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# DIFFERENTIAL LDPC CODED SYSTEMS WITH MULTIPLE SYMBOL DIFFERENTIAL DETECTION OVER AWGN CHANNELS

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# ДИФЕРЕНЦІАЛЬНІ СИСТЕМИ З LDPC-КОДУВАННЯМ І БАГАТОСИМВОЛЬНИМ ДИФЕРЕНЦІАЛЬНИМ ДЕТЕКТУВАННЯМ В AWGN-КАНАЛАХ

**Purpose.** In this paper, a scheme of differential LDPC coded systems with multiple symbol differential detection (MSDD) was studied to improve the performance of the differential LDPC coded systems with conventional differential detection (CDD).

**Methodology.** In order to make the system suit for iterative decoding, the metric of MSDD soft-input soft-output (SISO) module of the considered systems over AWGN channels was derived first. Extrinsic information transfer (EXIT) chart analysis and the computer simulations were then performed to analyze the characteristics and the performance of the considered systems.

**Findings.** It was shown that the considered systems could improve the performance of the system by extending the observation window size of MSDD SISO module and increasing the number of iterative decoding compared to the system with CDD.

**Originality.** It was proved that the proposed scheme could be used to resolve the problem of performance degradation of differential LDPC coded systems with CDD.

**Practical value.** Therefore, the proposed scheme may be applied into the wireless communication systems when it is expensive or infeasible to apply the coherent detection.

**Keywords:** *low-density parity-check (LDPC) codes, differential LDPC coded systems, conventional differential detection, multiple-symbol differential detection, soft-input soft-output, extrinsic information transfer (EXIT) charts*

**Introduction.** Low-density parity-check (LDPC) codes have gained significant attention due to its near-capacity error performance and relatively low complexity in decoding. LDPC codes have been shown to give amazing performance over additive white Gaussian noise (AWGN) channels with the ideal coherent detection. However, due to the performance of coherent detection relies on an accurate phase tracking and a good estimation of channel stat information (CSI), coherent detection becomes expensive or infeasible in some cases.

The classical solution of this problem is differential encoding combined with differential detection that does not require explicit knowledge or estimation of CSI. However, it is well known that a 3dB performance gap exists between the coherent detection and conventional differential detection. Multiple symbol differential detection (MSDD) has been proposed for mitigating this performance degradation [1].To further improve the system performance some pieces of literature have proposed the approach of combining error correcting codes with an inter leaver and the differential encoder in the transmitter, and using iterative decoding in the receiver [2–5].

Recently the performance of LDPC codes with general differential detection over Rayleigh fading channels was studied [6], but the performance with MSDD and iterative decoding were not discussed. The approach of the serial concatenation of LDPC codes and differential encoding was proposed in [7], which was mainly focused on how to design good LDPC codes for differential encoding over AWGN channel. More recently, in [8], a kind of differentially encoded LDPC codes and the detection schemes were proposed. Through the analysis of extrinsic information transfer (EXIT) chart, it is shown that conventional LDPC codes are not fitful for differential coding over flat Rayleigh fading channels, and does not in general deliver a desirable performance with differential detection. However, this paper also only discussed the conventional differential detection, and pilot symbol detection is used to estimate the channel information, which reduces the information rates and increases the complexity of the receiver. All the abovementioned papers studied the conventional differential detection. It was proved that the system has considerable performance loss comparing with the system with coherent detection.

On the other hand, EXIT chart is considered an effective tool for the analysis of iterative decoding systems in recent years [9]. EXIT chart analysis can visualize the transfer characteristics of the inner decoder and the outer decoder, and the convergence behaviour of iterative decoding based on tracking the exchange of mutual information between the component decoders.

In this paper, a differential LDPC coded systems with multiple symbol differential detection (MSDD) schemes is proposed. To make the system suit for iterative decoding, an appropriate MSDD soft-input soft-output (SISO) metric is derived first.

The characteristics and performance of the differential LDPC coded systems with MSDD over AWGN channels are then analysed by EXIT chart and computer simulations. Simulation results show that the considered systems can improve the performance by increasing the length of the observation window and the number of iterative decodings compared to the system with conventional differential detection.

The rest of this paper is organized as follows. In Section *2*, the system model is introduced. In Section *3*, the metric of MSDD SISO module is derived. Then, in Section *4*, we analyse the characteristics and performance of the considered system by EXIT chart and computer simulations. Finally, Section *5* concludes the paper.

**System model.** The system model is shown in fig.1. The LDPC encoder encodes a binary random message bit sequence $\boldsymbol{b}$ with length $K$ to a code sequence $\boldsymbol{c}$ with length $N$. The coding rate is $R=K/N$. Each $m=\log 2^M$ bits of the code sequence are then mapped to $M$-ary PSK symbols sequence x, $x_i \in (0,1,\ldots,M\text{-}1)$. Finally, sequence $\boldsymbol{x}$ is differentially encoded to $\boldsymbol{s}$, i.e., $s_i = x_i\, s_{i\text{-}1}$.

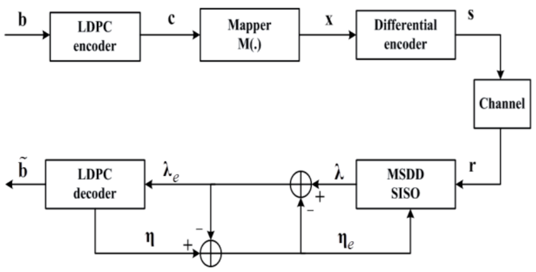


*Fig.1. System model*

We consider that the symbols are sent through AWGN channels. With differential detection, at time $k$, the received discrete-time baseband signal $r_k$ of the received signal vector $r$ can be represented as

$$r_k = s_k e^{j\theta_k} + n_k, \tag{1}$$

where $\theta_k$ is the unknown phase introduced by the channel with uniformly distributed over $[0, 2\pi)$, and $n_k$ is a sample of a zero-mean complex Gaussian noise with variance $\sigma_n^2$.

At the receiver, the MSDD SISO module outputs the posteriori probabilities (APPs) $\lambda$ of the bits of $\mathbf{c}$ based on the received signals and the prior probabilities outputted from the LDPC decoder in each iteration. The extrinsic information part $\lambda_e$ is then outputted to the LDPC decoder through subtracting the prior probabilities provided by the LDPC decoder. Based on $\lambda_e$, the LDPC decoder makes a tentative hard-decision and the resultant code word will be check by the LDPC code's parity-check matrix. If the resultant vector is an all zero vector, it represents a legitimate code word has been found. Otherwise, the extrinsic information $\eta_e$ which is obtained by $\eta$ subtracting the prior probabilities $\lambda_e$, will be fed back to MSDD SISO module as the prior probabilities, where $\eta$ is the APPs of the bits of c outputted from the LDPC decoder. This process is repeated until the pre-defined maximum number reached or a legitimate code word has been found. Finally, the hard decision $\tilde{b}$ is outputted from the LDPC decoder. In this paper, the LDPC decoder uses the sum-product algorithm to perform the decoding.

**Metric of MSDD SISO module.** Fig.1 shows that MSDD SISO module is the key element providing extrinsic information as an input of the LDPC decoder. It is known that the conventional MSDD makes a hard decision through maximum likelihood detection. Therefore, the metric computation method of MSDD SISO module used to output the soft information should be derived.

Here we assume that the observation window size of the MSDD SISO module is $L$, and $\theta_k$ remains constant over the entire received sequence. The received symbols are divided into sub-blocks of $L$ symbols each in such a way that the blocks overlap in one symbol. For the $k$-th sub-block, we can rewrite (1) into the following vector form

$$r_k = s e^{j\theta} + n_k, \tag{2}$$

where $r_k = [r_{k,0}, r_{k,1}, \ldots, r_{k,L\text{-}1}]^T$, $s_k = [s_{k,0}, s_{k,1}, \ldots, s_{k,L\text{-}1}]^T$, $n_k = [n_{k,0}, n_{k,1}, \ldots, n_{k,L\text{-}1}]^T$, and the superscript '$T$' denotes the transpose operation.

At the MSDD SISO module, the log-likelihood ratio (LLR) of each coded bits is computed. The LLR $\lambda_j$ of the $j$-th bit $c_j$ in the $L$ observation window size is given by

$$\lambda_j = \log \frac{P(c_j=0|r)}{P(c_j=1|r)}, \tag{3}$$

where $\log(\bullet)$ denotes the natural logarithm, $P(c_j=0|r)$ and $P(c_j=1|r)$ are *a* posteriori probabilities of $c_j=0$ and $c_j=1$, respectively.

Based on the Bayesian formula, and assuming that the coded bits are independent with each other due to the inherent interleaving nature of LDPC codes, (3) is equivalent to

$$\lambda_j = \log\frac{P(c_j=0)}{P(c_j=1)} + \log\frac{\sum\limits_{s:c_j=0} P(r|s) \prod\limits_{i=1,i\neq j}^{m(L-1)} P(c_i)}{\sum\limits_{s:c_j=1} P(r|s) \prod\limits_{i=1,i\neq j}^{m(L-1)} P(c_i)}, \tag{4}$$

where the sums in the numerator and denominator are take nover all sequences $s$ corresponding to the sequences $c$ whose bit in position $i$ is the value 0 or 1, respectively. $P(c_j)$ is the a priori probability of $c_j$ provided by the LDPC decoder. $P(r|s)$ is the conditional probability density function (pdf) of $r$ given s.

From (4), we can find that the LLR is the summation of a priori probability and extrinsic information. The first part of (4) is related to a priori probability of the coded bit $c_j$. The second part of (4) is related to the extrinsic information of the coded bit $c_j$, which is outputted into the LDPC decoder. In the first iteration, because no a priori probabilities of the coded bits are fed back, the transmitted bits are assumed to have equal a priori probabilities, i.e. $P(c_j=0)=$ $=P(c_j=1)=1/2$.

In the case of the AWGN channels, $P(\mathbf{r}|\mathbf{s})$ is given by [1]

$$\begin{aligned} P(r|s) &= \frac{1}{(2\pi\sigma_n^2)^L}\exp[-\frac{1}{2\sigma_n^2}\sum_{i=0}^{L-1}(|r_i|^2+|s_i|^2)]\text{Ч} \\ &\text{Ч} I_0\left(\frac{1}{\sigma_n^2}\left|\sum_{i=0}^{L-1} r_i s_i^*\right|\right), \end{aligned} \tag{5}$$

where $I_0(\bullet)$ is the zero-order modified Bessel function of the first kind, and the superscript '*' denotes the complex conjugation.

**Analysis of the system by exit chart and computer simulations.** The principle of how to obtain the EXIT chart is introduced as follows. In our considered systems, for MSDD SISO module, the mutual information between a priori information $\boldsymbol{\eta}_e$ and the coded sequence $\boldsymbol{c}$ is named as $I_{A1}$, while the mutual information between the extrinsic information $\boldsymbol{\lambda}_e$ and the coded sequence $\boldsymbol{c}$ is named as $I_{E1}$. $I_{E1}$ is the function of $I_{A1}$, which represents the transfer characteristics of the MSDD SISO module. Correspondingly, for LDPC decoder, $I_{A2}$ represents the mutual information between $\boldsymbol{\lambda}_e$ and $\boldsymbol{c}$, and $I_{E2}$ represents the mutual information between $\boldsymbol{\eta}_e$ and $\boldsymbol{c}$. And $I_{E2}$ is the function of $I_{A2}$, which represents the transfer characteristics of the LDPC decoder.

As shown in fig.1, the extrinsic information of the MSDD SISO module is the a priori information of the LDPC decoder and vice versa, which implies that $I_{A2}=I_{E1}$ and $I_{A1}=I_{E2}$. Therefore, the transfer characteristics of the MSDD SISO module and the LDPC decoder can be plotted into a signal diagram with axes of the LDPC decoder curve swapped. This diagram is referred to as EXIT chart.

In EXIT chart, the component decoders are characterized by the EXIT functions, which describe the output mutual information as a function of the input mutual information. The mutual information $I_R$ between the transmitted coded bits $C$ and respective LLR values $L(C)$ is defined as [9]

$$\begin{aligned} I_R = I(C;L(c)) &= \frac{1}{2}\sum_{c=-1,1}\int_{-\infty}^{\infty} p_R(l|C=c)\cdot \\ &\cdot\log_2\frac{2p_R(l|C=c)}{p_R(l|C=-1)+p_R(l|C=1)}dl, \end{aligned} \tag{6}$$

where $p_R(l|C=c)$ is the conditional probability density function of the LLR given the transmitted coded bits $C$, and $0\le I_R\le 1$.

The computation of (6) need the information of the transmitted coded bits and to obtain the condition probility $p_R(l|C)$ by simulation. To avoid these requirements, it is shown that (7) can be simplified as [10]

$$I_R \approx 1-\frac{1}{N}\sum_{n=1}^{N} H_b\left(\frac{e^{-|l_n|/2}}{e^{|l_n|/2}+e^{-|l_n|/2}}\right), \tag{7}$$

where $N$ is the code sequence length, $l_n$ is the LLR value of the $n$-th bit of the code sequence, $H_b$ is the binary entropy function

$$H_b(p) = -p\log_2 p-(1-p)\log_2(1-p),\quad 0\le p\le 1. \tag{8}$$

Using equation (6) or (7), the input mutual information $I_{A1}$ and the output mutual information $I_{E1}$ of the MSDD SISO module can be obtained. Viewing $I_{E1}$ as a function $T_1(\bullet)$ of $I_{A1}$ and the signal to noise ratio (SNR) $E_b/N_0$ of channel, the transfer characteristic of the MSDD SISO module is defined as

$$I_{E1} = T_1(I_{A1}, E_b/N_0). \tag{9}$$

Similarly, $I_{A2}$ and $I_{E2}$ of the LDPC decoder is computed, viewing $I_{E2}$ as a function $T_2(\bullet)$ of $I_{A2}$, the transfer characteristic of LDPC decoder is defined as

$$I_{E2} = T_2(I_{A2}). \tag{10}$$

In the following EXIT chart analysis and the computer simulations, regular rate-1/2 (3, 6) LDPC codes with length of $N$=10080 is used, and the coded bits is modulated using BPSK for simplicity.

Fig.2–4 shows the EXIT chart of the system with different observation window size. The curve which does not depart from the origin in the graph is the transfer characteristic of the MSDD SISO module while another curve in the graph is the transfer characteristic of the LDPC decoder. From [9–10], we know that if the curve of inner decoder has a steep slope, the strong potential performance improvement can be obtained by iterative decoding. And the bit error rate (BER) performance is determined by the location of the intersection of the two curves. If the intersection is at the left side of the EXIT chart, this means that the iterative decoding stops quickly and high BER will be achieved. On the contrary, if the intersection is at the very right side, it means that iterative decoding can converge at low BER.

From fig.2–4, we can observe that the slope of the MSDD SISO module curves increases with the increase of the observation window size. It implies that the system performance can be improved by extending the observation window size. And because the MSDD SISO module curves are horizontal lines, the iterative decoding is not valid with $L$=2. In addition, it can be observed that a tunnel opens between the two curves with the increase of SNR when $L$>2. For example, the tunnel opens at 3.5dB in fig.3, which means the iterative decoding can improve the performance effectively and can get low BER performance when $L$>2. Moreover, we also observe that the tunnel broadens when SNR is bigger than 3.5dB and the location of intersection is close to the point (1,1), therefore the lower BER than that at 3.5dB can be achieved. It implies that the turbo cliff appears near SNR= 3.5dB. Similarly, we can find that the turbo cliffs of $L$=2 and $L$=8 appear at SNR=5.0dB and 3.0dB respectively. The accuracy of this conclusion can be proved by fig.5, which is the BER performance of the considered system with $L$=2, 4, 8 respectively.

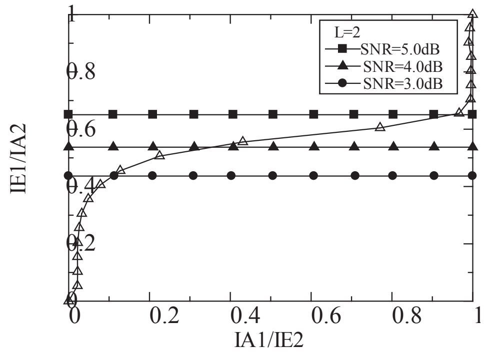


*Fig.2. EXIT chart of the system with L=2*

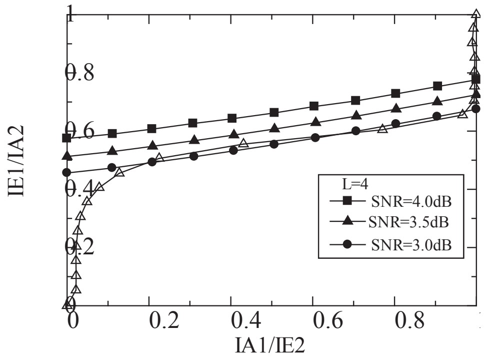


*Fig.3. EXIT chart of the system with L=4*

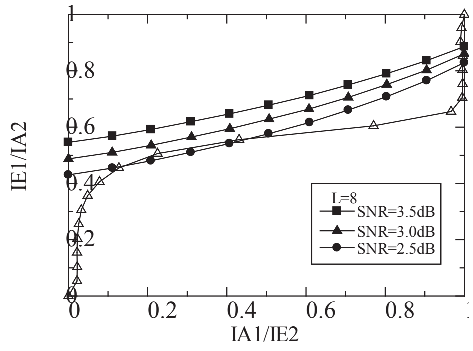


*Fig.4. EXIT chart of the system with L=8*

Fig. 6 shows the iterative decoding trajectory of the proposed system with $L$=4 at SNR=4.0dB, where the zigzag-path represents the iterative decoding trajectory. We can observe that the iterative decoding converges at the very right side after the 4th iteration. Fig.7, which is the relation of BER and iteration number at SNR=4.0dB, proves the accuracy of the conclusion obtained from EXIT chart.

In addition to obtaining the characteristic of the iterative decoding, we can also obtain the suggestion of information on how to improve the proposed system from the EXIT chart analysis. From fig.8, we observe that the output of the MSDD SISO module with $L$=2 is similar to that of $L$=4 and 8 in the first iteration at the same SNR. Moreover, we also observe that the MSDD SISO module curves of $L$=4 and 8 is close to each other in the first few iterations. This observation suggests that we can obtain the similar performance as fixed large observation window size used by using $L$=2 at the beginning of iteration and increasing $L$ with the increase of iteration. But how to control $L$ adaptively is needed to be studied in depth.

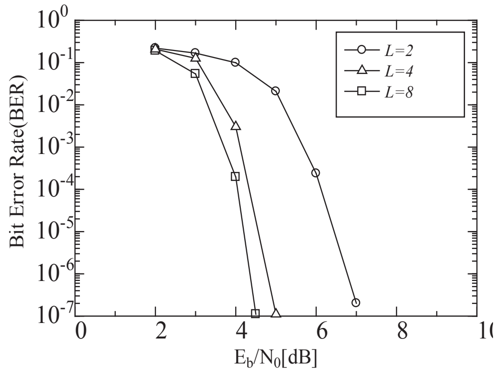


*Fig.5. BER performance of the system with L=2, 4, 8*

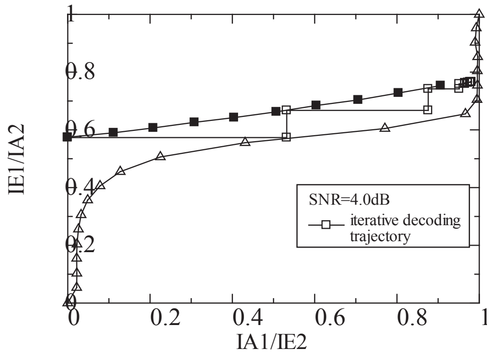


*Fig.6. Iterative decoding trajectory of the system with L=4*

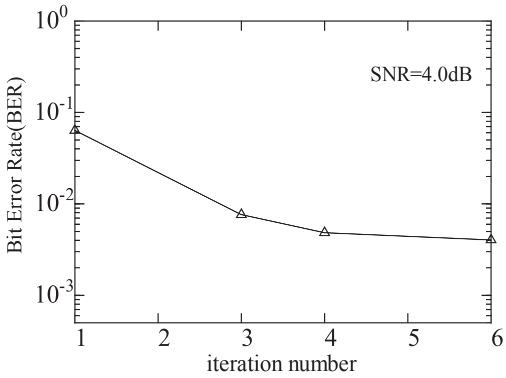


*Fig.7. Relation of BER and iteration number for the system with L=4*

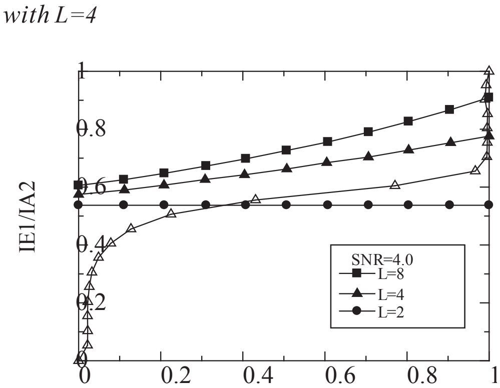


*Fig.8. EXIT chart of the system with L=2, 4, 8*

Moreover, we observed that the slope of MSDD SISO module curve is not large enough to intersect with LDPC decoder curve at the point (1,1). Increasing the slope of the MSDD curve and the steepness of the LDPC decoder curve can get turbo cliff at lower SNR. Therefore, the optimization design of LDPC codes and MSDD SISO module which makes the transfer characteristics of them fit with each other for the considered system is also worthy studying in the future research.

**Conclusion.** In this paper, we studied the differential LD-PC coded systems with MSDD scheme over AWGN channels. The metric of MSDD SISO module over AWGN channels was proposed for iterative decoding. EXIT chart analysis and the BER performance of the considered system were also evaluated over AWGN channels. Analysis results show that the performance of the considered systems can be effectively improved by increasing the observation window size and the iteration number compared to that of the system with conventional differential detection. In addition, analysis results also suggest that the MSDD SISO module with adaptive observation window size and the LDPC codes design of the considered systems should be further studied.

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**Мета.** Дослідження схеми диференціальних систем з LDPC-кодуванням і багатосимвольним диференціальним детектуванням для поліпшення ефективності цих систем шляхом диференціального детектування з перетворенням.

**Методика.** По-перше, системи пристосовані до ітеративного декодування з визначенням метрики для SISO-модуля багатосимвольного диференціального детектування цих систем в AWGN-каналах. По-друге, шляхом аналізу вихідних характеристик, виконаного за допомогою комп'ютерного моделювання, визначені особливості та оцінені показники роботи даних систем.

**Результати.** Встановлено, що поліпшення показників роботи досліджуваних систем можливе шляхом розширення ковзаючого вікна та збільшення кількості циклів декодування в порівнянні з диференціальними системами з LDPC-кодуванням і диференціальним детектуванням з перетворенням.

**Наукова новизна.** Доведено, що запропонована схема може застосовуватися для вирішення проблеми погіршення робочих показників диференціальних систем з LDPC-кодуванням і диференціальним детектуванням з перетворенням.

**Практична значимість**. Запропонована схема може використовуватися в системах безпровідного зв'язку, коли когерентне детектування дороге або нездійсненне.

**Ключові слова:** *LDPC-код, диференціальна система з LDPC-кодуванням, диференціальне детектування з перетворенням, багатосимвольне диференціальне детектування, SISO-модуль, вихідна характеристика*

**Цель.** Исследование схемы дифференциальных систем с LDPC-кодированием и многосимвольным дифференциальным детектированием для улучшения эффективности этих систем путем дифференциального детектирования с преобразованием.

**Методика.** Во-первых, системы приспособлены к итеративному декодированию с определением метрики для SISO-модуля многосимвольного дифференциального детектирования этих систем в AWGN-каналах. Во-вторых, путем анализа выходных характеристик, выполненного с помощью компьютерного моделирования, определены особенности и оценены показатели работы рассматриваемых систем.

**Результаты.** Установлено, что улучшение показателей работы исследуемых систем возможно путем расширения скользящего окна и увеличения количества циклов декодирования по сравнению с дифференциальными системами с LDPC-кодированием и дифференциальным детектированием с преобразованием.

**Научная новизна.** Доказано, что предлагаемая схема может применяться для решения проблемы ухудшения рабочих показателей дифференциальных систем с LDPC-кодированием и дифференциальным детектированием с преобразованием.

**Практическая значимость.** Предлагаемая схема может использоваться в системах беспроводной связи, когда когерентное детектирование дорого или неосуществимо.

**Ключевые слова:** *LDPC-код, дифференциальная система с LDPC-кодированием, дифференциальное детектирование с преобразованием, многосимвольное дифференциальное детектирование, SISO-модуль, выходная характеристика*

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# PERFORMANCE ANALYSIS IN A CALL CENTER WITH CALLS' ABANDONMENT AND OPTIONAL FEEDBACK

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# АНАЛІЗ ПРАЦЕЗДАТНОСТІ КОЛЛ-ЦЕНТРА З ФУНКЦІЯМИ ПОВОДЖЕННЯ З НЕТЕРПЛЯЧИМИ КЛІЄНТАМИ ТА ОПЦІЙНОГО ЗВОРОТНОГО ЗВ'ЯЗКУ

**Purpose.** With the increase of the call center business and the equipment update, the Interactive Voice Response Units (IVRU) become widely used in call centers. This research investigated two kinds of call problems (abandonment and optional feedback) and the role of the service channel (servers and the IVRU) in a call center. We have obtained some important performance measures, which are very helpful for optimizing call centers.

**Methodology.** We formulated the call center with the Interactive Voice Response Units by a two-stage queuing system. Applying the queueing theory, we discussed a call center with the customers' impatience, optional feedback, and part shutdown of the servers.

**Findings.** We first get the systems' Q-matrix, and then by using the Structured Gaussian Elimination method, we obtained the idle probability, the average number of customers in the second-level queue, the leaving probability due to customers' impatience and some other performance measures.

**Originality.** We made a study of a call center made up of trunk lines, interactive voice response units (IVRU) and agents. We discussed a partial closing rule, call abandonment and feedback in the center. The research on this aspect has not been found at present.

**Practical value.** We have also considered the fact that some customers who are dissatisfied with the service may return for service, and they may return to the Interactive Voice Response Units or the servers or both. Our model is more reasonable and close to widely used call centers.

**Keywords:** *call center, partial closing rule, call abandonment, optional feedback, the Interactive Voice Response Units*

**Introduction.** A call center is a place where agents handle a large volume of incoming and outgoing calls for various purposes. Call center business is developing rapidly in the past few years [1–9]. Based on queuing theory, some call centers in real life are discussed; this kind of analysis can be more accurate and closer to our life. Several researchers [3, 4] considered call abandonment and retrial. They obtained the stationary distribution of the system and other performance measures, but they did not consider the call center with the Interactive Voice Response Units (IVRU). Customers who enter the call center with the IVRU can receive the automatic service by the IVRU first,