up a system of vector equations of their closedness with further solving by a numerical method in the Mathcad software environment. The functions of the rotation angles of the mechanism links, their angular velocities and accelerations were obtained in analytical and graphical form depending on the rotation angle of the first driving crank of the mechanism, the calculation of the angular velocities and accelerations of all the driven links of the mechanism was performed.

Originality. A sequence was developed and a kinematic study of a complex planar mechanism of the third class with three leading links was done, the basis of which is a structural group of links of the third class of the fourth order. Schematic modeling of the mechanism with three leading links was performed, a visualization schedule of its kinematic scheme was built, and a valid version of its assembly was obtained from many possible ones, for which the values of kinematic parameters of all its links were determined.

Practical value. An analysis of the effect of the kinematic parameters of the three leading links of the third-class mechanism on the movement of the working body of the technological equipment was carried out. From the analysis of the obtained research results for the cycle of the mechanism the reason was found for the incomplete technological stoppage of the link, which should provide the working body of the machine for the required period of time. Recommendations of the elimination were made to identify the deficiency and proposals for the need to improve the drive of this equipment.

Keywords: *mechanism of the third class, kinematic analysis of the mechanism, Mathcad computer modeling, kinematic study*

Introduction. Technological machines with several driving links, whose working bodies perform rotational motion or reciprocating motion, move with their suspension in extreme positions, are used in various branches of mechanical engineering, for example, in mining. Recently, in global practice, there have been qualitative changes in the structure and technical level of machines and equipment from the point of view of the use of laser, optical, computer and other devices for controlling the movement and suspension of the working bodies. However, their use is not always justified due to the complications associated with the use of additional electronic sensors, equipment and, as a result, an increase in operating costs, an increase in the final cost of the machines.

The law of movement of the working bodies with the output link suspension can be provided by cam mechanisms, the wide application of which is restrained by a number of inherent disadvantages, including the low wear hardness of the working profile of the cams and the tendency of these mechanisms to open the kinematic pair "pusher – cam" at high angular speeds of cam rotation.

In contradistinction to cam mechanisms, lever mechanisms are highly reliable, durable, capable of long-term and significant loads, and are successfully used in high-speed technological equipment.

Flat complex mechanisms of the third and higher classes have always been in the "visual field" of specialists and scientists dealing with issues of improving existing and designing new technological equipment. That is how such mechanisms can ensure the movement of the working parts of the machine along geometrically complex trajectories and the necessary laws of motion, which is necessary for the technological process. This especially applies to equipment with uniform movement of the driving link (driving links) and non-uniform movement of the driven ones, which cause the cyclic movement of the working organs of the machine with certain technological suspensions and their being stationary for a certain period of time to ensure the possibility of high-quality execution of the technological process.

The use of flat complex mechanisms of the third and higher classes in the structure of technological machines requires their preliminary comprehensive analysis, which has a certain sequence of implementation from the theory of the structural structure [1] to kinematic [2] and dynamic [3] studies of such mechanisms for various branches of production [4].

Literature review. Kinematic analysis of complex mechanical systems is performed under the condition of carrying out their structural studies, which can be conditionally divided into two groups of tasks: the task of analysis [5] of existing mechanisms during their technological improvement and the task of synthesis [6] under the condition of designing new mechanisms for technological machines and processes. The creation of curved surfaces of folds is an actual and still insufficiently studied problem. Thus, for the formation of complex geometric surfaces, a flat multi-link mechanism was structurally synthesized, the lengths of its links were parameterized, and the directions of movement of the driving links were analyzed, which made it possible to determine the different trajectories of the points of the connecting rods of the mechanism in order to obtain the geometric surfaces necessary for the technology [7].

In the scientific works of recent years regarding the application of flat complex mechanisms in various mechanical systems and works, there is a trend of increasing the number of publications in which it is proposed to automate the processes of analysis and synthesis of complex mechanical systems using specially developed software and computer equipment.

The paper [8] presents a computerized method of synthesis of flat mechanisms, which allowed creating a database of synthesized mechanisms with up to sixteen moving links and eleven degrees of freedom. Another work [9] proposes a systematic method of synthesis, which made it possible to obtain a complete set of complex planar kinematic chains with the number of independent circuits up to six and up to nineteen moving links. The work [10] gives an overview of the existing methods of structural synthesis for various complex plane kinematic chains and mechanisms, presents the latest scientific achievements, predicts trends in this field of research in the future, and forms theoretical foundations for scientists who deal with issues of intelligent design of mechanisms. In the context of the automation of the processes of design, synthesis and analysis of flat mechanisms acquire a new, relevant content of the work [11, 12], in which the authors try to determine common patterns observed in the structure of such mechanisms in order to create software for their further research in the future, in particular, kinematic, which would be unified for various structures of lever mechanisms.

In [13], the authors give an example of solving the problems of synthesis of a third-class mechanism with a complex transfer function, which includes the stop of the output link, for which four independent vector equations of the closedness of the circuit were compiled using the vector method and its kinematic analysis was performed.

Complex multi-link mechanisms are used in various branches of mechanical engineering. In work [14], a multi-link mechanism with the presence of two complex links, which is used in heavy engineering in the structure of a forging manipulator, was analyzed, for which kinematic and power studies were performed. Works [15, 16] are devoted to a detailed analysis of the mechanisms of a barreling machine used for finishing parts, another publication [17] is devoted to a kinematic study of the mechanism used in the automotive industry.

From the above, it becomes clear why the authors in their publications try to carry out research, in particular, kinematics of complex mechanical systems using modern software products such as MathWorks Matlab and Dassault Systems Solidworks [18, 19], Mathcad [20], and others.

Unsolved aspects of the problem. Kinematic research on flat complex mechanisms of the third and higher class requires taking into account their individual structural features when conducting it. For mechanisms of the third class, methods have been developed that allow them to be carried out in a grapho-analytical manner under the condition of repeated repetition of research sequences, which, of course, affects the deterioration of the accuracy of their implementation and cannot be used in the presence of several leading links in the mechanism. For mechanisms of the fourth and higher classes, there is no universal method of kinematic analysis using the grapho-analytical method, which would allow such studies to be carried out for a large number of variants of such mechanisms; therefore, in order to carry them out, it is necessary to develop a strategy for their research in each individual case, if this is possible at all for specific complex mechanisms.

The best option for the kinematic analysis of complex mechanisms of the third and higher grades is the method of mathematical modeling, for example, in the Mathcad software environment, which requires the development of a certain sequence of actions for the implementation of the study, provided there is one input link. For a mechanism of the third class with three leading cranks and a structural group of links of the third class of the fourth order, kinematic studies have features in the development of the analysis sequence and certain specifics in mathematical modeling in the Mathcad software environment.

The purpose of the article. Using the mathematical modeling method in the Mathcad software environment, perform a kinematic analysis of a flat complex twelve-link hinged mechanism with three leading links, the basis of which is a structural group of links of the third class of the fourth order with rotational kinematic pairs. To perform an analysis of the influence of the kinematic parameters of the three leading links of the third-class mechanism on the movement of the working body of the technological equipment, to check the presence of a technological stop of the link, which should provide the working body of the machine with stability for the required period of time, draw conclusions about the results of the study.

Methods. The first part of the article gives the sequence of performing the analysis of a complex mechanism of the third class with three leading links using mathematical models that describe the functions of position, velocities, and accelerations of moving driven links with the help of vector algebra in the environment of Mathcad mathematical modeling. In the second part of the article, using the *Position* function, a visualization of the kinematic diagram of the mechanism was obtained and its certainty was checked, the influence of the movement of individual input links of the mechanism on the movement of the working body of the machine was studied, the results of the kinematic parameters of the driven links of the mechanism were calculated, and their graphic dependence was obtained as a function of the position of the drivers links, the results of the analysis were analyzed and recommendations were given regarding the operating parameters of the third-class mechanism with three leading links and the ability to ensure the stop of the working body in a certain time period necessary for the technological process.

Presentation of the main material and scientific results. ***Kinematic analysis of the mechanism of the third class with three leading cranks by the method of mathematical modeling in the Mathcad software environment.*** The mechanism is placed in the vertical plane XO_1Y of the right side coordinate system

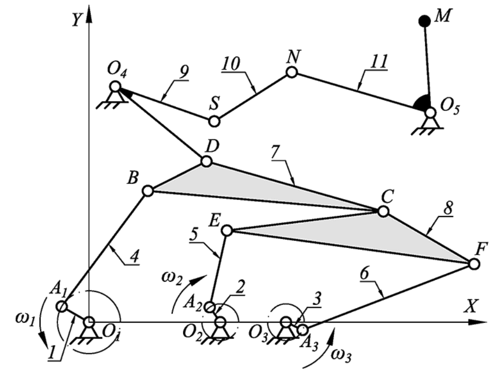

Fig. 1. Kinematic scheme of the mechanism with a structural group of links of the third class of the fourth order.

Let us denote the kinematic pairs of the mechanism: A1, A2, A3, B, C, D, E, F, M, N, and S. The radius vectors of the kinematic pairs in RSCS: P_{O1}, P_{O2}, P_{O3}, P_{O4}, P_{O5}, P_{A1}, P_{A2}, P_{A3}, P_B, P_C, P_D, P_E, P_F, P_M, P_N and P_S. Let us imagine the mechanism in the form of free vectors built on its links (Fig. 2): cranks 1 – P_{O1-A1}, 2 – P_{O2-A2}, 3 – P_{O3-A3}; connecting rods 4 – P_{A1-B}, 5 – P_{A2-E}, 6 – P_{A3-F}; 7 – P_{B-D}, P_{B-C}, P_{C-D}; 8 – P_{C-E}, P_{C-F}, P_{E-F}, 10 – P_{S-N}; rocker arm 9 – P_{O4-D}, P_{O4-S}; 11 – P_{O5-N}, P_{O5-M}. The first letter of the index in the notation of the vector indicates its beginning, the second – its end. The lengths of the specified vectors are li-j

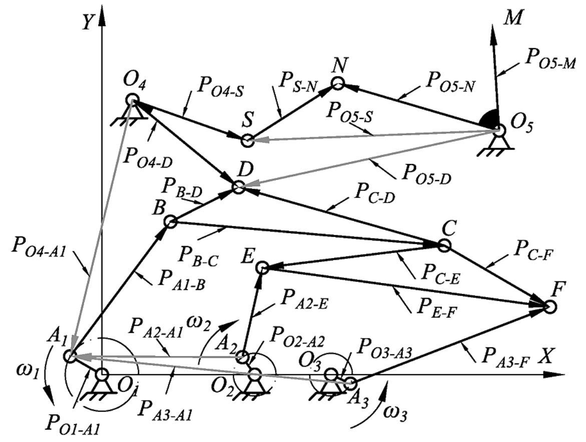

Fig. 2. Mechanism scheme in the form of free vectors

(hereinafter referred to as the RSCS) with the origin at point O_1 (Fig. 1). The mechanism has three leading links – cranks *1, 2* and *3*, the positions of which are determined by the corresponding angles f_1, f_2, and f_3, a group of links of the third class of the fourth order attached to them, which contains six links *4–9*, and a group of links of the second class of the second order , which contains two links – connecting rod *10* and rocker arm *11.* The group of links of the third class includes two complex basic links – connecting rods *7*, *8* and four leads – connecting rods *4*, *5* and *6* and rocker arm *9.* The kinematic pairs formed by the links of the mechanism are rotational kinematic pairs of the fifth class.

To perform a kinematic analysis of the mechanism, we consider the following kinematic parameters: link lengths and riser coordinates, etc., which are presented in Tables 1–4. As a generalized coordinate, we will take the rotation angle of the driving crank f_1.

In technological machines, the working bodies receive movements from the mechanisms, whose driving links are fixed on the main shaft and carry out rotational movement together with it. In the mechanism we are considering, the driving crank *1* is fixed on the main shaft. The laws of motion of driving cranks *2* and *3* depend on the angle of rotation of the main shaft f_1, that is, $f_2 = f(f_1)$; $f_3 = f(f_1)$.

The discrete variable that describes the main shaft movement is

$$f_1 := f_0, \quad f_0 + \Delta f_1 \cdot f_{\max}. \tag{1}$$

The variable that describes the movement of the driving crank *1*, taking into account its position on the main shaft, is

$$f_{M1}(f_1) := f_1 + f_{M1-0}\,. \tag{2}$$

The variable that describes the movement of the driving crank *2* depending on the angle f_1 is

$$f_2(f_1) := f_{M1}(f_1 - f_{M1-0}) \cdot \frac{Rot_2}{u_1} + f_{M2-0}. \tag{3}$$

The variable that describes the movement of the driving crank *3* depending on the angle f_1 is

$$f_3(f_1) := f_{M1}(f_1 - f_{M1-0}) \cdot \frac{Rot_3}{u_2} + f_{M3-0}. \tag{4}$$

In the expressions (1–4), the following notations are used: f_0 – the initial angle of the main shaft; Δf_1 – calculation step; $f_{\max}$ – final angle of the main shaft; f_{M1-0}, f_{M2-0} and f_{M3-0} – respectively, the initial angles of setting of the driving cranks *1*, *2* and *3*; Rot_1, Rot_2 and Rot_3 are, respectively, variables that determine the direction of rotation of the driving cranks *1*, *2* and *3* (the value "–1" corresponds to the direction of rotation against the clockwise direction, and "+1" to the clockwise); u_1, u_2 are, respectively, the variables that determine the transmission ratios in the case of using a gear for driving cranks *2* and *3.*

The structural group of links of the third class of the fourth order, which is used in the mechanism we are considering, can have a maximum of eighteen assembly options in the general case.

Solving the problem of determining the positions of the links of the mechanism is reduced to the determination of the valid points of intersection of the connecting rod curves of two four-links, which are obtained from the group by opening the central kinematic pair *C* and changing the conventionally generalized coordinates – the angles of its leads *4* and *9*, or *5* and *6.* The number of solutions to the problem of determining the positions of the members of the group corresponds to the number of options for its composition.

Since the connecting rod curve in the general case has a high order, it is possible to obtain the solution of the problem approximately by numerical methods.

To solve the problem of determining the positions of the mechanism links, we will select three vector contours

Table 1

Geometrical dimensions of the mechanism links

Parameter Marking	$l_{O1\text{-}A1}$	$l_{A1\text{-}B}$	$l_{B\text{-}D}$	$l_{C\text{-}D}$	$l_{O2\text{-}A2}$	$l_{A2\text{-}E}$	$l_{E\text{-}F}$	$l_{C\text{-}E}$
Parameter value, mm	12	55	25	70	7	30	95	60
Parameter Marking	$l_{C\text{-}F}$	$l_{O3\text{-}A3}$	$l_{A3\text{-}F}$	$l_{O4\text{-}D}$	$l_{O4\text{-}S}$	$l_{S\text{-}N}$	$l_{O5\text{-}N}$	$l_{O5\text{-}M}$
Parameter value, mm	40	7	70	45	40	35	55	35

Table 2

Values of angular parameters

Parameter Marking	$f_{M1\text{-}0} = \angle XO_1A_1$	$f_{M2\text{-}0} = \angle XO_2A_2$	$f_{M3\text{-}0} = \angle XO_3A_3$	$\alpha_3 = \angle DO_4S$	$\alpha_4 = \angle NO_5M$
Parameter value, degrees	150	125	–25	20	–70

Table 3

Coordinates of fixed kinematic pairs of the mechanism

Parameter Marking	O_1		O_2		O_3		O_4		O_5	
Coordinate making	*X*	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
Parameter value, mm	0	0	50	0	75	0	10	90	130	80

Table 4

The values of the angular velocities of the leading links of the mechanism

Parameter Marking	ω_1	ω_2	ω_3
Parameter value, s^{-1}	100	–200	300

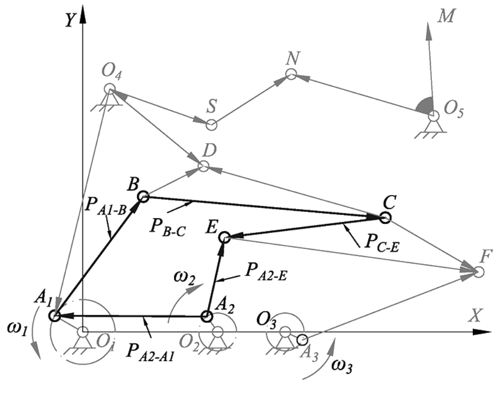

Fig. 3. Scheme of vector circuit 1

1) $P_{A2-A1} \to P_{A1-B} \to P_{B-C} \to P_{C-E} \leftarrow P_{A2-E}$ (Fig. 3);
2) $P_{A3-A1} \to P_{A1-B} \to P_{B-C} \to P_{C-F} \leftarrow P_{A3-F}$ (Fig. 4);
3) $P_{O4-A1} \to P_{A1-B} \to P_{B-D} \leftarrow P_{O4-D}$ (Fig. 5).

We compose a system of three vector equations of the closure of vector contours (5, 6 and 7) with six unknown angles f_{A1-B}, f_{B-C}, f_{C-E}, f_{A2-E}, f_{A3-F}, f_{O4-D}, which characterize provisions of the relevant sections 4, 7, 8, 5, 6, 9

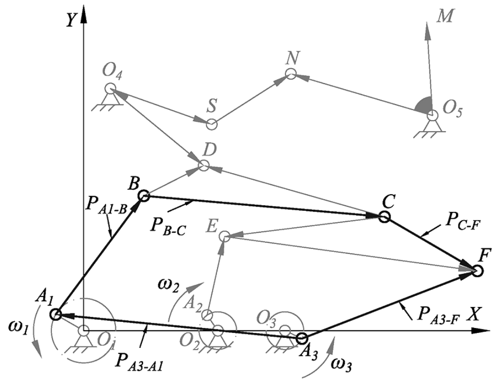

Fig. 4. Scheme of vector circuit 2

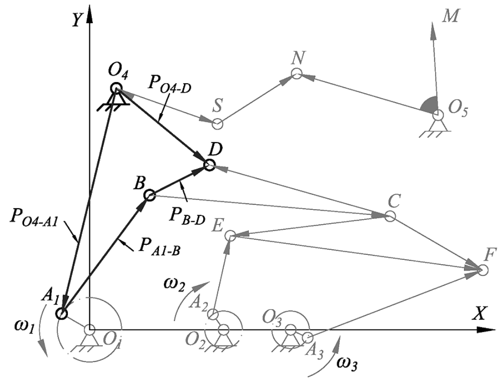

Fig. 5. Scheme of vector circuit 3

$$P_{A2-A1}(f_1)+\begin{bmatrix} l_{A1-B}\cdot\cos(f_{A1-B}) \\ l_{A1-B}\cdot\sin(f_{A1-B}) \\ 0 \end{bmatrix}+\begin{bmatrix} l_{B-C}\cdot\cos(f_{B-C}) \\ l_{B-C}\cdot\sin(f_{B-C}) \\ 0 \end{bmatrix}+\begin{bmatrix} l_{C-E}\cdot\cos(f_{C-E}) \\ l_{C-E}\cdot\sin(f_{C-E}) \\ 0 \end{bmatrix}=\begin{bmatrix} l_{A2-E}\cdot\cos(f_{A2-E}) \\ l_{A2-E}\cdot\sin(f_{A2-E}) \\ 0 \end{bmatrix}; \quad (5)$$

$$P_{A3-A1}(f_1)+\begin{bmatrix} l_{A1-B}\cdot\cos(f_{A1-B}) \\ l_{A1-B}\cdot\sin(f_{A1-B}) \\ 0 \end{bmatrix}+\begin{bmatrix} l_{B-C}\cdot\cos(f_{B-C}) \\ l_{B-C}\cdot\sin(f_{B-C}) \\ 0 \end{bmatrix}+\begin{bmatrix} l_{C-F}\cdot\cos(f_{C-E}+\alpha_1) \\ l_{C-F}\cdot\sin(f_{C-E}+\alpha_1) \\ 0 \end{bmatrix}=\begin{bmatrix} l_{A3-F}\cdot\cos(f_{A3-F}) \\ l_{A3-F}\cdot\sin(f_{A3-F}) \\ 0 \end{bmatrix}; \quad (6)$$

$$P_{O4-A1}(f_1)+\begin{bmatrix} l_{A1-B}\cdot\cos(f_{A1-B}) \\ l_{A1-B}\cdot\sin(f_{A1-B}) \\ 0 \end{bmatrix}+\begin{bmatrix} l_{B-D}\cdot\cos(f_{B-C}+\alpha_2) \\ l_{B-D}\cdot\sin(f_{B-C}+\alpha_2) \\ 0 \end{bmatrix}=\begin{bmatrix} l_{O4-D}\cdot\cos(f_{O4-D}) \\ l_{O4-D}\cdot\sin(f_{O4-D}) \\ 0 \end{bmatrix}, \quad (7)$$

where in expressions (5–7) we have

$$P_{A2-A1}(f_1):=\begin{bmatrix} P_{O1X}+l_{O1-A1}\cdot\cos(f_{M1}(f_1))-P_{O2X}-l_{O2-A2}\cdot\cos(f_2(f_1)) \\ P_{O1Y}+l_{O1-A1}\cdot\sin(f_{M1}(f_1))-P_{O2Y}-l_{O2-A2}\cdot\sin(f_2(f_1)) \\ 0 \end{bmatrix};$$

$$P_{A3-A1}(f_1)\cdot\begin{bmatrix} P_{O1X}+l_{O1-A1}\cdot\cos(f_{M1}(f_1))-P_{O3X}-l_{O3-A3}\cdot\cos(f_3(f_1)) \\ P_{O1Y}+l_{O1-A1}\cdot\sin(f_{M1}(f_1))-P_{O3Y}-l_{O3-A3}\cdot\sin(f_3(f_1)) \\ 0 \end{bmatrix};$$

$$P_{O4-A1}(f_1)\cdot\begin{bmatrix} P_{O1X}+l_{O1-A1}\cdot\cos(f_{M1}(f_1))-P_{O4X} \\ P_{O1Y}+l_{O1-A1}\cdot\sin(f_{M1}(f_1))-P_{O4Y} \\ 0 \end{bmatrix};$$

$$\alpha_1:=\arccos\left(\frac{l_{C-E}^2+l_{C-F}^2-l_{E-F}^2}{2\cdot l_{C-E}\cdot l_{C-F}}\right);$$

$$\alpha_2:=\arccos\left(\frac{l_{B-C}^2+l_{B-D}^2-l_{C-D}^2}{2\cdot l_{B-C}\cdot l_{B-D}}\right).$$

Solving the system of vector equations (5, 6 and 7) is carried out using the numerical method in Mathcad. For this purpose, we set the initial values of six unknown angles f_{A1-B}, f_{B-C}, f_{C-E}, f_{A2-E}, f_{A3-F}, f_{O4-D}, in the locality of which Mathcad performs a numerical solution. From the choice of the initial values of the angles of the position of the links, the structurally correct version of the assembly of the mechanism depends on the eighteen possible versions of the assembly of its structural group of the third class of the fourth order. For the mechanism under investigation, we will accept the following initial values of the angles of the links in accordance with its version of assembly: $f_{A1-B-0} = 80°$, $f_{B-C-0} = -10°$, $f_{C-E-0} = 190°$, $f_{A2-E-0} = 80°$, $f_{A3-F-0} = 30°$, $f_{O4-D-0} = -40°$.

It is convenient to represent the solutions of the system of equations (5), (6), and (7) in the form of a solution vector

$$\begin{bmatrix} f_{A1-B}(f_1) \\ f_{B-C}(f_1) \\ f_{C-E}(f_1) \\ f_{A2-E}(f_1) \\ f_{A3-F}(f_1) \\ f_{O4-D}(f_1) \end{bmatrix} := Find(f_{A1-B-0}, f_{B-C-0}, f_{C-E-0}, f_{A2-E-0}, f_{A3-F-0}, f_{O4-D-0}). \tag{8}$$

In order to check the option of assembly of the mechanism, we carry out circuit-technical modeling of the mechanism with the construction of a visualization graph of its kinematic scheme.

In circuit modeling, we define vector-links $P_{A1\text{-}B}(f_1)$, $P_{B\text{-}C}(f_1)$, $P_{C\text{-}E}(f_1)$, $P_{A2\text{-}E}(f_1)$, $P_{A3\text{-}F}(f_1)$, and $P_{O4\text{-}D}(f_1)$ in the function of the crank angle f_1 according to the expression calculated (8) by the angles of the position of the links included in the structural group of the third class of the fourth order

$$P_{A1-B}(f_1) := T_Z(f_{A1-B}(f_1)) \cdot (l_{A1-B} \quad 0 \quad 0)^T;$$

$$P_{B-C}(f_1) := T_Z(f_{B-C}(f_1)) \cdot (l_{B-C} \quad 0 \quad 0)^T;$$

$$P_{C-E}(f_1) := T_Z(f_{C-E}(f_1)) \cdot (l_{C-E} \quad 0 \quad 0)^T;$$

$$P_{A2-E}(f_1) := T_Z(f_{A2-E}(f_1)) \cdot (l_{A2-E} \quad 0 \quad 0)^T;$$

$$P_{A3-F}(f_1) := T_Z(f_{A3-F}(f_1)) \cdot (l_{A3-F} \quad 0 \quad 0)^T;$$

$$P_{O4-D}(f_1) := T_Z(f_{O4-D}(f_1)) \cdot (l_{O4-D} \quad 0 \quad 0)^T,$$

where $T_Z(f_i) := \begin{bmatrix} \cos(f_i) & -\sin(f_i) & 0 \\ \sin(f_i) & \cos(f_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is rotation matrix of the vector in the plane XO_1Y, the argument of which is the angle of rotation f_i vector.

We define the radius vectors of the kinematic pairs of the structural group of the third class of the fourth order $P_{A1}(f_1)$, $P_{A2}(f_1)$, $P_{A3}(f_1)$, $P_B(f_1)$, $P_C(f_1)$, $P_D(f_1)$, $P_E(f_1)$ and $P_F(f_1)$ in the function of the crank angle f_1 in the accepted PSC

$$P_{A1}(f_1) := [P_{O1X} + l_{O1-A1} \cdot \cos(f_{M1}(f_1)) \quad P_{O1Y} + l_{O1-A1} \cdot \sin(f_{M1}(f_1)) \quad 0]^T; \tag{9}$$

$$P_{A2}(f_1) := [P_{O2X} + l_{O2-A1} \cdot \cos(f_2(f_1)) \quad P_{O2Y} + l_{O2-A2} \cdot \sin(f_2(f_1)) \quad 0]^T; \tag{10}$$

$$P_{A3}(f_1) := [P_{O3X} + l_{O3-A1} \cdot \cos(f_3(f_1)) \quad P_{O3Y} + l_{O3-A2} \cdot \sin(f_3(f_1)) \quad 0]^T; \tag{11}$$

$$P_B(f_1) := P_{A1}(f_1) + P_{A1-B}(f_1); \tag{12}$$

$$P_C(f_1) := P_B(f_1) + P_{B-C}(f_1); \tag{13}$$

$$P_D(f_1) := P_{O4}(f_1) + P_{O4-D}(f_1); \tag{14}$$

$$P_E(f_1) := P_C(f_1) + P_{C-E}(f_1); \tag{15}$$

$$P_F(f_1) := P_{A3}(f_1) + P_{A3-F}(f_1). \tag{16}$$

For attaching to the structural group of the links of the third class of the fourth order of the group of the second class of the second order of the first type it will be determined the free vectors of the links $P_{O4\text{-}S}(f_1)$, $P_{S\text{-}N}(f_1)$, $P_{O5\text{-}N}(f_1)$, the vector of the working point of the mechanism $P_{O5\text{-}M}(f_1)$ and the radius vectors of the characteristic points $P_S(f_1)$, $P_N(f_1)$, working point $P_M(f_1)$ as a function of the angle of rotation of the crank f_1.

$$P_{O4-S}(f_1) := T_Z(\alpha_3) \cdot \frac{P_{O4-D}(f_1)}{l_{O4-D}} \cdot l_{O4-S};$$

$$P_S(f_1) := P_{O4} + P_{O4-S}(f_1); \tag{17}$$

$$P_{O5-N}(f_1) := T_Z\left(\arccos\left(\frac{\left(\left|P_S(f_1) - P_{O5}\right|\right)^2 + l_{O5-N}^2 - l_{S-N}^2}{2 \cdot \left|P_S(f_1) - P_{O5}\right| \cdot l_{O5-N}}\right)\right) \times$$

$$\times \frac{P_S(f_1) - P_{O5}}{\left|P_S(f_1) - P_{O5}\right|} \cdot l_{O5-N};$$

$$P_{\mathrm{N}}(f_1) := P_{O5} + P_{O5-S}(f_1); \tag{18}$$

$$P_{S-N}(f_1) := P_N(f_1) - P_S(f_1);$$

$$P_{O5-M}(f_1) := T_Z(\alpha_4) \cdot \frac{P_{O5-N}(f_1)}{l_{O5-N}} \cdot l_{O5-M};$$

$$P_M(f_1) := P_{O5} + P_{O5-M}(f_1). \tag{19}$$

We build the graph of visualization of the kinematic scheme of the mechanism using the radius vectors determined from expressions (9–19). At the same time, we use function (20) to represent free vectors on 2D graphics in Mathcad

$$Link(RV_1, RV_2, K) := [RV_{1K} \quad RV_{1K}]^T, \tag{20}$$

where RV_1 and RV_2 are the radius vectors, respectively of the beginning and end of the vector f, which depicts a link on the visualization graph of the kinematic diagram of the mechanism in Mathcad.

We build the graph of the mechanism positions using the Position function (Fig. 6).

The function argumentation *Position* is: P – the vector of radius vectors of the characteristic points of the mechanism; $f_{\min}$ – the angle of initial crank position; n – the number of positions of the mechanism (the number of calculation steps); *Rotation* – a variable that determines the direction of crank rotation; K – a variable that defines the axis of the radius vector projection of the characteristic point of the mechanism.

Results of a kinematic study of a third-class mechanism with three leading cranks and their analysis. With the use of function (20) and the *Position* function, a visualization graph of the kinematic scheme of the mechanism (Fig. 7) is built, combined with its 12 positions and trajectories of characteristic points, by which it is possible to monitor the correct version of the assembly of the mechanism and the absence of a branching defect of the kinematic scheme during the cycle of operation.

The vectors of the angular velocities of the links are defined in general by the expression

$$\omega_i(f_1) := f'(P_{i-j}(f_1), r'(P_{i-j}, f_1, \Delta\alpha)) \cdot \omega_1, \tag{21}$$

where $f'(r, r') = \dfrac{r \times r'}{(|r|)^2}$ is a function for determining the analogue of the angular velocity of the link;

```
Position(P, f_min, n, Rotation, K) := | j ← 1
                                      | i ← 1
                                      | k ← rows(P)
                                      | for i ∈ 1..k
                                      |   | for j ∈ 1..n + 1
                                      |   |   | P1 ← P_i
                                      |   |   | M_i,j ← P1[f_min + (Rotation·360°/n)·(j − 1)]_K
                                      |   |   | j ← j + 1
                                      |   | i ← i + 1
                                      | M
```

Fig. 6. Position Function

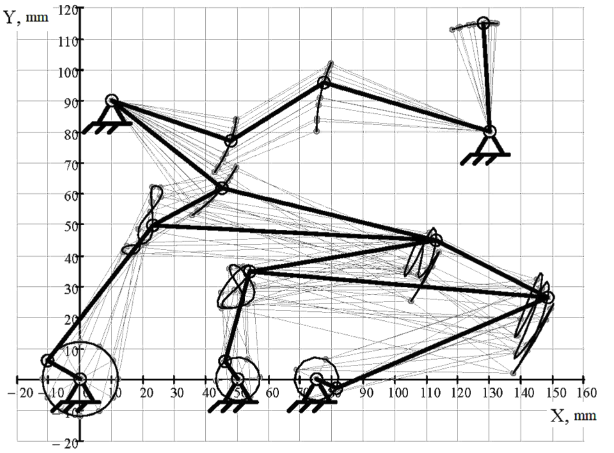

Fig. 7. Visualization graph of the mechanism's kinematic scheme

$$r'(r,\alpha,\Delta\alpha) := \begin{bmatrix} \dfrac{r(\alpha+\Delta\alpha)_X - r(\alpha)_X}{\Delta\alpha} \\ \dfrac{r(\alpha+\Delta\alpha)_Y - r(\alpha)_Y}{\Delta\alpha} \\ \dfrac{r(\alpha+\Delta\alpha)_Z - r(\alpha)_Z}{\Delta\alpha} \end{bmatrix}$$ – a function for numerical calculation of the first derivative in Mathcad.

The vectors of angular accelerations are defined in general by the expression

$$\varepsilon_i(f_1) := f''(P_{i-j}(f_1), r''(P_{i-j}, f_1, \Delta\alpha)) \cdot \omega_1^2 + r'(P_{i-j}, f_1, \Delta\alpha) \cdot \varepsilon_1, \quad (22)$$

where $f''(r,r'') := \dfrac{r \times r''}{(|r|)^2}$ is a function for determining the analogue of angular acceleration of the link;

$$r''(r,\alpha,\Delta\alpha) = \begin{bmatrix} \dfrac{r(\alpha+\Delta\alpha)_X - 2r(\alpha)_X + r(\alpha-\Delta\alpha)_X}{\Delta\alpha} \\ \dfrac{r(\alpha+\Delta\alpha)_Y - 2r(\alpha)_Y + r(\alpha-\Delta\alpha)_Y}{\Delta\alpha} \\ \dfrac{r(\alpha+\Delta\alpha)_Z - 2r(\alpha)_Z + r(\alpha-\Delta\alpha)_Z}{\Delta\alpha} \end{bmatrix}$$ – a function for numerical calculation of the second derivative in Mathcad.

With the use of function (8), the values of the functions of the angles of rotation of the links were obtained and their graphs were constructed (Fig. 8). When constructing a graph of the angular movements of the links, the values of the angles were determined from the initial positions of the links, which they occupy at the angle of the position of the driving crank $f_{M1\text{-}0}$. The graphs show angles $\varphi_4 = f_{A1-B}$, $\varphi_5 = f_{A2-E}$, $\varphi_6 = f_{A3-F}$, $\varphi_7 = f_{B-C}$, $\varphi_8 = f_{C-E}$, $\varphi_9 = f_{O4-D}$, $\varphi_{10} = f_{S-N}$, $\varphi_{11} = f_{O5-N}$.

With the use of function (21), the values of the angular velocity vectors of the links were obtained and their graphs were constructed (Fig. 9).

By the use of function (22), the values of the vectors of the angular accelerations of the links are obtained and their graphs are constructed (Fig. 10).

The results of the calculation of the angular velocities and accelerations of the links for one of the positions of the mechanism at the crank rotation angles $f_1 = 150°$; $f_2 = 125°$; $f_3 = -25°$, corresponding to their positions in fig. 1 and under the condition of their uniform rotation with the corresponding angular speeds $\omega_1 = 100\ s^{-1}$; $\omega_2 = -200\ s^{-1}$; $\omega_3 = 300\ s^{-1}$ are presented in Table 5.

Consider a large-scale image of the kinematic diagram of the third-class mechanism (Fig. 7). At point M (Fig. 1) of the rocker arm 11, there is a working body, which technologically must stop for a certain time during the cycle of its working

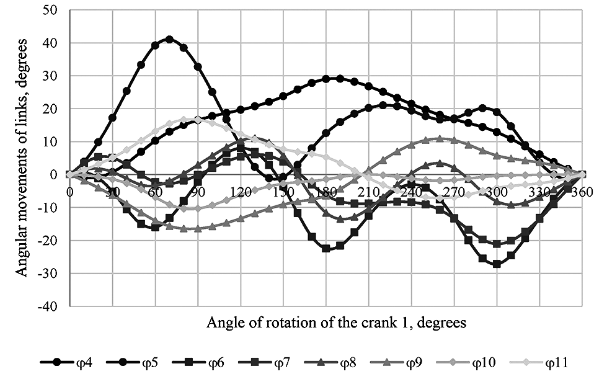

Fig. 8. Graphic dependences of the rotation angles of the mechanism links on the rotation angle of crank 1

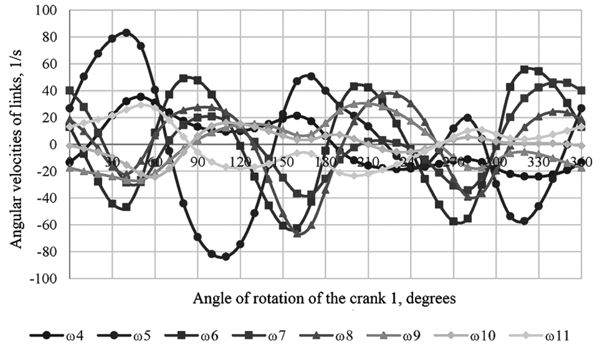

Fig. 9. Graphical dependences of the angular velocities of the mechanism links on the angle of rotation of crank 1

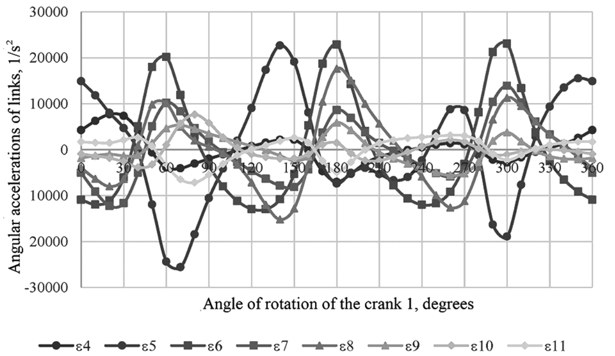

Fig. 10. Graphical dependences of angular accelerations of the mechanism links on the angle of rotation of crank 1

Table 5

The results of the kinematic study of the mechanism

Link number	Angular speed parameter designation	The value of angular velocity, s^{-1}	Angular acceleration parameter designation	The amount of angular acceleration, s^{-2}
4	ω_4	71.6	ε_4	4,006.9
5	ω_5	26.1	ε_5	13,876.3
6	ω_6	11.1	ε_6	–10,476.9
7	ω_7	38.7	ε_7	–4,525.0
8	ω_8	17.6	ε_8	–2,834.8
9	ω_9	–16.9	ε_9	–1,768.1
10	ω_{10}	–1.4	ε_{10}	–922.9
11	ω_{11}	12.8	ε_{11}	1,659.9

movement, which will contribute to the improvement of the conditions for the execution of the technological process. According to the cyclogram of the machine operation, the stop of the working body should take place during the time period corresponding to the rotation of the crank *1* (Fig. 1) by angles from 145 to 180 degrees, which are measured from the horizontal coordinate axis O_1X opposite clockwise direction.

According to the results of the research given in Table 5 and Fig. 11, it is seen that the absolute suspension of the rocker arm *11* and, accordingly, the working body of the mechanism is not observed for the period of time required by the technological requirements, and its angular velocity during this time changes and has a minimum value that corresponds to the rotation angle of the crank *1* of approximately 165 degrees. From the structural diagram (Fig. 1), it is seen that the movement of the rocker arm *11* is determined by the connecting rod *7*, which has the appearance of a complex link. Movement of its two kinematic pairs *B* and *C*, whose trajectories have the form of closed curves of the second order (Fig. 7) that should ensure the suspension of the third pair *D* and lead to the suspension of point *M* in the required time period. In turn, the movement of kinematic pairs *B* and *C* is the result of the movement of connecting rods *4* and *8*, which are driven on one side by crank *1*, and on the other – by cranks *2, 3* using, respectively, connecting rods *5* and *6*.

It became necessary to model and analyze the influence of the movement of each of the three driving cranks on the possible suspension of the rocker arm *11* together with the machine's working body. The results of such studies are shown in Fig. 11, where graphical dependences of the movement of point *M* depending on the position of the crank *1* are presented under the condition that the values of the angular velocities of some driving cranks will change while the values of the angular velocities of the others remain unchanged.

Angular velocities of cranks *2* and *3* varied modulo from 100 to 300 s^{-1} (with the step100 s^{-1}) under the condition of constant angular velocity of crank *1* (100 s^{-1}), while the "negative" sign corresponds to the direction of movement crank, which coincides with the direction of clockwise movement.

From the analysis of the obtained study results, we conclude that that the best of the options for the conditions of the idle working body of the technological machine, which corresponds to the rotation of crank 1 to an angle of 145 to 180 degrees for a mechanism with a structural group of links of the third class of the fourth order and three leading cranks 1–3 is observed if they have, respectively, the following angular velocities: 100, –300, 300 s^{-1}. The obtained results do not coincide with those used on this type of equipment; therefore, it is necessary to improve the drive of the technological equipment, which would allow to increase the angular speed of the crank *2* of the mechanism by 33 %.

Conclusions. A kinematic analysis was performed for a flat complex mechanism with a structural group of links of the third class of the fourth order, which is used in this machine under the condition that the working body is stationary to ensure the process. With the help of mathematical models describing the functions of the position, velocities and accelerations of moving driven links and vector algebra methods in the environment of Mathcad mathematical modeling, the values of the kinematic parameters of all links of the third-class mechanism with three leading cranks were obtained in numerical and graphical forms.

From the analysis of the obtained research results for the cycle of the mechanism, the reason for the incomplete stop of the link was found, which ensures the working body of the mechanism to stand still for the period of time required by technological requirements. In order to improve the technological conditions of the interaction of the working bodies of the technological machine and increase the reliability of its operation, recommendations were made for the improvement of the drive of the technological equipment, which would allow increasing the angular speed of the second crank of the mechanism by 33 %, which would make it possible to create optimal conditions of the technological working body of the machine, which must take place during the period of time of turning the first crank to an angle of 145 to 180 degrees.

References.

1. Morlin, F. V., Carboni, A. P., & Marti, D. (2023). Synthesis of Assur groups via group and matroid theory. *Mechanism and Machine Theory*, *184*. https://doi.org/10.1016/j.mechmachtheory.2023.105279.

2. Diwan, N. (2020). Kinematic analysis of a new designed eight-link riveter mechanism. *International Journal of Advanced Research in Engineering and Technolog*, *11*(9), 380-386. https://doi.org/10.34218/IJARET.11.9.2020.039.

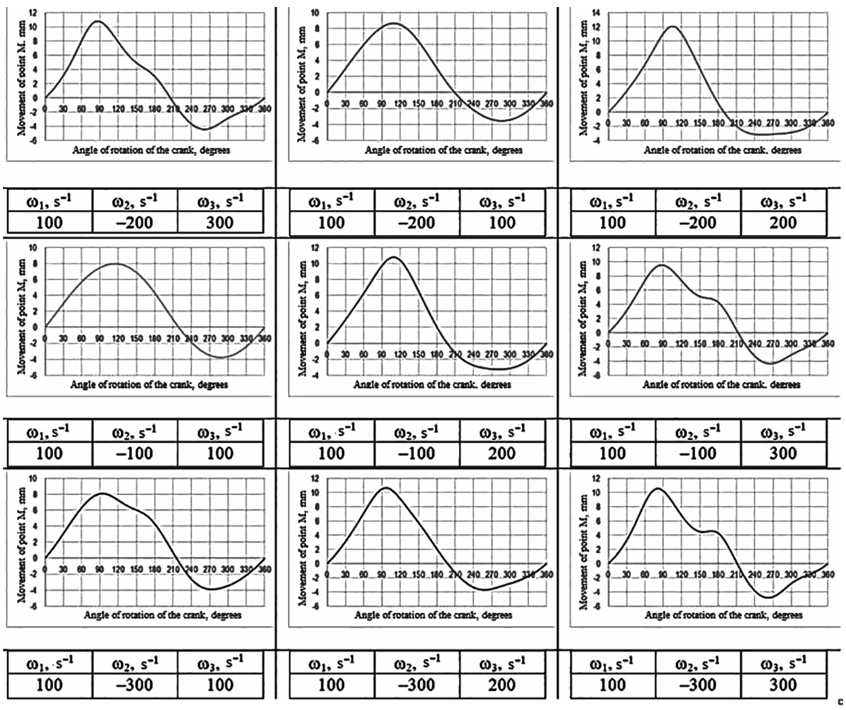

Fig. 11. Graphs of the movement of the working body of the mechanism depending on the angular speeds of the driving cranks

3. Kemper, D., Fimbinger, E., Antretter, T., Egger, M., & Flachberger, H. (2023). The dynamics of an impact swing mechanism as an analytical mathematical model. *Results in Engineering, 21*, 101694. https://doi.org/10.1016/j.rineng.2023.101694.
4. Roussev, R., & Paleva-Kadiyska, B. (2015). Determination of the kinematic features of the feed dog of mechanisms for transportation of material of the sewing machines. *Journal of Textiles and clothing, 3*, 58-63.
5. Saxena, T., & Singh, V. P. (2017). Structural Analysis of Twelve Link Kinematic Chain using Graph Theory. *International Journal for Scientific Research & Development, 5*(10). ISSN (online): 2321-0613.
6. Jia, G., Li, B., Huang, H., & Zhang, D. (2020). Type synthesis of metamorphic mechanisms with scissor-like linkage based on different kinds of connecting pairs. *Mechanism and Machine Theory, 151*, 103848. https://doi.org/10.1016/j.mechmachtheory.2020.103848.
7. Duta, A., Popescu, I., Cretu, S.-M., Corzanu, A., Corzanu, V., & Popa, D.-L. (2022). The Manufacture of Curved Crease Surfaces Starting from Kinematic Analysis of Planar Mechanisms. *Processes, 10*, 2344. https://doi.org/10.3390/pr10112344.
8. Huang, P., Liu, T., Ding, H., & Zhao, Y. (2022). Computer-aided synthesis of planar mechanisms with one multiple-joint. *Mechanism and Machine Theory*, *177*, 105044. https://doi.org/10.1016/j.mechmachtheory.2022.105044.
9. Ding, H., Huang, P., Yang, W., & Kecskeméthy, A. (2016). Automatic generation of the complete set of planar kinematic chains with up to six independent loops and up to 19 links. *Mechanism and Machine Theory*, 96(part 1), 75-93. https://doi.org/10.1016/j.mechmachtheory.2015.09.00.
10. Yanga, W., Dinga, H., & Kecskeméthy, A. (2022). Structural synthesis towards intelligent design of plane mechanisms: Current status and future research trend. *Mechanism and Machine Theory*, *171*, 104715. https://doi.org/10.1016/j.mechmachtheory.2021.104715.
11. Han, J., & Shi, S. (2020). A novel methodology for determining the singularities of planar linkages based on Assur groups. *Mechanism and Machine Theory*, *147*, 103751. https://doi.org/10.1016/j.mechmachtheory.2019.103751.
12. Petrescu, R.V.V., Aversa, R., Akash, B., Abu-Lebdeh, T., Apicella, A., Ion, F., & Petrescu, T. (2018). Some Aspects of the Structure of Planar Mechanisms. *American Journal of Engineering and Applied Sciences*, *11*(1), 245.259. https://doi.org/10.3844/ajeassp.2018.245.259.
13. Zhauyt, A., Zhahanova, I., Smailova, G., Murzakhmetova, U., Kurmangalieva, L., & Kurenbay, A. (2017). The kinematic analysis of the third class mechanism. *Vibroengineering Procedia*, *12*, 208-212. https://doi.org/10.21595/vp.2017.18678.
14. Aversa, R., Petrescu, R.V.V., Akash, B., Bucinell, R., Corchado, J., Chen, G., Li, S., ..., & Petrescu, T. (2017). Kinematics and Forces to a New Model Forging Manipulator. *American Journal of Applied Sciences*, *14*(1), 60-80. https://doi.org/10.3844/ajassp.2017.60.80.
15. Zalyubovs'kyi, M.G., Panasyuk, I.V., Koshel', S.O., & Koshel', G.V. (2021). Synthesis and analysis of redundant-free seven-link spatial mechanisms of part processing machine. *International Applied Mechanics*, *57*(4), 466-476. https://doi.org/10.1007/s10778-021-01098-y.
16. Zalyubovsky, M.G., Panasyuk, I.V., Koshel', S.O., & Lychov, D.O. (2022). Design Parameters of the Four-Link Hinged Mechanism of Barreling Machine Drive. *International Applied Mechanics*, *58*(6), 725-731. https://doi.org/10.1007/s10778-023-01196-z.
17. Pramanika, S., & Thipsec, S.S. (2020). Kinematic synthesis of central-lever steering mechanism for four wheel vehicles. *Acta Polytechnica*, *60*(3), 252-258. https://doi.org/10.14311/AP.2020.60.0252.
18. Teutan, E., Stan, S., Oarcea, A., & Cobilean, V. (2022). Kinematic Analysis of the Cyclo-Variator Transmission Mechanism. *Mechanika*, *28*(2), 139-144. ISSN 1392-1207.
19. Bogdanof, G.C., Moise,V., Visan, A.L., & Ciobanu, G.V. (2017). Kinematic analysis of soil drilling mechanism used in afforestation. *Latvia University of Agriculture, 131*, 653-658. https://doi.org/10.22616/ERDev2017.16.N131.
20. Koshel', S.O., Dvorzhak, V.M., Koshel', G.V., & Zalyubovskyi, M.G. (2022). Kinematic Analysis of Complex Planar Mechanisms of Higher Classes. *International Applied Mechanics*, *58*(1), 111-122. https://doi.org/10.1007/s10778-022-01138-1.

Аналіз механізму третього класу методом моделювання у програмному середовищі Mathcad

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Мета. Виконати кінематичний аналіз складного плоского дванадцятиланкового шарнірного механізму із трьома ведучими ланками, основою якого є структурна група ланок третього класу четвертого порядку з обертальними кінематичними парами, що застосовується в технологічному обладнанні.

Методика. Кінематичне дослідження плоского складного механізму третього класу із трьома ведучими ланками виконано з використанням методу математичного моделювання у програмному середовищі Mathcad.

Результати. За допомогою методу математичного моделювання механізм було представлено у вигляді вільних векторів, що побудовані на його ланках, для яких, з урахуванням наявності трьох ведучих ланок, було виділено три векторні контури, що дозволило скласти системи векторних рівнянь їх замкнутості з подальшим розв'язуванням чисельним методом у програмному середовищі Mathcad. Отримані функції кутів повороту ланок механізму, їх кутових швидкостей і прискорень в аналітичному та графічному вигляді в залежності від кута повороту першого ведучого кривошипа механізму, виконано розрахунок кутових швидкостей і прискорень усіх ведених ланок механізму.

Наукова новизна. Розроблена послідовність і виконане кінематичне дослідження складного плоского механізму третього класу із трьома ведучими ланками, основою якого є структурна група ланок третього класу четвертого порядку. Виконане схемотехнічне моделювання механізму із трьома ведучими ланками, побудовано графік візуалізації його кінематичної схеми та отримано дійсний її варіант складання із багатьох можливих, для якого визначені величини кінематичних параметрів усіх його ланок.

Практична значимість. Проведено аналіз впливу кінематичних параметрів трьох ведучих ланок механізму третього класу на рух робочого органу технологічного обладнання. З аналізу отриманих результатів дослідження за цикл роботи механізму виявлена причина не повної технологічної зупинки ланки, що повинна забезпечувати робочому органу машини вистій у необхідний за вимогами період часу. Надані рекомендації щодо усунення виявленого недоліку та пропозиції про необхідність проведення удосконалення приводу даного обладнання.

Ключові слова: *механізм третього класу, кінематичний аналіз механізму, комп'ютерне моделювання Mathcad, кінематичне дослідження*

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