

парата на некоторые физико-механические свойства цементных растворов и цементного камня, получаемого из них после твердения. Проведенные экспериментальные исследования позволили выявить оптимальные параметры для улучшения этих свойств, что применительно к струйной технологии дает возможность существенно ее усовершенствовать. Возможно уменьшить расход цемента при сохранении заданной прочности закрепления,

или увеличить прочность закрепления при таком же количестве закрепляющего раствора.

**Ключевые слова:** *индукция магнитного поля, магнитная обработка, струйная технология закрепления, цементные растворы*

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## EFFECT OF HARMONIC OSCILLATIONS ON A CRACK INITIATION IN THE ROCK MASS

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## ВПЛИВ ГАРМОНІЙНИХ КОЛИВАНЬ НА СТРАГУВАННЯ ТРІЩИНИ В ПОРОДНОМУ МАСИВІ

**Purpose.** The purpose is to improve, develop and generalize the criterion of a crack initiation taking into account the simultaneous effect of the rock stress state and oscillatory processes. The influence of the oscillation amplitude-frequency characteristics on the critical crack length has to be studied as well.

**Methodology.** The methodological basis of the problem solution is a comprehensive approach that includes the mathematical analysis, numerical methods for solving transcendental equations, analysis of the results in the mathematical package Mathcad.

**Findings.** A crack initiation criterion has been developed considering both static stresses (tensile and compressive) and dynamic stress in an elastic wave spreading in the rock mass. The critical length of the initiated crack is determined depending on oscillation amplitude and frequency, static stress and rock crack resistance. The abrupt change in the critical crack length is defined as an indication of the dynamic rock failure. The values of oscillation frequency provoking a decrease in the critical crack length and dynamic rock failure are determined for different rocks.

**Originality.** New approach is used to describe the rock destruction considering the elastic oscillations in the rock mass. The criterion of a crack initiation has been generalized to consider the action of both tensile and compressive stress near by the crack and harmonic stress in an elastic wave. So, based on a common time-space failure criterion, we take into account the stress which is a sum of quasi-stationary and harmonic components. In this paper we focused on the critical crack length that provokes the crack initiation. The influence of oscillation amplitude and frequency on critical length altering has been estimated for different values of the crack resistance.

**Practical value.** The results obtained in the study of criterion crack initiation, can be used to improve the acoustic forecast of gas-dynamic phenomena by the amplitude-frequency characteristics of oscillations. For this purpose, we defined ranges of harmonic oscillation frequencies in some rocks (fine-grained sandstone, limestone, siltstone, coal) in which the initiating of "short" cracks occurs. And we got the real conditions (for coal), in which a slight change in the oscillation amplitude in the array of rocks leads to a jump in the length of the cracks.

**Keywords:** *crack initiation, the amplitude, crack resistance, critical crack length*

**Introduction.** Dynamic phenomena originating in mines, such as outburst and rockburst have been known for about 250 years. The problem of combating these phenomena in coal mines has been relevant in Ukraine for the past 85 years. The need to predict, prevent and avoid the negative effects provides the development of various methods focused on forecasting the outburst at certain stage of mining. One of the ways to deal with the

gas-dynamic phenomena is the diagnosis of the rock stress-strain state and identification of the areas of potentially hazardous emissions of coal, rock and gas. The acoustic monitoring of a coal seam state as a source of a gas-dynamic phenomena is an effective diagnosis method. It includes the following basic stages: the seam probing by artificial acoustic signal, receiving a part of signal that passes through the stressed rock mass and subsequent analysis of the return signal. Based on this analysis the conclusion concerning the rock mass state can be

made regarding two types of rock inhomogeneity: structural faults and dramatic stress concentration (strength inhomogeneity). Specific features of structural inhomogeneity prediction are given in [1]. Locating the strength inhomogeneity can result in the outburst prediction.

The effectiveness and reliability of the forecast depends on the validity of an evaluation criterion. There have been developed acoustic control systems based on processing the amplitude-frequency characteristics of the return signal. The outburst prediction involves a criterion developed empirically based on statistical data processing. However, theoretical substantiation of the criterion has not been sufficiently developed. The lack of theoretical assumptions hinders the improvement of forecasting technique and increasing the prediction reliability. That is why the correlation between the rock stress state components and harmonic oscillations in rock mass should be considered. The rock destruction is always associated with a crack propagation. N. Morozov and Yu. Petrov [2] developed a criterion of a crack initiation under stress acting. The criterion is based on a general space-time approach to studying a solid failure. In this paper we consider the effect of elastic oscillation in rock mass on a crack initiation.

**Presentation of the main research.** Generally, the space-time changing in rock stress-strain state is caused by the rock excavation and openings creation. But there are not only quasi-static stresses induced in rock mass. The alternating stresses occur in elastic waves inside the rocks caused by mining mechanisms operating. The elastic waves are generated by the rock brittle failure as well.

A stress tensor  $T_{\sigma(t)}$  acting in a crack vicinity is a sum of a quasi-static stress tensor  $T_{qs(t)}$  and a wave stress tensor  $T_{w(t)}$  occurring at a particular point of the rock mass

$$T_{\sigma(t)} = T_{qs(t)} + T_{w(t)}.$$

The stress that is normal to the crack plane should be taken in the form

$$\sigma_1(t) = \sigma_0 + k \cdot (t - t_0) + a \cdot \cos[2\pi\vartheta \cdot (t - t_0) + \varphi_0]. \quad (1)$$

Here the expression  $\sigma_0 + k \cdot (t - t_0)$  is the stress component not related to oscillations in (1);  $k = \left(\frac{d\sigma}{dt}\right)_{t=t_0}$  is the rate of quasi-static stress  $\sigma_0$  altering; the expression  $a \cdot \cos[2\pi\vartheta \cdot (t - t_0) + \varphi]$  determines the vibration load, where  $a$ ,  $\vartheta$ ,  $\varphi_0$  are the oscillation amplitude, frequency and phase respectively.

The crack initiation (the start, readiness to propagate) would be possible at the critical combination of static and harmonic loads. The condition of crack initiation is defined by the crack resistance of rocks.

The criterion of a crack initiation under joined impact of harmonic oscillation and static tensile stress in crack vicinity has been developed in [3]. In this paper, the generalization of crack initiation condition is carried out considering the existence in rock mass of both compressive and tensile stresses. The generalized criterion of a crack initiation looks like

$$\bar{a} \cdot \text{sinc}(\pi \cdot \bar{l}) \cdot \cos(\pi \cdot \lfloor \bar{l} \rfloor) \geq \frac{K_{cv}}{\sqrt{\bar{l}}} + \frac{\alpha \cdot \bar{l}}{2} - \text{sign}(\sigma_0). \quad (2)$$

The inequality (2) contains the following dimensionless variables:  $\bar{l} = \frac{l \cdot \vartheta}{C_R}$  is a relative crack length, where  $l$  is a specific dimension (a radius) of a circular cracks,  $C_R$  is a rate of the Rayleigh wave;  $\bar{a} = \frac{a}{|\sigma_0|}$  is a relative oscillation amplitude;  $K_{cv} = \frac{K_{lc}}{2|\sigma_0|} \cdot \sqrt{\frac{\pi \vartheta}{C_R}}$  is a complex parameter determined by the ratio of a rock crack resistance  $K_{lc}$  and acting stress  $\sigma_0$ ;  $\alpha = \frac{k}{|\sigma_0| \cdot \vartheta}$  is a coefficient characterizing the rate of the quasi-stationary stress change;  $\lfloor \bar{l} \rfloor$  is an integer part of the value  $\bar{l}$  (the floor of  $\bar{l}$ ). The term  $\text{sign}(\sigma_0) = 1$  means the existence of tensile stress and the term  $\text{sign}(\sigma_0) = -1$  points at existence of compressive stresses.

Moreover, the inequality must be true

$$\frac{K_{cv}}{\sqrt{\bar{l}}} + \frac{\alpha \bar{l}}{2} - \text{sign}(\sigma_0) \geq 0. \quad (3)$$

If the inequality (3) is not true the crack can be initiated (started) under action of only static stress without impact of oscillation load.

Let only static tensile stress and vibration load act in the crack vicinity, that is, in equation (1)  $\sigma_0 > 0$  and  $k = 0$ . Then  $\text{sign}(\sigma_0) = 1$  and  $\alpha = 0$  therefore, the crack initiation criterion (2) takes the form

$$\bar{a} \cdot \text{sinc}(\pi \cdot \bar{l}) \cdot \cos(\pi \cdot \lfloor \bar{l} \rfloor) \geq \frac{K_{cv}}{\sqrt{\bar{l}}} - 1. \quad (4)$$

**Impact of amplitude increasing on the critical crack length.** Let the relative amplitude  $\bar{a}$  be expressed from (4)

$$\bar{a}(\bar{l}) = \frac{\frac{K_{cv}}{\sqrt{\bar{l}}} - 1}{\text{sinc}(\pi \cdot \bar{l})} \cdot \cos(\pi \cdot \lfloor \bar{l} \rfloor), \quad (5)$$

where  $\bar{l} \neq 1, 2, \dots$  and  $\frac{K_{cv}}{\sqrt{\bar{l}}} - 1 \geq 0$  or  $0 < \bar{l} \leq K_{cv}^2$ .

The function  $\bar{a}(\bar{l})$  means that if a crack has a length  $\bar{l}_0$ , then it can initiate at amplitude value  $\bar{a}(\bar{l}_0)$  under given condition defined by the complex parameter  $K_{cv}$ . The diagram of function  $\bar{a}(\bar{l}_0)$  is shown in Fig. 1. The graphics are built formally according to expression (4), but they should be corrected to satisfy the physical sense of the problem. This means the condition  $0 < \bar{l} \leq K_{cv}^2$  should be met.

Then cracks of length  $\bar{l}_1 \in (0; K_{cv}^2]$  can be initiated at the amplitude value  $\bar{a}(\bar{l}_1)$ , but cracks of length  $\bar{l}_2 \in (K_{cv}^2; +\infty)$  can be initiated under the action of only static stress without impact of oscillation load. So, the diagram part corresponding to negative value of amplitude  $\bar{a}$  should be replaced by a horizontal half-line  $\bar{a} = 0$  within the interval  $\bar{l}_2 \in (K_{cv}^2; +\infty)$  (Fig. 2).

At  $K_{cv} \geq 1$  the  $\bar{a}(\bar{l})$  diagram consists of two branches (Fig. 1). This means that different values  $\bar{l}$  correspond to the same value of the amplitude  $\bar{a}$ . In this case we should put the smallest value of  $\bar{l}$  as a critical crack length under given amplitude value.

Discontinuities of the function  $\bar{a}(\bar{l})$  are of particular interest. The critical crack value changes abruptly (by a jump) in this case. The situation matches to the horizontal segment on the corrected function  $\bar{a}(\bar{l})$  diagram (Fig. 2). The horizontal segment connects two points, located on two branches of function  $\bar{a}(\bar{l})$  (5). The coordinates of the left end of the segment  $(\bar{l}_{left}, \bar{a}_*)$  can be determined as a point at which a derivative of the left function branch equals zero.

After differentiation one can derive the transcendental equation to determine  $\bar{l}_{left}$

$$\sin(\pi \cdot \bar{l}_{left}) \cdot (K_{cv} - 2 \cdot \sqrt{\bar{l}_{left}}) - 2\pi \cdot \bar{l}_{left} \cdot (K_{cv} - \sqrt{\bar{l}_{left}}) \cdot \cos(\pi \cdot \bar{l}_{left}) = 0. \quad (6)$$

We should take the smallest positive root of the equation (6) as the  $\bar{l}_{left}$  value. Then let us substitute the found

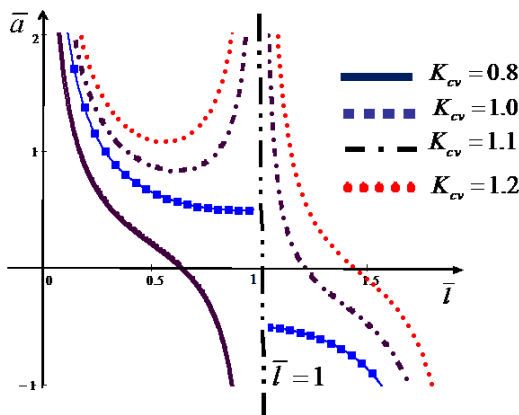


Fig. 1. Graph of the function  $\bar{a}(\bar{l})$  according to equation (5)

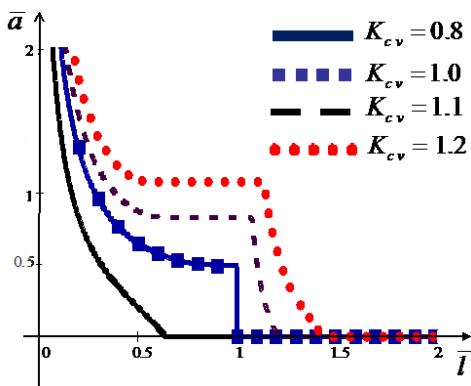


Fig. 2. Corrected diagram of the function  $\bar{a}(\bar{l})$  at different values of complex parameter  $K_{cv}$

value  $\bar{l}_{left}$  into (5) and obtain the amplitude value  $\bar{a}_*$ , at which the jump of the crack critical length occurs. To determine the right end of the horizontal segment  $\bar{l}_{right}$  we should find the intersection of the line  $\bar{a} = \bar{a}_*$  with the second branch of the function  $\bar{a}(\bar{l})$  (5), i. e. to find the greatest positive root of the transcendental equation (6).

Calculations carried out in this way make it possible to find the desired function  $\bar{a}(\bar{l})$  at any value of the complex parameter  $K_{cv}$  (Fig. 2).

The corrected diagram in Fig. 2 shows that under condition defined by parameter  $K_{cv} = 0.8$  the cracks with relative length  $\bar{l}_3 \approx 0.64$  can be initiated without oscillation impact, i. e. only under acting of the static stress. But if the amplitude increases, the cracks of smaller length became ready to start.

For example, in case of fine-grained sandstone the crack resistance is  $K_{Ic} = 1.47 \text{ MPa} \cdot \sqrt{m}$ , the rate of the

Rayleigh wave is  $C_R = 2400 \frac{m}{s}$  [4]. Then at tensile stress

$\sigma_0 = 0.9 \text{ MPa}$  and frequency of oscillation  $\vartheta = 800 \text{ Hz}$  the cracks of real length  $l = 1.92 \text{ m}$  can be initiated at a very small value of amplitude (that is close to zero). If the amplitude of oscillation increases the cracks of smaller length ( $l < 1.92 \text{ m}$ ) can be initiated as well. Precisely initiating the cracks of a small length can be interpreted as a risk of dynamic phenomena in the rock mass.

At parameter  $K_{cv} = 1$  and relative amplitude altering from  $\bar{a} = 0$  to  $\bar{a} \approx 0.5$ , the cracks of relative length  $\bar{l} \approx K_{cv}^2 = 1$  can be initiated.

At parameter  $K_{cv} = 1.1$  the transcendental equation (6) should be used to determine the point  $(\bar{l}_{left}, \bar{a}_*)$ , mentioned above

$$\sin(\pi \cdot \bar{l}_{left}) \cdot (1.1 - 2 \cdot \sqrt{\bar{l}_{left}}) - 2\pi \cdot \bar{l}_{left} \cdot (1.1 - \sqrt{\bar{l}_{left}}) \cdot \cos(\pi \cdot \bar{l}_{left}) = 0.$$

The root of this equation is  $\bar{l}_{left} = 0.618$ .

Similar actions give result  $\bar{l} = 0.558$  at  $K_{cv} = 1.2$ . So the increase in parameter  $K_{cv}$  reduces the length  $\bar{l}$  of initiated cracks. The amplitude values at the moment of the crack initiating equal  $\bar{a}(0.618) = 0.832$  and  $\bar{a}(0.558) = 1.081$ .

The inverse function  $\bar{l}(\bar{a})$  is more interesting practically. It can be derived from the equation (4) directly and by inversion of  $\bar{a}(\bar{l})$  function. Analysis carried out above showed that diagram of  $\bar{l}(\bar{a})$  function has horizontal segments at  $K_{cv} \geq 1$  similar to  $\bar{a}(\bar{l})$  diagram. Critical lengths of cracks are represented in Fig. 3 depending on relative amplitude at different values of the parameter  $K_{cv}$ .

The function  $\bar{l}(\bar{a})$  also demonstrates that the increase in oscillation amplitude causes the instability in rock mass structure that means initiating and propagating the “short” cracks. However, the degree of critical crack length reduction (the crack ability to start and propagate) alters significantly depending on the stress

static and harmonic components and crack resistance of rock mass. That is why a derivative  $\frac{d\bar{a}}{d\bar{l}}$  should be used

as a local characteristic of  $\bar{a}(\bar{l})$  function (Fig. 4).

The derivative is not defined at the points corresponding to the jumps of crack length. The greatest sensitivity of critical crack length to the amplitude change takes place at those points. It should be noted, however, that in situ there are oscillations with more than one amplitude and frequency. In most cases an oscillation can be represented as a set of harmonic oscillations with different frequencies and amplitudes. Besides, in most cases the oscillation amplitude does not exceed a static stress in the crack vicinity ( $\bar{a}$  does not exceed  $\sigma_0$ ). That is why the additional characteristic should be introduced to clarify how the critical crack length depends on the oscillation amplitude. Let us consider the average value of length derivative with respect to the amplitude within an interval  $[0, \bar{a}_0]$

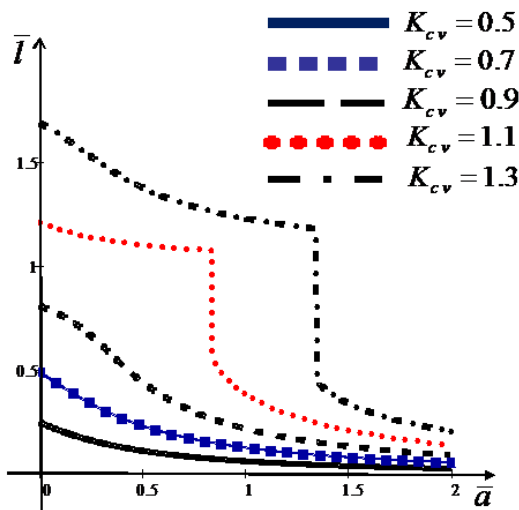


Fig. 3. Critical crack length  $\bar{l}$  depending on the relative amplitude  $\bar{a}$  at different values of the parameter  $K_{cv}$

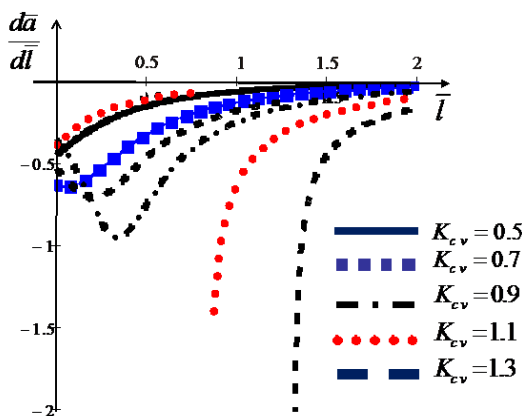


Fig. 4. Derivative  $\frac{d\bar{a}}{d\bar{l}}$  at different values of the parameter  $K_{cv}$

$$s(\bar{a}_0, K_{cv}) = \frac{1}{\bar{a}_0} \cdot \int_0^{\bar{a}_0} \frac{\partial \bar{l}}{\partial \bar{a}} \cdot d\bar{a} = \frac{1}{\bar{a}_0} \cdot (\bar{l}(\bar{a}_0, K_{cv}) - \bar{l}(0, K_{cv})). \quad (7)$$

The function  $s(\bar{a}_0, K_{cv})$  characterizes the sensitivity of the critical crack length to the amplitude variation. It depends on the length of the averaging interval  $\bar{a}_0$ . The most typical variation of the amplitude corresponds to the interval  $[0; 0.5]$ . Then at  $\bar{a}_0 = 0.5$  the sensitivity of critical crack length with respect to the amplitude, variation  $s(\bar{a}_0, K_{cv})$  depends on the parameter  $K_{cv}$  as it is shown in Fig. 5.

Diagrams in Fig. 5 show that abrupt change in the critical crack length caused by an increase in oscillation amplitude occurs in the interval  $K_{cv} \in (0.8; 1.0)$ . At lower values of  $K_{cv}$  the amplitude impact on the critical crack length is significantly less. At larger values of parameter  $K_{cv}$  the abrupt length change becomes possible only in the area of practically unrealizable amplitude values.

So, based on the results above, we should put the value  $K_{cv} = 1$  as a critical one. At such  $K_{cv}$  value the amplitude  $\bar{a}$  increase by 20 % (from 0.5 to 0.6) reduces the critical crack length by 2 times. Besides, such an oscillation amplitude value exists, that before reaching up this value the critical crack length hardly changes under increasing amplitude. But after reaching up this value the critical crack length reduces very abruptly.

**Comparing theoretical results and experimental data.**

Mirer and Maslennikov (article “About the control of the outburst of the slaughtering by the spectral characteristics of acoustic signals”) studied in situ the amplitude – frequency specters of acoustic signal generated in rock mass by mining cutting mechanism. They proved that during an outburst the amplitude of registered signal increased twice in the interval of frequency 800–1200 Hz. This fact is recognized normative [5]. That is why the theoretical study of the crack length altering under the amplitude increase should be focused just on this particular frequency interval.

All previous research has been carried out using relative dimensionless values. So, the real absolute magnitudes should be estimated to verify their identity to real conditions of mining. The properties of rock mass are characterized by a crack resistance  $K_{1c}$  and a rate of the Rayleigh wave  $C_R$ . These parameters accumulated from different sources are represented in Table.

The results of calculations for fine-grained sandstone are given in article [3]. Numerical estimation has

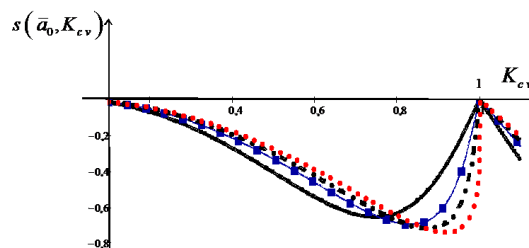


Fig. 5. Sensitivity of the critical crack length with respect to the amplitude variation  $s(\bar{a}_0, K_{cv})$  depending on the parameter  $K_{cv}$  at different values of averaging interval  $\bar{a}_0$



shown that insignificant increasing in the relative amplitude (from 0.4 to 0.8) at frequency of 1145 Hz, provokes the abrupt reducing the length of initiated cracks (from 4 to 1.6 m). Taking into account new results represented in Fig. 2 the frequency diapasons should be determine at which the amplitude increase causes the initiation of short cracks in different types of rock. The relative length values  $\bar{l}$  are obtained from transcendent equation

$$\bar{a} \cdot \sin c(\pi \bar{l}) - \left( \frac{K_{cv}}{\sqrt{\bar{l}}} - 1 \right) = 0.$$

The oscillation frequency  $\vartheta$  is calculated with respect to the crack resistance  $K_{lc}$ , and rate of the Rayleigh wave  $C_R$  and static tensile stress  $\sigma_0$  that define the parameter  $K_{cv}$  by equality  $K_{cv} = \frac{K_{lc}}{2|\sigma_0|} \cdot \sqrt{\frac{\pi \vartheta}{C_R}}$ .

In calculations the static tensile stress is assumed to be  $\sigma_0 = 0.9$  MPa, the relative amplitude  $\bar{a}$  changes from 0.4 to 0.8.

In case of limestone the increase in the amplitude by 2 times causes reducing the initiated crack by 2–2.6 times. This fact takes place at frequency varied from 890 to 1400 Hz.

In case of aleurolite (argillite) the increase in the amplitude by 2 times causes reducing the initiated crack by 2.3 times at frequency varied from 2000 to 3000 Hz. So, at tensile stress  $\sigma_0 = 0.9$  MPa the oscillations with frequency in the diapason 900–1200 Hz are not dangerous from point of view jump the crack length in case of aleurolite (argillite). In case of coal the increase in the amplitude by 2 times causes reducing the initiated crack by 1.67 times at frequency varied from 600 to 3600 Hz. Therefore, the frequency diapason 600–3600 Hz should not be considered dangerous from point of view jump of the crack length for coal seam if tensile stress is  $\sigma_0 = 0.9$  MPa. But situation changes dramatically if the tensile stress is  $\sigma_0 = 0.25$  MPa. At such value of the static stress the increase in the amplitude by 2 times causes reducing the initiated crack by 4 times.

The important fact should be noted in this case. At oscillation frequency  $\vartheta = 1340$  Hz the critical crack length changes insignificantly ( $\bar{l} \in [0.9; 1.2]$ ) before reaching up the amplitude value  $\bar{a} = 0.8$ . But after ex-

ceeding this amplitude value the critical crack length reduces very abruptly by 4 times down to the value  $\bar{l} = 0.29$ .

Hence, in coal case the frequency diapason 700–1400 Hz at static tensile stress  $\sigma_0 = 0.25$  MPa should be considered dangerous from point of view of jump of the crack length.

**Conclusions.**

1. The criterion of a crack initiation in rocks under joined impact of harmonic oscillation and static stress has been developed. It has been generalized to consider both types of normal stresses acting in crack vicinity: tensile stress and compressive one.

2. The oscillation amplitude is derived as a function of critical crack length  $\bar{a}(\bar{l})$  and the inverse function  $\bar{l}(\bar{a})$  has been obtained. Increasing the oscillation amplitude causes reducing the length of cracks that are capable to initiate. The degree of the amplitude impact on crack initiation is defined by a ratio of rock cracking resistance and the level of stress acting in the rock mass (by complex parameter  $K_{cv}$ ).

3. Both  $\bar{l}(\bar{a})$  and  $\bar{a}(\bar{l})$  diagrams demonstrate that the increase in oscillation amplitude causes the instability in rock mass structure that means initiating and propagating the “short” cracks.

4. Analysis of the crack critical length derivative with respect to amplitude  $\frac{d\bar{l}}{d\bar{a}}(\bar{a})$  shows that there are points

of derivative discontinuity. The biggest sensitivity of crack critical length occurs at these points. The increase in oscillation amplitude causes altering the  $\bar{l}(\bar{a})$  diagram by jump at these points as well.

5. A crack initiating is most probable if a complex parameter  $K_{cv}$  approaches approximately  $K_{cv} \approx 1$ . So, the value  $K_{cv} = 1$  is considered as a critical combination of the static stress and oscillation amplitude and frequency under given geological conditions defined by the rock crack resistance and the rate of the Rayleigh wave.

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Table

Crack resistance  $K_{lc}$  and a rate of the Rayleigh wave  $C_R$  related to different type of rocks

Rock	$K_{lc}$ , MPa· $\sqrt{m}$	$C_R$ , $\frac{m}{s}$	Source
Sandstone	0.36–1.42 (1.4*)	1954	[4, 6]
Limestone	0.36–1.24 (1.2*)	2073	[4, 6]
Aleurolite	0.53	1786	[4, 7]
Coal	0.27	1012	[4, 6, 7]

\* – values taken into calculation

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**Мета.** Удосконалити, розвинути та узагальнити критерій старту тріщини в попередньо напруженому породному масиві з урахуванням знакозмінних статичних напружень. Встановити вплив амплітудно-частотних характеристик пружних коливань на критичну довжину тріщин.

**Методика.** Методологічною основою вирішення поставленого завдання є комплексний підхід, що включає математичний аналіз, використання чисельних методів розв'язання трансцендентних рівнянь, аналіз результатів у прикладному математичному пакеті.

**Результати.** Розроблено й досліджено узагальнену умову страгування (старту) тріщини внаслідок сумісної дії статичних напружень (розтягування й стискання) і динамічних напружень у пружній хвилі, що розповсюджується у вугільному пласті та породі. Отримана залежність критичної довжини тріщини, за якої відбувається її страгування, від амплітуди й частоти пружних коливань, статичного напруження в околі тріщини та тріщиностійкості порід. Виявлені умови стрибкоподібної зміни критичної довжини, що свідчить про початок динамічного руйнування в породному масиві. Для різних порід і вугілля визначені діапазони частот пружних коливань, що обумовлюють стрибкоподібну зміну критичної довжини тріщини й провокують динамічне явище.

**Наукова новизна.** Використано новий підхід до опису руйнування породного середовища з урахуванням пружних коливань, що розповсюджуються в гірському масиві. Критерій старту (страгування) тріщини розвинений та узагальнений на випадок дії в околі тріщини як напружень розтягування, так і стискання, а також гармонійної складової напружень у пружній хвилі. Встановлено вплив амплітуди й частоти коливань на критичну довжину тріщини за різних значень статичних напружень і коефіцієнта тріщиностійкості порід.

**Практична значимість.** Визначені діапазони частот гармонійних коливань у деяких гірських породах (дрібнозернистий пісковик, вапняк, алевроліт, кам'яне вугілля), в яких створюються умови для старту „коротких“ тріщин. Дані умови є необхідними для початку динамічного руйнування порід. Визначення критичних частотних діапазонів є основою для вдосконалення методики акустичного прогнозу динамічних явищ за аналізом амплітудно-частотних характеристик сигналів у породному масиві.

**Ключові слова:** тріщина, амплітуда, коефіцієнт тріщиностійкості, критична довжина тріщини

**Цель.** Усовершенствовать, развить и обобщить критерий старта трещины в предварительно напряженном породном массиве с учетом знакопеременных статических напряжений. Установить влияние амплитудно-частотных характеристик упругих колебаний на критическую длину трещин.

**Методика.** Методологической основой решения поставленной задачи является комплексный подход, включающий математический анализ, использование численных методов решения трансцендентных уравнений, анализ результатов в прикладном математическом пакете.

**Результаты.** Разработано и исследовано обобщенное условие страгивания (старта) трещины в результате совместного действия статических напряжений (растяжения и сжатия) в породном массиве и динамических напряжений в упругой волне, распространяющейся в угольном пласте и породе. Получена зависимость критической длины трещины, при которой происходит ее страгивание, от амплитуды и частоты упругих колебаний, знакопеременного статического напряжения в окрестности трещины и трещиностойкости пород. Выявлены условия скачкообразного изменения критической длины трещины, которые свидетельствуют о начале динамического разрушения в породном массиве. Для различных пород и угля определены диапазоны частот упругих колебаний, обуславливающих скачкообразное изменение критической длины трещины и провоцирующих динамическое явление.

**Научная новизна.** Использован новый подход к описанию разрушения породной среды с учетом упругих колебаний, распространяющихся в горном массиве. Критерий старта (страгивания) трещины развит и обобщен на случай действия в окрестности трещины как напряжений растяжения, так и сжатия, а также гармонической составляющей напряжения в упругой волне. Установлено влияние амплитуды и частоты колебаний на критическую длину трещины при различных значениях статических напряжений и коэффициента трещиностойкости пород.

**Практическая значимость.** Определены диапазоны частот гармонических колебаний для некоторых горных пород (мелкозернистый песчаник, известняк, алевролит, каменный уголь), в которых создаются условия для старта „коротких“ трещин. Данные условия являются необходимыми для начала динамического разрушения пород. Определение таких критических диапазонов частот является основой для совершенствования методики акустического прогноза динамических явлений по анализу амплитудно-частотных характеристик сигналов в породном массиве.

**Ключевые слова:** трещина, амплитуда, коэффициент трещиностойкости, критическая длина трещины

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