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## DEFINING THE LIMITS OF APPLICATION AND THE VALUES OF INTEGRATION VARIABLES FOR THE EQUATIONS OF TRAIN MOVEMENT

Railway transportation is an integral part in the transport infrastructure of our country. They cover passenger and cargo transportations by Ukrzaliznytsia, industrial enterprises, including transportation of the mining sector, which is characterized by heavy loads on the traction rolling stock due to large gradients of the track profile. Railway transport management is always preceded by traction calculations, the center of which is to solve the equation of train movement.

**Purpose.** To determine the rational values of the variables in solving the equation of train movement, as well as relevant limits in their applicability.

**Methodology.** To achieve the purpose, methods of system analysis, nonlinear programming, numerical methods for solving differential equations, namely the classical, Runge-Kutta-Feelberg, and Rosenbrock methods, are used. Computational accuracy was verified using simulation methods and compared with experimental data.

**Findings.** The results of the research involve increasing the calculating speed when solving the equation of train movement without loss of accuracy, which allowed using the proposed method in on-board systems of locomotives.

**Originality.** During the research, new scientifically grounded results were obtained that solve the scientific task in improving the energy efficiency of train operation, and are of great importance for railway transport. The obtained results constitute the originality, which consists in determining the rational limits of applicability and the value in a step of integration variables for the equations of the train movement.

**Practical value.** The research results allow reducing the cost of energy consumed by hauling operations due to the prompt recalculation of rational control modes when changing the train situation.

**Keywords:** *railway transportation, hauling operations, equation, integration, step, accuracy*

**Introduction.** Increasing the level of automation is one of the directions in the railway transport development, concerning various components of the railway automation systems both in our country and abroad [1]. At the same time, the vital task is improvement of on-board locomotive control systems in order to increase the safety of traffic, effectiveness of the locomotive control in general, enhancing the working conditions for the locomotive brigade, etc.

The tasks that involve calculating the parameters of the bodies motion, in most cases, are reduced to the integration of differential equations. The role of numerical methods for solving differential equations in such engineering and scientific problems is not only necessary, but also obligatory.

Solution of set tasks in accordance with all rules of mathematics, physics and mechanics in some cases may not correspond to the safety of transportation, comfort of passenger transportation, criteria of energy efficiency, etc. Obviously, the problem of operational reliability and optimality of transportation is also common to all methods of calculating the train movement and should be solved in traction calculations. The specified requirements and approaches facilitate the solution of the task and are fundamental to any of the listed methods for calculating the train movement.

If among several forces there are speed-dependent ones and they determine the motion of a system, it is impossible to calculate the parameters of motion by methods and techniques of classical mechanics, since these forces change in motion and depend on speed themselves.

The above tasks can be solved only by methods of integration of the differential equation of motion.

All main forces that determine the train movement are set in dependence on speed. Therewith, additional resistance forces from track gradient and curvature also affect the speed of the train, and the degree of impact depends on the combinations of elements in the track profile and their length.

Calculation of the trajectories is the most important task in the simulation of dynamic systems, algorithms of which provide time as a continuous quantity. The aim of the algorithm to solve differential equations is approximation of a system behavior with continuous time. Since computing at digital calculations is inherently discrete in terms of time, the integration algorithm performs the simulation of a system, in which the time is considered to be continuous, a system with discrete time. The actual system, obtained in such simulation, is not a differential equation. Often, this is a complex system with discredited time. The integration algorithms are characterized by the fact that they reflect the same differential equation in different systems with discrete time.

Modern mathematics provides extremely powerful and universal research methods. Practically every concept in mathematics, every mathematical object, starting from the concept of a number, is a mathematical model [2]. When constructing a mathematical model of an object being studied, those peculiarities, features and details are distinguished, which, on the one hand, describe the object to the full extent, and on the other, admit some mathematical formalization. This means that when there is a mathematical description of an object, its features and details one can assign mathematical concepts into a certain correspondence: numbers, functions, functionals, matrices, etc. Then connectivities and correspondences detected and predicted in the object under study between its individual parts and components can be written down using mathematical relations: equalities, inequalities, equations. As a result, we have a mathematical description of the investigated process or phenomenon, that is, its mathematical model. However, the constructed mathematical model must meet such requirements as universality, adequacy, accuracy, efficiency, etc.

Universality characterizes the completeness in showing the properties of a real object being studied during simulation. For the model of the train movement, universality is expressed in the possibility of its use for various sections profiles, variational masses of the train formation, series of locomotives, etc.

The adequacy of the train movement model shows the mapping of the desired properties of an object, namely, technical-economic characteristics, with an error not higher than the specified one.

Accuracy can be estimated by the coincidence values in the characteristics of a real object and the corresponding values of the characteristics obtained by the models. Accuracy can be determined by comparing the results of a real trip with calculated ones: speed gauge data, energy consumption, travel time, etc.

Economical efficiency is determined by the cost of memory resources in the electronic computer (EC) and the time for implementation and operation of the model. For models that operate on stationary EC, economy is not a limiting factor comparing with the models that are software-monitored for operational decision-making (on-board), because they have to provide information concerning the change in such factors as traffic signals, current traffic speed, electric machines capacity, overheating temperature, etc.

The usage of mathematical models to describe physical processes and objects is universally accepted and in demand. To construct a model based on physical laws and calculate the exact value of any magnitude, it is reasonable to use, at any time, a deterministic model. However, due to certain unknown factors such as technical condition and technical-economic parameters of the traction rolling stock, net train resistance, the situation on a track, etc., this task cannot be classified as deterministic one. Consequently, the mathematical model of the train should take into account the probabilistic parameters of the fact that variables in the equation of train movement will lie in a certain definite interval. Therefore, this model must be classified as stochastic one.

Mathematical simulation while in operation of traction rolling stock is performed at the stages of designing the locomotives both in general and in separate assembly units, operation, including simulation of trains movement, technical maintenance [3], and others.

In the simulation of the train movement, the latter is considered as a dynamic system, which operates in steady state only at separate intervals of time. Transient processes lead to a change in all internal parameters of the system, including power, current, fuel consumption, etc., and some external ones: kinetic energy of the train, resistance to movement, etc. [4]. Therefore, the mathematical model of a train is dynamic by nature.

Graphical methods for traction calculations involve the use of discrete values in some variables, but the value of phase

coordinates does not always meet these requirements. The numerical integration of the train movement equation provides determining the phase coordinates at particular points of time or a track. The accuracy of the definition increases with decreasing track  $\Delta s$  and time  $\Delta t$  intervals. But at the same time, such a requirement as economical efficiency worsens. Consequently, in order to improve the accuracy of the calculations, the model should correspond to a continuous type.

**Literature review.** In traction calculations, the following methods are used to solve the differential equation of the train movement: analytical, graphical, numerical and machine. The common theoretical basis for these methods is that they involve solution of the equation for the train movement in the form of Cauchy problem. This implies the use of certain theoretical approaches known in technical cybernetics, mechanics and applied mathematics. They include linearization rules for nonlinear functions; the principle of small deviations of the variables of the object state; calculating the current coordinates of a moving object by the observability method by Thaler J.

The graphical and constant speed methods are used for calculations that do not require high accuracy and do not provide the definition of energy-efficient modes in driving the train.

A characteristic feature of the analytical method is the large amount of calculations, but it provides high precision under certain conditions. For a more detailed review, the following methods for solving the equation of the train movement with the definition of energy-efficient modes can be identified. They are:

- variational calculus;
- Pontryagin's maximum principle;
- nonlinear programming;
- dynamic programming;
- vector optimization and others.

It is advisable to use these methods in accordance with certain factors that affecting the train movement [5].

One of the methods most often used to solve the equation of train movement is Euler's method. The set task in it is finding a program for changing the speed  $v(t)$ , which would provide a given train movement along the section  $s$ , that is, the definition of the integral

$$s = \int_0^T v(t) dt,$$

where  $s$  is the section length;  $t$  is the period (time) of the train movement;  $T$  is the total period of the train movement along a section.

Optimization of the equation with account of the moment equilibrium on the moving wheel sets

$$\mu = \dot{v} + \mu_o,$$

where  $\mu_o$  is the moment of train rolling resistance, reduced to the wheelsets, is performed by finding the minimum integral value

$$Q = \int_0^T \mu^2 dt = \int_0^T (\dot{v} + \mu_o)^2 dt.$$

The condition is that the initial equation  $s$  must comply with the Euler's equation

$$\frac{\partial l}{\partial v} - \frac{d}{dt} \frac{\partial l}{\partial \dot{v}} = 0,$$

where  $l$  is auxiliary Lagrange function, which is from the expression

$$l = (\dot{v} + \mu_o)^2 + \lambda v,$$

where  $\lambda$  is Lagrange multiplier.

After solving the equation, we obtain the Euler's equation in the form

$$\ddot{v} - \lambda = 0.$$

With the substitution of the initial data, we obtain the equation of the optimized locomotive control program

$$v(t) = \frac{6s}{T^2} \left( t - \frac{t^2}{T} \right),$$

where  $T$  is train movement time.

Proceeding from this, the parabolic train speed control program is the optimal one.

During the operation of traction rolling stock in real conditions, traction characteristics of diesel locomotives, electric locomotives and multiple units do not allow realizing the speed change curve in trapezoidal form. Therefore, the operation of electric traction machines should be used as a nonlinear acceleration limitation.

This method has the following disadvantages. They include a large number of limitations, imposed on the train movement in real conditions. Transient processes and operation modes of power transmission elements must be considered by additional algorithms. The variables of the accelerating-retardation effort caused by the track profile are difficult to consider when solving the equation of the train movement.

The basis of the method of dynamic programming in the theory of hauling operations is Bellman's optimality principle, according to which the reduction of energy consumption for hauling operations is performed by minimizing the value of the objective function, describing the capacity control in a locomotive. To simplify this task, the number of limitations on the equation of train movement increases, which leads to a decrease in the number of iterative approximations.

$$f(v, t, p) = \sum_{i=1}^n \Delta e_i(v_i, t_i, p_i) \rightarrow \min,$$

where  $p$  is the capacity factor of the traction rolling stock;  $n$  is the number of iterative approximations in the solution for the equation of train movement;  $\Delta e_i$  is energy resources consumption for  $i$  solution spacing (step).

The objective function at dynamic programming for energy cost reduction on hauling operations is dependence

$$f_i(v_i, t_i) = \frac{\min}{p_i} [f_{i-1}(v_{i-1}, t_{i-1}) + \Delta e(v_i, t_i, p_i)],$$

where  $f_{i-1}(v_{i-1}, t_{i-1})$ ,  $f_i(v_i, t_i)$  is minimum of the values relatively for the  $(i-1)^{th}$  and  $i^{th}$  solution spacing (step).

When applying the time limits of the train movement along a section in the form of integral dependence [6, 7]

$$t_i = \int_{s_0}^{s_i} \frac{ds}{v},$$

the objective function involves the introduction of Lagrange multiplier  $\lambda$ . This partially simplifies the task of energy cost reduction

$$f_i(v_i) = \frac{\min}{p_i} [f_{i-1}(v_{i-1}) + \Delta e(v_i, t_i, p_i) + \lambda t_i].$$

When solving the equation of train movement by the dynamic programming method it is expedient to use the method of finite increments.

There are deficiencies in this method. They are as follows: when the integration step is reduced, the number of options for solving the equation increases exponentially, which leads to diseconomies of the model and necessitates some amount of computer time. This raises certain limitations in the application of this method in on-board systems of operational decision-making when changing the train situation during traffic.

When comparing solutions in the equation of train movement depending on expenses of energy resources consumption from the time of a train  $q(t)$  is possible only at certain discrete points

$$\left\{ \begin{matrix} t_i & t_{i+1} & \dots & t_{n-1} & t_n \\ q_i & q_{i+1} & \dots & q_{n-1} & q_n \end{matrix} \right\},$$

which makes it impossible to determine the mode of driving that corresponds to the intermediate values.

When solving the equation of train movement according to the method by L. S. Pontryagin, the principle of maximum is used. This method is used in systems that provide high-speed, including in on-board software and hardware complexes. The algorithms of the Pontryagin method are based on the method of dynamic programming.

In problems of the locomotive traction theory at solving the equation of movement, a set of control impacts is determined from the tolerance region, which consists of vector piecewise continuous function  $u(t)$ , defined at a certain time interval

$$t_0 \leq t \leq t_1,$$

in every time moment from  $U$  area. Problem statement is reduced to the choice of such control  $u(t)^*$  from a set of control impacts from the admissible region that reset the phase point from the position  $x_0$  into  $x_1$ , which minimizes the functional

$$\int_{t_0}^{t_1} f^0[x(t), u(t)] dt \rightarrow \min.$$

To solve the equation of train movement, we set a system of equations

$$\left. \begin{matrix} \frac{de}{dt} = \frac{p(u)}{\eta} \\ \frac{ds}{dt} = v \\ \frac{dv}{dt} = \frac{R(u)}{Q} \end{matrix} \right\},$$

where  $e$  is energy resources consumption;  $p$  is realized power converted to the control notches;  $\eta$  is total efficiency of the locomotive;  $s$  is the distance covered;  $R$  is resulting force;  $u$  is traction control function;  $Q$  is train weight.

To find the roots in this system of equations, Hamiltonian function  $H$  is formed

$$H = \psi_0 \frac{p(u)}{\eta} + \psi_1 \cdot v + \psi_2 \frac{R(u)}{m},$$

where  $\psi_i$  is Hamiltonian conjugate variables.

Hamiltonian multipliers are determined from a system of equations with partial derivatives

$$\left. \begin{matrix} -\frac{\partial H}{\partial e} = \frac{d\psi_0}{dt} \\ -\frac{\partial H}{\partial s} = \frac{d\psi_1}{dt} \\ -\frac{\partial H}{\partial v} = \frac{d\psi_2}{dt} \end{matrix} \right\}.$$

This gives some disadvantages of this method, namely the need for a large number of calculations of differential equations, both in solving the equation of train movement, and in determining the Hamiltonian conjugate function.

In order to increase the accuracy of calculations by reducing the integration step, which leads to an increase in the number of computations, A. M. Kostromin suggested choosing the integration step proportionally to constant train time  $T_{const}$

$$T_{\text{const}} = \frac{1}{\xi \cdot k_s},$$

where  $\xi$  is the coefficient, which takes into account the mass of a train and the coefficient of inertia of the rotating masses;  $k_s$  is the stabilization coefficient of the train speed at the change of resultant force

$$k_s = -\frac{\partial F}{\partial v}.$$

In this case, with an increase in the constant time, when the speed and the values dependent on it, change more slowly, the integration step can be increased. This does not affect the established accuracy of calculations, but significantly reduces the computing time and improves the efficiency of the mathematical model. In case of taking into account the potential energy of a train, when increasing the positive value of the gradient, it is expedient to reduce the integration step [8].

When determining the minimum value of energy resources consumption by the method of variational solution for the equation of the train movement, proposed by Yu. P. Petrov, it is assumed that the equation of the train movement is integrated. The train travel time is chosen as the variable of integration. At that, limitations are imposed on the execution of the scheduled time of movement

$$\frac{d}{dt} \left( \frac{Qv^2}{2} \right) + \omega v = \eta p.$$

Energy resources consumption  $G$  when driving along a section, length  $s_0$  during period  $T$  in this method is calculated by integrating the equation

$$G = \int_0^T q dt = \frac{1}{\eta} \int_0^T (s''s' + \omega_1 s' + ks'^2 + k_1 s'^3) dt + \int_0^T a dt,$$

where  $\omega_1$  is train rolling resistance caused by the change in the gradient of a track profile and depending on the current track coordinate;  $k$ ,  $k_1$  are coefficients of train rolling resistance, depending on the speed;  $a$  is the coefficient of fuel consumption intensity, depending on the realized power.

When determining the optimal control law for traction rolling stock, it is enough to find such a function  $s(t)$ , which is the minimum of functional  $G$

$$J = \int_0^T (s''s' + \omega_1 s' + ks'^2 + k_1 s'^3) dt \rightarrow \min.$$

When solving the equation, the following limitations should be introduced: power of the primary engine  $P \leq P_{\text{max}}$ ; maximum acceleration  $s'' \leq s''_{\text{max}}$ ; maximum speed, factoring in track state, as well as time limits  $s' \leq s'_{\text{max}}$ ; the impossibility of recovery for diesel locomotives.

The disadvantages of this method include: accounting for energy resources consumption, linked linearly to the power-plant output, which affects the adequacy of the model; complexity of solving the target braking problem, since the braking start coordinate is set in advance; transmission efficiency of power is described by the constant, that is, an additional error is created in the calculations.

Bosov A. A. proposed a method for determining the optimal energy resources consumption by an additive criterion. The overheating temperature of windings in traction electric machines is included as an integral part in the equation of train movement

$$\frac{d\tau}{dt} = \frac{-\tau + \tau_{\infty}(v, u)}{T(v, u)},$$

where  $\tau_{\infty}$  is overheating temperature at the set mode;  $T(v, u)$  is time constant, depending on the speed of movement and control mode.

The algorithm for finding the control mode that is optimal by energy resources consumption consists of elementary operations, indexed  $\Delta_c$ . They form an area  $B$ , for which the optimal trajectory is determined  $X$ . It is checked for quality control in accordance with the set task. According to the minimum value of the time in train movement  $t$  and work of resistance  $A$  the function of optimization is as follow

$$t = \int_{s_1}^{s_2} \frac{ds}{v(s)} \rightarrow \min;$$

$$A = \int_{s_1}^{s_2} \omega(v) ds \rightarrow \min.$$

There are a number of methods for solving the equation of train movement. They take into account more variable parameters of the train movement, traffic situation and constraints. Increasing the number of variation parameters leads to improved accuracy of calculations, which in turn reduces the speed of calculations.

**Unsolved aspects of the problem.** The need to increase the speed of trains to ensure the competitiveness of railway transport, the technical condition of locomotives and motor-vehicle rolling stock and the constant increase in the cost of fuel and energy resources lead to the need for analysis and scientific substantiation of control modes for the traction rolling stock, their rationalization, calculation of individual energy-saving mode maps, correction of train schedules. One of the main directions of rationalization for the modes of trains is their operational calculation directly during the trip. Therefore, under these conditions, there is a need to increase the speed of traction calculations without loss of accuracy. This issue can be partially disclosed through the use of modern computer technology, but the main direction in the solution of this issue is the improving the algorithms of traction calculation.

**Problem statement.** The advantage of using a numerical method for solving the differential equation of train movement in traction calculations is to achieve a high accuracy of them. The numerical values of integration variables in the equation of train movement play an important role in the performance of traction calculations on electronic computers. While increasing the value of the integration variable, the number of iterations decreases when determining the trajectories of the train. This leads to an increase in computing speeds. However, large values of the integration variables reduce the accuracy of the calculations. Therefore, it is necessary to determine the values of integration variables that will satisfy the high-speed computations and accuracy of the calculations.

To perform the operational traction calculations directly during the trip, it is necessary to determine such an integration step for the equation of the train movement, which without loss of accuracy will enable to accelerate calculations. Therefore, the purpose of the article is to determine the rational values of the variables in solving the equation of train movement, as well as the corresponding limits of their applicability.

**Description of the research structure.** To achieve the purpose, the methods of system analysis are used when compiling the mathematical model of the train movement, nonlinear programming in determining the rational values of integration variables, numerical methods for solving differential equations:

- classical;
- Runge-Kutti-Feelberg;
- Rosenbrock when constructing a mathematical model of

the train movement and checking the accuracy of the model.

The accuracy of the calculations was also checked by the simulation methods and compared with the experimental data of the research trips.

**Results.** The equation of train movement, which is considered as a chain of distributed masses on the basis of Newton's second law, can be represented as a system of differential equations

$$\begin{cases} \frac{dv}{ds} = \frac{R \cdot g}{v \cdot (P + Q)}, \\ \frac{dt}{ds} = \frac{1}{v} \end{cases} \quad (1)$$

where  $v$  is movement speed;  $R$  is resultant of external forces;  $g$  is free-fall acceleration;  $P$ ,  $Q$  are mass of a locomotive and a train, respectively.

To solve the system of differential equations by numerical methods, each element of the profile is divided into integration steps  $ds$  (in case of integration along the track) or  $dt$  (when integrating over time). Under the step of varying the modes of train driving, one assumes the track section  $ds$ , on which the mode of movement is stable.

In the process of solving the differential equations of train movement, using a mathematical model, we determine the final speed  $v_f$ , time  $dt$  or a track  $ds$  and fuel consumption  $dq$  for each integration step, based on the initial speed  $v_n$ .

Consequently, in order to calculate the train movement, it is necessary to adopt the method for integrating the differential equation of movement (1), and to transform it so as to find the law of movement

$$\frac{dv}{dt} \frac{ds}{ds} = \xi \cdot r, \quad (2)$$

where  $\xi$  is acceleration of the train under the action of the specific force of 1 N/kN;  $r$  is specific resultant force.

Total resultant of forces  $R$ , acting on the train is an additive force consisting of rail tractive effort

$$F_t = f(v, I_{tem}, \eta_\Sigma, u(x)),$$

braking forces

$$B = f(v, u(x)),$$

and the train rolling resistance

$$W = f(v, x),$$

where  $I_{tem}$  is current of traction electric motors;  $\eta_F$  is full efficiency of traction transmission;  $u$  is control of the locomotive under the appropriate mode of operation (traction, run-up or braking);  $x$  is the center-of-mass coordinate of the train.

Specific resultant force can be determined by the formula

$$r = \frac{F_t(v, I_{tem}(u), \eta_F) - B(v, u(x), I_f) - W(v)}{P + Q} - \tilde{i}(x),$$

where  $\tilde{i}(x)$  is straightened profile factoring in the length and mass distribution of the train.

Let us consider the trajectory of the train acceleration in coordinates  $s - v$  with different independent integrals of the equation (2) in the range of 1 km [9]. According to one of the recommendations for performing traction calculations for integrating the equation of train movement we perform the following:

- at speed up to 20 km/h, to integrate over time variable  $t$ ;
- in the range of speeds from 20 km/h to the maximum permissible – along the track variable  $s$ .

The values of the integration step in time  $t$  and a track  $s$  are selected at speed increment  $\Delta v \leq 3 \div 5$  km/h.

The above condition during the train movement at uniform speed provides large values of interval  $\Delta s$ , and with intensive acceleration – small values of the interval of time  $\Delta t$ , which leads to degradation in the accuracy of calculations (Fig. 1). The combination of both conditions in the general algorithm results in the imposition of supplemental checks at

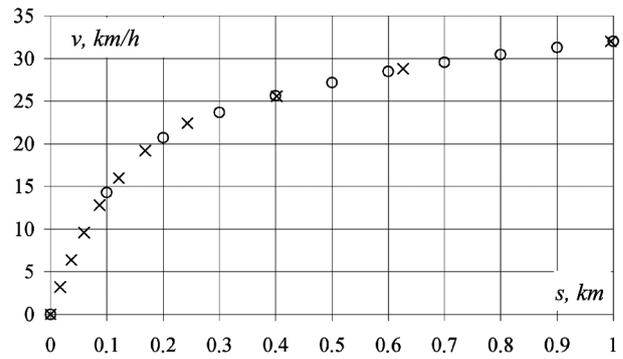


Fig. 1. Comparison of the train acceleration trajectories with different independent integration variables for the equation of train movement:

$\times$  – speed  $v$ ;  $\circ$  – track  $s$

each step in the solution of the equation, which worsens such a property of the model as economical efficiency.

To improve the economical efficiency while maintaining the accuracy of calculations, it is proposed to choose the step of variables when integrating the differential equation on the basis of equality of distances between adjacent points of the trajectory in the track-speed coordinates (Fig. 2).

In the course of the research, the parameters and phase coordinates of the trajectory were obtained, which showed that at the step of integration variables for the track  $\Delta s = 50$  m and speed  $\Delta v = 1$  km/h at dynamic simulation and  $\Delta s = 25$  m and speed  $\Delta v = 0.5$  km/h sufficient model accuracy is provided at static simulation [10].

In case of integrating the equation of train movement along the track  $s$  and speed  $v$  and accentuation of a real number using function  $\Re$  at dynamic simulation

$$\int_{s_{j-1}}^{s_j} ds = \int_{v_{j-1}}^{v_j} \frac{v dv}{\xi \cdot r};$$

$$v_j = \Re \sqrt{\frac{\Delta s \cdot \xi \cdot r}{500} + v_{j-1}^2},$$

integration step  $\Delta s_j$  in meters of the current iteration, factoring in the previous one, is calculated by the formula

$$\Delta s_j = 50 \sqrt{1 - (v_{j-2} - v_{j-1})^2},$$

in case of integration by speed  $v$  and time  $t$  by equation

$$\int_{t_{j-1}}^{t_j} dt = \int_{v_{j-1}}^{v_j} \frac{dv}{\xi \cdot r};$$

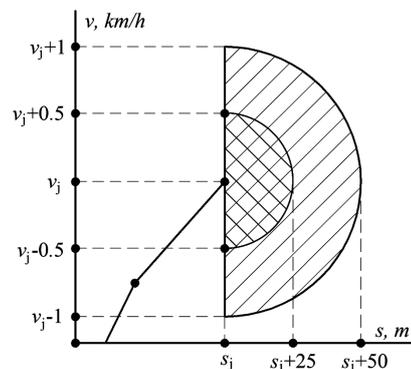


Fig. 2. Determining the range of permissible values for the step of variables when integrating the equation of train movement

$$t_j = 60 \cdot \frac{\Delta v}{\xi \cdot r} + t_{j-1},$$

step integration value  $\Delta v_j$  in km/h is calculated by the formula

$$\Delta v_j = \sqrt{1 - \left( \frac{s_{j-2} - s_{j-1}}{50} \right)^2}.$$

The greatest accuracy of the model is achieved by integrating the equation of train movement with certain variables under the following conditions:

- when starting and accelerating up to 40 km/h – by integration over variables of speed  $v$  and time  $t$ ;
- when moving in traction mode with power regulation and in run down mode – by integration over variables of track  $s$  and speed  $v$ ;
- in braking mode – by integration over speed  $v$  and time  $t$ .

Verification in the accuracy and adequacy of the proposed algorithm for determining the step of solving the equation of train movement is performed by the classical, Runet-Kutti-Feelberg and Rosenbrock methods of integrating the differential equations. The results of the solution for the equation of train movement in terms of the above methods for integrating the differential equations indicate the possibility of using the algorithm in traction calculations, including on-board systems of the locomotive. The average absolute error value is:

- track coordinates – 0.69 %;
- speed – 0.02 %;
- train movement time – 0.55 %.

**Conclusions.** The impact of the interval for integration variables of the equation of train movement on the accuracy and economical efficiency of the mathematical model is analyzed. The accuracy of calculations within the engineering error for a dynamic model is provided with the integration step for variable track – 50 m, speed – 1 km/h; static model for the variable track – 25 m, speed – 0.5 km/h. The determined step for a dynamic model allows using the model of train movement in on-board systems of locomotives for the operative recalculation of rational control modes when changing the train situation.

To improve the economical efficiency while maintaining the accuracy of calculations, it is proposed to choose the step of variables for integrating the differential equation on the basis of equality in distances between adjacent points of the trajectory in the track-speed coordinates.

The solution of the equation of train movement is performed by separate integration variables for different speed modes and phases in train movement. This increases the reliability of the calculation results in transient processes.

The accuracy and adequacy of the mathematical model in certain modes of the train movement are checked by comparing the results with known methods of numerical integration of differential equations. From the results of the test, one can affirm that the average error value does not exceed 0.7 % for the track covered, 0.02 % for speed and 0.55 % for the train travel time. Consequently, the adopted model can be used for operational definition of the energy-saving modes in driving the trains in on-board systems.

**Title and number of the project in which the obtained results are presented.** The work is performed in accordance with the plans of research works at Dnipro National University of Railway Transport named after Academician V. Lazaryan, in particular within the framework of taxpayer-funded theme “Analysis of possible causes in deviation of diesel fuel consumption from existing standards and preparing the recommendations for reducing the fuel consumption”, Contract No. ОД/Т-15-787НЮ (No 79.21.15.15) from 18.09.2015, (No ДР 0115U007071). Performers of the theme: Martyshevskiy M.I., Kapitsa M.I., Bobyr D.V., Ochkasov O.B., Kyslyi D.M., Koreniuk R.O.

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## Визначення меж застосування та значень змінних інтегрування рівняння руху поїзда

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Залізничні перевезення займають важливе місце у транспортній інфраструктурі нашої країни. Вони охоплюють пасажирські й вантажні перевезення Укрзалізниці, промислових підприємств, у тому числі перевезення гірничо-добувного сектору господарства, що характеризуються великими навантаженнями на тяговий рух мий склад за рахунок великих ухилів профілю колії. Організації залізничних перевезень завжди передують тягові розрахунки, осередком яких є розв'язання рівняння руху поїзда.

**Мета.** Визначення раціональних значень змінних при розв'язанні рівняння руху поїзда, а також відповідних меж їх застосування.

**Методика.** Для досягнення мети використані методи системи аналізу, нелінійного програмування, числових методів розв'язання диференціальних рівнянь, а

само класичного, Рунге-Кутти-Фельберга, Розенброка. Точність розрахунків перевірялася за допомогою методів імітаційного моделювання та порівнювалася з експериментальними даними.

**Результати.** Результатами дослідження є підвищення швидкодії розрахунків при розв'язанні рівняння руху поїзда без втрати точності, що дозволило використовувати запропоновану методику в бортових системах локомотива.

**Наукова новизна.** Під час виконання дослідження отримані нові науково обґрунтовані результати, які розв'язують завдання підвищення енергоефективності ведення поїздів, що має істотне значення для залізничного транспорту. Отримані результати складають наукову новизну, що полягає у визначенні раціональних меж застосування та значення кроку змінних інтегрування рівняння руху поїзда.

**Практична значимість.** Результати дослідження дозволяють зменшити витрати енергоресурсів на тягу поїздів унаслідок оперативного перерахунку раціональних режимів керування при зміні поїзної ситуації.

**Ключові слова:** залізничні перевезення, тяга поїздів, рівняння, інтегрування, крок, точність

### **Определение границ применения и значений переменных интегрирования уравнения движения поезда**

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Железнодорожные перевозки занимают важное место в транспортной инфраструктуре нашей страны. Они охватывают пассажирские и грузовые перевозки Укрзалізнични, промислових підприємств, в том числе перевозки горнодобывающего сектора хозяйства, который

характеризуется большими нагрузками на тяговый подвижной состав за счет больших уклонов профиля пути. Организации железнодорожных перевозок всегда предпочитают тяговые расчеты, основой которых является решение уравнения движения поезда.

**Цель.** Определение рациональных значений переменных при решении уравнения движения поезда, а также соответствующих границ их применения.

**Методика.** Для достижения цели использованы методики системного анализа, нелинейного программирования, численных методов решения дифференциальных уравнений, а именно классического, Рунге-Кутты-Фельберга, Розенброка. Точность расчетов проверялась с помощью методов имитационного моделирования и сравнивалась с экспериментальными данными.

**Результаты.** Результатами исследования является повышение быстродействия расчетов при решении уравнения движения поезда без потери точности, что позволило использовать предложенную методику в бортовых системах локомотива.

**Научная новизна.** Во время выполнения исследования получены новые научно обоснованные результаты, которые решают задачу повышения энергоэффективности ведения поездов, что имеет существенное значение для железнодорожного транспорта. Научная новизна полученных результатов заключается в определении рациональных границ применения и значения шага переменных интегрирования уравнения движения поезда.

**Практическая значимость.** Результаты исследования позволяют уменьшить затраты энергоресурсов на тягу поездов в результате оперативного пересчета рациональных режимов управления при изменении поезда ситуации.

**Ключевые слова:** железнодорожные перевозки, тяга поездов, уравнения, интегрирование, шаг, точность

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