Научная новизна. Впервые показано, что системы автоматического управления секцией обогащения и обогатительной фабрикой в целом по вышеуказанному каналу должны строиться по реакции на отклонение характеристик товарного концентрата от заданных. Впервые выполнен анализ параметров спектров дисперсий содержания общего железа в исходной руде и концентрате и амплитудно-частотных характеристик по каналу управления, полученных в процессе промышленной эксплуатации рудо-обогатительной фабрики.

Практическая значимость. Полученные результаты могут быть использованы для построения автоматической системы ситуационного управления процессом обогащения руд чёрных и цветных металлов как на отдельных секциях с шаровым измельчением, так и на фабриках, в состав которых эти секции входят.

Ключевые слова: *автоматизация, индикативные события, система ситуационного управления, спектральный анализ, частотный анализ, обогащение железной руды, рудо-обогатительная фабрика с шаровым измельчением*

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ANALYTICAL SOLUTION OF THE DIRICHLET GENERALIZED BOUNDARY PROBLEM OF HEAT-EXCHANGE IN THE FINITE CYLINDER WITH HOMOGENEOUS LAYERS

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АНАЛІТИЧНИЙ РОЗВ’ЯЗОК УЗАГАЛЬНЕНОЇ КРАЙОВОЇ ЗАДАЧІ ДІРІХЛЕ ТЕПЛООБМІНУ ДВОСКЛАДОВОГО СКІНЧЕННОГО ЦИЛІНДРА

Purpose. To develop a new generalized 3D mathematic model for calculating temperature fields in the solid finite cylinder with homogeneous layers in the form of the mathematical physics boundary problem for hyperbolic equations by the Dirichlet conditions (temperature on the cylinder surface is a continuous function of the coordinate axis), and to solve the obtained boundary problem.

Methodology. Use of the known Laplace and Fourier integral transformations and application of the new integral transformation to the space with homogeneous layers.

Findings. A nonstationary temperature field in the rotating double-layer solid finite cylinder in the cylindrical coordinate system with taking into account finite velocity of the heat conduction was defined. Heat-transfer properties of the cylinder in either layer are constant at an ideal heat contact between the layers while no internal sources of the heat are available. At the initial moment of time, the cylinder temperature is constant, and temperature on the outside surface of the cylinder is known.

Originality. It is the first a mathematical 3D model for calculating temperature fields in the rotating double-layer solid finite cylinder has been created in the form of the physicomathematical boundary problem for the heat conduction hyperbolic equations by the Dirichlet conditions and with taking into account finite velocity of the heat conduction. A new integral transformation was created for the space with homogeneous layers, with the help of which it became possible to present a temperature field in the finite cylinder with homogeneous layers in the form of convergence orthogonal series by Bessel and Fourier functions.

Practical value. The obtained analytical solution of the generalized boundary problem of heat exchange in the rotating double-layer cylinder, which takes into account the known time period of the heat-conduction relaxation, can be used for detecting temperature fields, which occur in different technical systems (forming rolls, satellites, turbines, etc.).

Keywords: *Dirichlet boundary value problem, integral transformation, relaxation time, double-layer finite cylinder*

Introduction. Analysis of the recent research and publications. In the world, an essential part of the melt metal is subject to further processing in the rolling-mill shops [1]. Engineering-and-economic performance of the mill greatly depends on the roller life. Interdependence between the static and dynamic profiling methods cannot ensure proper control of the section without proper regulation of the roller heat state. The key task of the thermal-state control system is to reduce thermal stress of the mills in the process of their operation and, hence, to stabilize their heat profile, which is characterized by the "thermal convexity" along a length of the body of roll and a width of the strip. Cooling of the mills not only influences the heat profile but also is a stabilizing factor, which reduces the thermal stresses. The thermal state of a locally heated rotating cylinder is needed to be determined for calculating the rollers for the mills, roller-beds for furnaces and continuous casting machines.

In this paper, a double -layer cylinder is presented as a model of the forming roll under the action of the heat flow. The heal flow is a consequence of interaction between the roller and metal strip heated up to 1200°.

In scientific literature, issues of heat exchange in the rotating cylinders with taking into account finite velocity of the heat conduction have not been studied fully yet [2, 3]. As it is shown in [1], numerical methods are not always effective for studying heat-exchange of cylinders considering finite velocity of the heat conduction, if the cylinders rotate at the high rates.

Thus, it is stated in [1] that conditions for reliable calculations by the finite element method and finite equation method for calculating nonstationary non-axis-symmetrical temperature fields of the rotating cylinders are described by the same characteristics and can be expressed in the following way

$$1-\frac{\Delta F_0}{\Delta\varphi^2}\ge 0;\quad \frac{1}{\Delta\varphi}-\frac{Pd}{2}\ge 0,$$

where F_0 is Fourier's criterion; Pd is Predvoditelev's criterion.

If $Pd = 10^5$ and consequently, corresponds to the angular velocity $\omega = 1.671$ sec^{-1} of rotation of metal cylinder with a radius of 100 mm, then variables $\Delta\varphi$ and ΔF_0 should comply with the following conditions

$$\Delta\varphi\le 2\cdot 10^{-5};\quad \Delta F_o\le 2\cdot 10^{-10}.$$

For the uniformly cooled cylinder when Bi = 5 (Bi is Bio criterion) time period needed for temperature to reach 90 % of stationary state is equal to $F_0 \approx 0.025$ [4]. It means that within this period of time at least $1.3\cdot 10^8$ operations should be fulfilled in order to reach the stationary temperature distribution.

Moreover, it would be necessary to make $3.14\cdot 10^5$ calculations within one cycle of computation as the inside state of the ring should be characterized by $3.14\cdot 10^5$ points. It is obvious that this number of calculations is unrealistic.

Therefore, the known (Laplace and Fourier) and new integral transformations are employed in this paper for solving the boundary problem.

The problem statement. Let us consider calculation of non-axis-symmetrical nonstationary temperature field of the rotating solid double-layer cylinder with finite length L and outside radius R in the cylindrical coordinate system (r, φ, z); the cylinder is heterogeneous along its radius, and calculation takes into account known time period of the heat-conduction relaxation. The cylinder rotates with the angular velocity ω around the axis OZ. The heat-transfer properties of each of the layers are constant on the assumption that heat contact between the layers is ideal and if no internal sources of the heat are available. At the initial moment of time, the cylinder temperature is constant G_0 and temperature on the outside surface of the cylinder is known $G(\varphi,z)$.

Relative temperature $\theta(\rho,\varphi,z,t)$ of the cylinder can be expressed in the following way

$$\theta(\rho,\varphi,z,t)=\begin{cases}\theta_1(\rho,\varphi,z,t) & \text{if}\quad \rho\in(\rho_0,\rho_1)\\ \theta_2(\rho,\varphi,z,t) & \text{if}\quad \rho\in(\rho_1,\rho_2)\end{cases}.\qquad (1)$$

Relative temperatures $\theta_s(\rho,\varphi,z,t)$ of the *s-th* layer of the cylinder is calculated by the formulas

$$\theta_s(\rho,\varphi,z,t)=\frac{T_s(\rho,\varphi,z,t)-G_0}{T_{\max}-G_0},$$

where $\rho=\dfrac{r}{R}$; $s = 1, 2$.

Solving of the problem. In the [1], a generalized heat-conduction equation is presented for the moving element of a solid medium with taking into account finiteness of the velocity and heat-conduction values. According to the [1], a generalized equation for the energy balance of a solid body, which rotates with the constant angular velocity ω around the axis OZ, and whose transfer properties are constant, and internal sources of the heat are not available, can be written in the following way

$$\gamma_c\left\{\frac{\partial T}{\partial t}+\omega\frac{\partial T}{\partial\varphi}+\tau_r\left[\frac{\partial^2 T}{\partial t^2}+\omega\frac{\partial^2 T}{\partial\varphi\partial t}\right]\right\}=$$

$$=\lambda\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}+\frac{\partial^2 T}{\partial z^2}\right],\qquad (2)$$

where γ is density of the medium; c is specific heat capacity; λ is heat conductivity coefficient; $T(\rho,\varphi,z,t)$ is temperature of the medium; t is time; τ_r is relaxation time.

The mathematical physics boundary problem of defining relative temperature $\theta_s(\rho,\varphi,z,t)$ for the cylinder consists of integration of hyperbolic differential equations (2) of heat conduction into domains

$$D_s=\Big\{(\rho,\varphi,z,t)\ \Big|\rho\in(\rho_{s-1},\rho_s),\ \varphi\in(0,2\pi),$$

$$z\in(0,1),\ t\in(0,\infty)\Big\},$$

which are expressed as

$$\frac{\partial\theta_s}{\partial t}+\omega\frac{\partial\theta_s}{\partial\varphi}+\tau_r\frac{\partial^2\theta_s}{\partial t^2}+\tau_r\omega\frac{\partial^2\theta_s}{\partial\varphi\partial t}=$$

$$=\alpha_s^2\left[\frac{\partial^2\theta_s}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta_s}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\theta_s}{\partial\varphi^2}+\chi\frac{\partial^2\theta_s}{\partial z^2}\right],\qquad (3)$$

with initial conditions

$$\theta_s(\rho,\varphi,z,0)=0; \quad \frac{\partial\theta_s(\rho,\varphi,z,0)}{\partial t}=0, \quad (4)$$

with boundary conditions

$$\theta_2(1,\varphi,z,t)=V(\varphi,z); \quad (5)$$

$$\theta_s(\rho,\varphi,1,t)=0; \quad \theta_s(\rho,\varphi,0,t)=0, \quad (6)$$

with ideal heat contact

$$\theta_1(\rho_1,\varphi,z,t)=\theta_2(\rho_1,\varphi,z,t); \quad (7)$$

$$\lambda_1\frac{\partial\theta_1(\rho_1,\varphi,z,t)}{\partial\rho}=\lambda_2\frac{\partial\theta_2(\rho_1,\varphi,z,t)}{\partial\rho}, \quad (8)$$

and with two restrictions on the cylinder axis

$$\theta_1(0,\varphi,z,t)<\infty, \quad (9)$$

where $\rho_1=\frac{R_1}{R}$; $\rho_0 = 0$; $\rho_2 = 1$; R_1 is the radius of the layer boundary; λ_s is the heat conductivity coefficient; γ_s is density; c_s is specific heat capacity; $a_s=\frac{\lambda_s}{c_s\gamma_s}$ is thermal diffusivity of the s^{th} layer of the cylinder; $\alpha_s^2=\frac{a_s}{R^2}$; $s = 1, 2$; $z=\frac{z}{L}$; $\chi=\left(\frac{R}{L}\right)^2$; $V(\varphi,z)\in C(0,2\pi)$.

In this case, solution of the boundary problem (3–9) $\theta_s(\rho,\varphi,z,t)$ is twice differentiated by ρ, φ, z, t in the domain D_s and continuous $\bar{D}_s$ on the [4], i.e. $\theta_s(\rho,\varphi,z,t)\in C^{2,1}(D_s)\cap C(\bar{D}_s)$, and functions $V(\varphi,z)$, $\theta_s(\rho,\varphi,z,t)$ can be expressed by the Fourier complex series [4]

$$\begin{Bmatrix}\theta_s(\rho,\varphi,z,t)\\ V(\varphi,z)\end{Bmatrix}=\sum_{n=-\infty}^{+\infty}\begin{Bmatrix}\theta_{s,n}(\rho,z,t)\\ V_n(z)\end{Bmatrix}\cdot\exp(in\varphi), \quad (10)$$

where

$$\begin{Bmatrix}\theta_{s,n}(\rho,z,t)\\ V_n(z)\end{Bmatrix}=\frac{1}{2\pi}\int_0^{2\pi}\begin{Bmatrix}\theta_s(\rho,\varphi,z,t)\\ V(\varphi,z)\end{Bmatrix}\cdot\exp(-in\varphi)d\varphi;$$

$$\theta_{s,n}(\rho,z,t)=\theta_{s,n}^{(1)}(\rho,z,t)+\mathrm{I}\theta_{s,n}^{(2)}(\rho,z,t);$$

$$V_n(z)=V_n^{(1)}(z)+\mathrm{I}V_n^{(2)}(z),$$

where I is an imaginary unit.

In view of the fact that $\theta_s(\rho,\varphi,z,t)$ ρ is a real-valued function, let us confine ourselves by considering only $\theta_{s.n}(\rho,z,t)$ for n = 0,1,2,..., because $\theta_{s,n}(\rho,z,t)$ and $\theta_{s,-n}(\rho,z,t)$ are complexly conjugate [4]. By putting values of functions from (10) into (3–9) we obtain the following system of differential equations

$$\frac{\partial\theta_{s,n}^{(i)}}{\partial t}+\vartheta_n^{(i)}\theta_{s,n}^{(m_i)}+\tau_r\frac{\partial^2\theta_{s,n}^{(i)}}{\partial t^2}+\tau_r\vartheta_n^{(i)}\frac{\partial\theta_{s,n}^{(m_i)}}{\partial t}=$$

$$=\alpha_s^2\left[\frac{\partial^2\theta_{s,n}^{(i)}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta_{s,n}^{(i)}}{\partial\rho}-\frac{n^2}{\rho^2}\theta_{s,n}^{(i)}+\chi\frac{\partial^2\theta_{s,n}^{(i)}}{\partial z^2}\right], \quad (11)$$

with initial conditions

$$\theta_{s,n}^{(i)}(\rho,z,0)=0; \quad \frac{\partial\theta_{s,n}^{(i)}(\rho,z,0)}{\partial t}=0, \quad (12)$$

with boundary conditions

$$\theta_{2,n}^{(i)}(1,z,t)=V_n^{(i)}(\mathrm{z}); \quad (13)$$

$$\theta_{s,n}^{(i)}(\rho,0,t)=0; \quad \theta_{s,n}^{(i)}(\rho,1,t)=0, \quad (14)$$

with an ideal heat contact

$$\theta_{1,n}^{(i)}(\rho_1,z,t)=\theta_{2,n}^{(i)}(\rho_1,z,t); \quad (15)$$

$$\lambda_1\frac{\partial\theta_{1,n}^{(i)}(\rho_1,z,t)}{\partial\rho}=\lambda_2\frac{\partial\theta_{2,n}^{(i)}(\rho_1,z,t)}{\partial\rho}, \quad (16)$$

and with two restrictions on the cylinder axis

$$\theta_{1,n}^{(i)}(0,z,t)<\infty, \quad (17)$$

where $\vartheta_n^{(1)}=-\omega n$; $\vartheta_n^2=\omega n$; $m_1=2$; $m_2=1$; $i,s = 1, 2$.

Let us employ the Fourier integral transformation [5] for the system of differential equations (11)

$$\bar{f}(\lambda_m)=\int_0^1 f(x)\sin(\pi\cdot\mathrm{m}\cdot x)\mathrm{d}x,$$

where $\lambda_m = \pi \cdot m$; $m = 1,2,...,$ and formula of inverse transformation is expressed as

$$f(x)=2\sum_{m=1}^{\infty}\sin(\pi\cdot\mathrm{m}\cdot x)\cdot\bar{f}(\lambda_m). \quad (18)$$

As a result, we receive the following system of differential equations

$$\frac{\partial\tilde{\theta}_{s,n}^{(i)}}{\partial t}+\vartheta_n^{(i)}\tilde{\theta}_{s,n}^{(m_i)}+\tau_r\frac{\partial^2\tilde{\theta}_{s,n}^{(i)}}{\partial t^2}+\tau_r\vartheta_n^{(i)}\frac{\partial\tilde{\theta}_{s,n}^{(m_i)}}{\partial t}=$$

$$=\alpha_s^2\left[\frac{d^2\tilde{\theta}_{s,n}^{(i)}}{d\rho^2}+\frac{1}{\rho}\frac{d\tilde{\theta}_{s,n}^{(i)}}{d\rho}-\frac{n^2}{\rho^2}\tilde{\theta}_{s,n}^{(i)}-\chi\lambda_m^2\tilde{\theta}_{s,n}^{(i)}\right], \quad (19)$$

with initial conditions

$$\tilde{\theta}_{s,n}^{(i)}(\rho,0)=0; \quad \frac{\partial\tilde{\theta}_{s,n}^{(i)}(\rho,0)}{\partial t}=0, \quad (20)$$

with boundary conditions

$$\tilde{\theta}_{2,n}^{(i)}(1,t)=\tilde{V}_n^{(i)}, \quad (21)$$

with an ideal heat contact

$$\tilde{\theta}^{(i)}_{1,n}\left(\rho_1,t\right)=\tilde{\theta}^{(i)}_{2,n}\left(\rho_1,t\right); \quad (22)$$

$$\lambda_1\frac{\partial\tilde{\theta}^{(i)}_{1,n}\left(\rho_1,t\right)}{\partial\rho}=\lambda_2\frac{\partial\tilde{\theta}^{(i)}_{2,n}\left(\rho_1,t\right)}{\partial\rho}, \quad (23)$$

and with two restrictions on the cylinder axis

$$\tilde{\theta}^{(i)}_{1,n}\left(0,t\right)<\infty. \quad (24)$$

In order to solve the boundary equation (19–24) let us make an integral transformation

$$\bar{f}\left(\mu_{n,k}\right)=\int_{\rho_0}^{\rho_2}\frac{Q_0\left(\mu_{n,k}\rho\right)}{\alpha\left(\rho\right)}\rho f\left(\rho\right)d\rho=$$

$$=\sum_{s=1}^{2}\int_{\rho_{s-1}}^{\rho_s}\frac{Q_s\left(\mu_{n,k}\rho\right)}{\alpha_s^2}\rho f\left(\rho\right)d\rho, \quad (25)$$

where

$$Q_0\left(\mu_{n,k}\rho\right),\alpha\left(\rho\right)=\begin{cases}Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right), & \alpha_1^2 \text{ if } \rho\in\left(\rho_0,\rho_1\right)\\ Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right), & \alpha_2^2 \text{ if } \rho\in\left(\rho_1,\rho_2\right)\end{cases}.$$

Eigen functions $Q_s(\mu_{n,k}\rho)$ and eigenvalues $\mu_{n,k}$ can be defined by solving the Sturm-Liouville problem

$$\frac{d^2Q_s}{d\rho^2}+\frac{1}{\rho}\frac{dQ_s}{d\rho}-\frac{n^2}{\rho^2}+\frac{\mu^2_{n,k}}{\alpha_s^2}Q_s=0; \quad (26)$$

$$Q_1(0)<0;\ Q_2\left(\frac{\mu_{n,k}}{\alpha_s}\right)=0; \quad (27)$$

$$\begin{cases}Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)=Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)\\ \lambda_1\frac{\partial Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\partial\rho}=\lambda_2\frac{\partial Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\partial\rho}\end{cases}, \quad (28)$$

where $s = 1, 2$.

By solving the Sturm-Loviull problem (26–28), we obtain

$$Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)=\frac{J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)}{J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)};$$

$$Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)},$$

where

$$\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=Y_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)J_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)-$$
$$-J_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)Y_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right);$$

$J_n(x)$, $Y_n(x)$ are the Bessel functions of the 1st and 2nd types of the nth order, correspondingly [4].

Eigenvalues of $\mu_{n,k}$ are defined by solving the transcendental equations

$$\frac{\mu_{n,k}J'_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\alpha_1J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}=\sigma\frac{H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}, \quad (29)$$

where

$$H\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_2}\left[Y_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)J'_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)-\right.$$
$$\left.-J_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)Y'_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)\right];$$

$$\sigma=\frac{\lambda_2}{\lambda_1}.$$

Formula of the inverse transformation can be written as

$$f\left(\rho\right)=\sum_{n=1}^{\infty}\frac{Q_0\left(\mu_{n,k}\rho\right)}{\left\|Q_0\left(\mu_{n,k}\rho\right)\right\|^2}\bar{f}\left(\mu_{n,k}\right), \quad (30)$$

where

$$\left\|Q_0\left(\mu_{n,k}\rho\right)\right\|^2=\frac{\rho_1^2}{2\alpha_1^2}\left\{\left(1-\frac{n^2\alpha_1^2}{\mu^2_{n,k}\rho_1^2}\right)+\left[\frac{\mu_{n,k}J'_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\alpha_1J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}\right]^2\right\}+$$

$$+\frac{\rho_2^2}{2\alpha_2^2}\left[\frac{\alpha_2H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)}{\mu_{n,k}\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]^2-$$

$$-\frac{\rho_1^2}{2\alpha_2^2}\left\{\left(1-\frac{n^2\alpha_2^2}{\mu^2_{n,k}\rho_1^2}\right)+\left[\frac{\alpha_2H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\mu_{n,k}\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]^2\right\}.$$

By applying the integral transformation (25) to the system of integral equations (19) and by taking into account (1) we obtain the following system of ordinary differential equations

$$\frac{d\bar{\tilde{\theta}}_n^{(i)}}{dt}+\vartheta_n^{(i)}\left[\bar{\tilde{\theta}}_n^{(m_i)}+\tau_r\frac{d\bar{\tilde{\theta}}_n^{(m_i)}}{dt}\right]+\tau_r\frac{d^2\bar{\tilde{\theta}}_n^{(i)}}{dt^2}=$$

$$=\mu_{n,k}\Omega_{\text{n,k}}^{(i)}-\mu_{n,k}^2\bar{\tilde{\theta}}_n^{(i)}\ \text{-}\chi\lambda_m^2\bar{\tilde{\theta}}_n^{(i)}, \qquad (31)$$

where

$$\Omega_{n,k}^{(i)}=\frac{1}{\alpha_2}Q_2{}'\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)\tilde{V}_n^{(i)},$$

with initial conditions

$$\bar{\tilde{\theta}}_n^{(i)}\left(\mu_{n,k},t\right)=0;\quad \frac{\partial\bar{\tilde{\theta}}_n^{(i)}\left(\mu_{n,k},t\right)}{\partial t}=0,\ (i=1,2). \qquad (32)$$

Let us apply the Laplace integral transformation [5] with conditions of (32) to the system of integral equations (31)

$$\hat{f}(s)=\int_0^\infty f(\tau)\,e^{-s\tau}\,\mathrm{d}\tau.$$

As a result we obtain a system of algebraic equations relatively to the $\bar{\tilde{\hat{\theta}}}_n^{(i)}$

$$s\bar{\tilde{\hat{\theta}}}_n^{(i)}+\vartheta_n^{(i)}\left(\bar{\tilde{\hat{\theta}}}_n^{(m_i)}+\tau_r s\bar{\tilde{\hat{\theta}}}_n^{(m_i)}\right)+\tau_r s^2\bar{\tilde{\hat{\theta}}}_n^{(i)}=$$

$$=q_{n,k}\left(\frac{\mu_{n,k}\hat{\Omega}_{n,k}^{(i)}}{q_{n,k}}-\bar{\tilde{\hat{\theta}}}_n^{(i)}\right), \qquad (33)$$

where i = 1, 2; $q_{n,k}=\mu_{n,k}^2+\chi\lambda_m^2$.

By solving the system of equations (33) we obtain

$$\bar{\tilde{\hat{\theta}}}_n^{(i)}=\mu_{n,k}\frac{\hat{\Omega}_{n,k}^{(i)}\left(\tau_r s^2+s+q_{n,k}\right)+}{\left(\tau_r s^2+s+q_{\text{n,k}}\right)^2+}\frac{+(-1)^{i+1}\omega n\hat{\Omega}_{n,k}^{(m_i)}\left(1+s\tau_r\right)}{+\omega^2n^2\left(1+s\tau_r\right)^2},$$

where i = 1, 2.

By applying the Laplace formulas of inverse transformation [6] to the function we obtain the original functions

$$\bar{\tilde{\theta}}_n^{(1)}(t)=\sum_{j=1}^{2}\zeta_{n,k}(s_j)\Big\{\hat{\Omega}_{\text{n,k}}^{(1)}(s_j)\cdot\left[\left(2\tau_r s_j+1\right)+\tau_r\omega nI\right]+$$
$$+\hat{\Omega}_{n,k}^{(2)}(s_j)\cdot\left[\tau_r\omega n-\left(2\tau_r s_j+1\right)I\right]\Big\}\left(e^{s_j t}-1\right)+$$
$$+\sum_{j=3}^{4}\zeta_{n,k}(s_j)\Big\{\hat{\Omega}_{\text{n,k}}^{(1)}(s_j)\cdot\left[\left(2\tau_r s_j+1\right)-\tau_r\omega nI\right]+$$
$$+\hat{\Omega}_{\text{n,k}}^{(2)}(s_j)\cdot\left[\tau_r\omega n+\left(2\tau_r s_j+1\right)I\right]\Big\}\left(e^{s_j t}-1\right); \qquad (34)$$

$$\bar{\tilde{\theta}}_n^{(2)}(t)=\sum_{j=1}^{2}\zeta_{n,k}(s_j)\Big\{\hat{\Omega}_{\text{n,k}}^{(2)}(s_j)\cdot\left[\left(2\tau_r s_j+1\right)+\tau_r\omega nI\right]-$$
$$-\hat{\Omega}_{\text{n,k}}^{(1)}(s_j)\cdot\left[\tau_r\omega n-\left(2\tau_r s_j+1\right)I\right]\Big\}\left(e^{s_j t}-1\right)+$$
$$+\sum_{j=3}^{4}\zeta_{n,k}(s_j)\Big\{\hat{\Omega}_{\text{n,k}}^{(2)}(s_j)\cdot\left[\left(2\tau_r s_j+1\right)-\tau_r\omega nI\right]-$$
$$-\hat{\Omega}_{\text{n,k}}^{(1)}(s_j)\cdot\left[\tau_r\omega n+\left(2\tau_r s_j+1\right)I\right]\Big\}\left(e^{s_j t}-1\right), \qquad (35)$$

where $\zeta_{n,k}(s_j)=\dfrac{0.5s_j^{-1}\mu_{n,k}}{\left(2\tau_r s_j+1\right)^2+\left(\tau_r\omega n\right)^2}$, and values s_j for j = 1, 2, 3, 4 are defined by formulas

$$s_{1,2}=\frac{\left(\tau_r\omega ni-1\right)\pm\sqrt{\left(1+\tau_r\omega ni\right)^2-4\tau_r q_{n,k}}}{2\tau_r};$$

$$s_{3,4}=\frac{\left(\tau_r\omega ni+1\right)\pm\sqrt{\left(1-\tau_r\omega ni\right)^2-4\tau_r q_{n,k}}}{2\tau_r}.$$

By this way, and by taking into account the formulas of inverse transformation (10, 18 and 30), we receive a temperature field of the rotating solid double-layer finite cylinder with taking into account the finite velocity of the heat conductivity

$$\theta(\rho,\varphi,z,t)=\sum_{n=-\infty}^{+\infty}\left\{\sum_{k=1}^{\infty}\left\langle 2\sum_{m=1}^{\infty}\left[\bar{\tilde{\theta}}_n^{(1)}(t)+I\cdot\bar{\tilde{\theta}}_n^{(2)}(t)\right]\times\right.\right.$$
$$\left.\left.\times\sin(\pi\cdot\mathrm{m}\cdot\mathrm{z})\right\rangle\frac{Q_0\left(\mu_{n,k}\rho\right)}{\left\|Q_0\left(\mu_{n,k}\rho\right)\right\|^2}\right\}\cdot\exp(in\varphi), \qquad (36)$$

where values $\bar{\tilde{\theta}}_n^{(1)}(t)$ and $\bar{\tilde{\theta}}_n^{(2)}(t)$ are defined by the formulas (34, 35).

With the view of calculating the temperature fields $\theta(\rho,\varphi,z,t)$ by formula (36), new software was created in the object-oriented programming language C#, which was integrated into the development framework Microsoft Visual Studio 2010, and which can operate with any operational system on the basis of the version Microsoft. NET Framework or higher. In the numerical calculation s, sums of series were replaced by the partial sums with the accuracy of 10^{-4}.

In order to test the mathematical model, the temperature fields were calculated with the help of the created software (37).

Temperature on the cylinder surface was given as

$$\theta(1,\varphi,z,t)=\Theta_1(\varphi)\eta(l_1-z)+$$
$$+\Theta_2(\varphi)\left[\eta(1-l_2-z)-\eta(l_1-z)\right]+$$
$$+\Theta_3(\varphi)\cdot\left[\eta(1-z)-\eta(1-l_2-z)\right],$$

where

$$\Theta_2(\varphi) = 2 \cdot \pi^{-1} \cdot \varphi \cdot \eta(0{,}5\pi - \varphi) +$$

$$+\eta(1{,}5\pi - \varphi) - \eta(0{,}5\pi - \varphi) + \left[1 + 2 \cdot \pi^{-1} \cdot \eta(1{,}5\pi - \varphi)\right] \times$$

$$\times \left[\eta(2 \cdot \pi - \varphi) - \eta(1{,}5\pi - \varphi)\right];$$

$$\Theta_1(\varphi) = \Theta_3(\varphi) = 0.03;$$

$$\theta(\rho, \varphi, z, t) = \theta(\rho, \varphi, 1, t) = 0;\ \rho_0 = 0;\ \rho_1 = 0.75;\ l = 1;$$

$$l_1 = l_2 = 0.25;\ \eta(z) = \begin{cases} 1 & \text{if} \quad z \geq 0 \\ 1 & \text{if} \quad z \geq 0 \end{cases}.$$

The following values are needed for calculating the cylinder material properties:

$\lambda_1 = 34.8$ W/(m·K); $\tau_r = 10^{-11} s$; $c_1 = 560$ J/(kg·K); $c_2 =$ J/(kg·K); $\gamma_1 = 7800$ kg/m^3; $\gamma_1 = 7820$ kg/m^3; $\sigma = 1.5$.

Results of the numerical experiments are shown in Figure in the form of temperature distribution curves at the following parameters z = 0.5; $Pd = 10$($Pd = \dfrac{\omega R^2}{a_1}$), where $F_0 = a_1 t \cdot R^{-2}$.

As it is seen in the Figure, the temperature field at $\rho < 0.2$, is axially symmetrical, and amplitude of the temperature fluctuation becomes less with time.

Conclusions. The first mathematic model has been created for calculating temperature fields in the rotating solid double-layer finite cylinder, in the form of the mathematical physics boundary problem for hyperbolic equations by the Dirichlet conditions and with taking into account the finite velocity of the heat conduction.

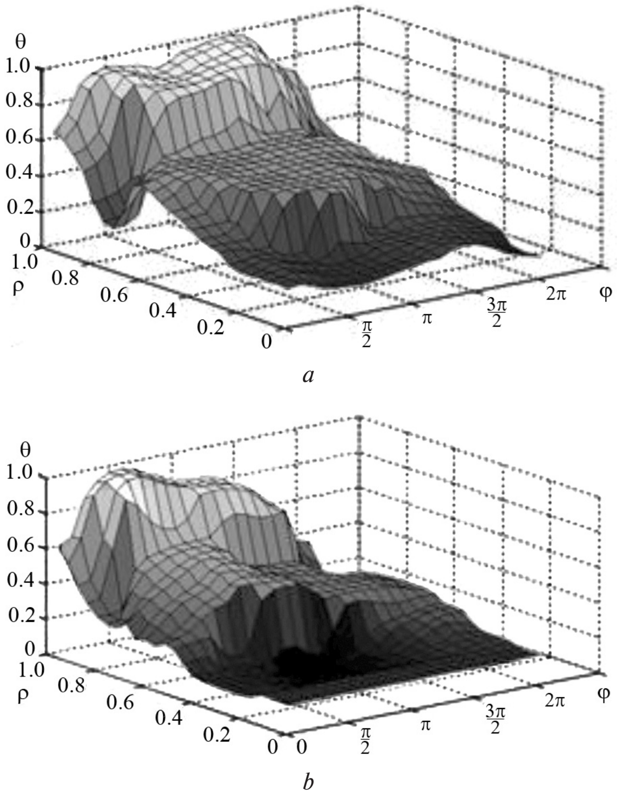

Fig. Distribution of temperature field in the cylinder at different value of the Fourier criteria:
a – $F_0 = 0.1$; *b* – $F_0 = 0.1$

The new integral transformation was created for the solid space with homogeneous layers, with the help of which it became possible to present a temperature field in the double-layer finite cylinder in the form of convergence orthogonal series by Bessel and Fourier functions.

The obtained analytical solution of the generalized boundary problem of heat exchange in the solid double-layer finite cylinder, which takes into account the value finiteness of the heat conduction velocity, can be used for calculating temperature fields, which occur in different technical systems (forming rolls, satellites, turbines, etc.).

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Мета. Побудова нової узагальненої просторової математичної моделі розрахунку температурних полів у суцільному, двоскладовому скінченному циліндрі у вигляді крайової задачі математичної фізики для гіперболічних рівнянь теплопровідності з умовами Діріхле (на поверхні циліндра температура є неперервною функцією від координат) і знаходження розв'язку отриманої крайової задачі.

Методика. Використання відомих інтегральних перетворень Лапласа, Фур'є, а також розробленого нового інтегрального перетворення для кусково-однорідного простору.

Результати. Знайдено нестаціонарне температурне поле в суцільному двошаровому скінченному циліндрі, що обертається, у циліндричній системі координат, з урахуванням кінцевої швидкості поширення тепла. Теплофізичні властивості циліндрі в кожному шарі є сталі за ідеального теплового контакту між шарами, а внутрішні джерела тепла відсутні. У початковий момент часу температура циліндра є стала, а на зовнішній поверхні циліндра температура відома.

Наукова новизна. Уперше побудована просторова математична модель розрахунку температурних полів у двошаровому скінченному циліндрі, що обертається, з урахуванням кінцевої швидкості поширення тепла, у вигляді крайової задачі математичної фізики для гіперболічних рівнянь теплопровідності із граничними умовами Діріхле. У роботі побудоване інтегральне перетворення для кусково-однорідного простору, із застосуванням якого знайдено температурне поле двошарового скінченного циліндра у вигляді збіжних рядів за функціями Бесселя й Фур'є.

Практична значимість. Знайдене аналітичне рішення узагальненої крайової задачі теплообміну двошарового циліндра, що обертається, з урахуванням відомого часу релаксації поширення тепла, може застосовуватися при знаходженні полів температури, які виникають у багатьох технічних системах (у прокатних валках, супутниках, турбінах і т. і.).

Ключові слова: *крайова задача Діріхле, інтегральне перетворення, час релаксації, двошаровий скінченний циліндр*

Цель. Построение новой обобщенной пространственной математической модели расчета температурных полей в сплошном, двухслойном конечном цилиндре в виде краевой задачи математической физики для гиперболических уравнений теплопроводности с условиями Дирихле (на поверхности цилиндра температура есть непрерывная функция от координат) и нахождения решений полученной краевой задачи.

Методика. Применение известных интегральных преобразований Лапласа, Фурье, а также разработанного нового интегрального преобразования для кусочно-однородного пространства.

Результаты. Найдено нестационарное температурное поле в сплошном, двухслойном конечном цилиндре, который вращается, в цилиндрической системе координат, с учетом конечной скорости распространения тепла. Теплофизические свойства цилиндра в каждом слое постоянные при идеальном тепловом контакте между слоями, а внутренние источники тепла отсутствуют. В начальный момент времени температура цилиндра является постоянной, а на внешней поверхности цилиндра температура известна.

Научная новизна. Впервые построена пространственная математическая модель расчета температурных полей в двухслойном конечном вращающемся цилиндре, с учетом конечной скорости распространения тепла, в виде краевой задачи математической физики для гиперболических уравнений теплопроводности с граничными условиями Дирихле. В работе построено интегральное преобразование для кусочно-однородного пространства, с применением которого найдено температурное поле двухслойного конечного цилиндра в виде сходящихся рядов по функциям Бесселя и Фурье.

Практическая значимость. Найденное аналитическое решение обобщенной краевой задачи теплообмена двухслойного вращающегося цилиндра, с учетом известного времени релаксации распространения тепла, может найти применение при определении полей температуры, которые возникают во многих технических системах (в прокатных валках, спутниках, турбинах и т. д.).

Ключевые слова: *краевая задача Дирихле, интегральное преобразование, время релаксации, двухслойный конечный цилиндр*

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