Методика. При выполнении работы использовались как общенаучные, так и специальные методы исследований, включая научное обобщение, методы комплексной оценки технического уровня, математическое моделирование и аппарат линейного программирования. Решение данной задачи базируется на составлении дифференциального уравнения движения, для чего были использованы уравнения Лагранжа второго рода, а также соответствующие выражения кинетической, потенциальной энергии и диссипативной функции.

Результаты. С помощью математических расчетов в программном продукте математического анализа и расчета – "Wolfram Mathematica", получена методика расчета тяговых и динамических характеристик автосамосвала в процессе движения по дороге с продольным уклоном.

Научная новизна. Составлены расчетные схемы и уравнения движения при прямолинейном движении машины с учетом упругих и рассеивающих характеристик упругих связей, продольного уклона и профиля дороги, изменения конструктивных характеристик, что позволит дать близкую к реальной картину динамики движения.

Практическая значимость. Разработаны методики расчета динамических характеристик автосамосвала в процессе движения, а также выполнен анализ параметров конструкции автосамосвалов, на основании чего можно дать рекомендации по сокращению капитальных затрат на разработку карьеров.

Ключевые слова: *расчетная схема, автосамосвал*

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ROTATING MOMENT FOR STATICALLY UNBALANCED ROTOR WITH ELASTIC SHAFT

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ПОВНИЙ ОБЕРТАЮЧИЙ МОМЕНТ ДЛЯ СТАТИЧНО НЕВРІВНОВАЖЕНОГО РОТОРА З ПРУЖНИМ ВАЛОМ

Purpose. Study of total rotational moment components for statically unbalanced system of the "rotor – supports" type with a bent elastic shaft.

Methodology. In this paper, theoretical approaches of classical mechanics were used to study a process of unbalanced rigid body rotation. These approaches are based on a methodology to determine axial moments of inertia of statically unbalanced system of the "rotor – supports" type with a rigid (non-deformable) shaft and an elastic (deformable) shaft.

Findings. The conducted researches show that a statically unbalanced rotating system of the "rotor – supports" type, with an elastic shaft, is a system with variable moments of inertia that depend on the rotor's rotation velocity. Therefore, moments that impede the rotor's rotation occur in the system. The obtained general equation of moments makes it possible to find total rotational moment of the drive in each particular case. Total rotational moment is defined as the sum of rotational moments needed to ensure the "rotor –supports" system's rotation and the production process.

Originality. The equation of rotation for the "rotor – supports" deformed system, which is given in the paper, reflects the behavioral features of a statically unbalanced rotor with a bent elastic shaft at different rotation velocities, such as the bent shaft's middle line position with respect to the rotational axis, the rotor's center of mass position with respect to the bent shaft's middle line and the rotational axis, and the rotor's angle of turn around the shaft's middle line. The equation of total rotational moment, which is obtained in this paper, confirms and explains why the rotational moment, with rotor's shaft bend, exceeds the rotational moment sufficient for rotor rotation with no shaft's bend.

Practical value. The equation of total rotational moment can be used as a master equation in computer software to calculate required rotational moment of the drive or to process experimental results.

Key words: *moment of inertia; "rotor – supports" system; damping forces; deformed system; total rotational moment*

Introduction. Various rotating assemblies and parts are widely used in the up-to-date mining machine designs. Even a small unbalance of the rotating assemblies and parts (rotors) may lead to the occurrence of undesirable vibrations, which are sometimes hazardous to the whole machine integrity. Hazardous vibrations cause increased wear of the machine parts, power consumption boost, drive power reduction, degradation of processing operations accuracy, and service life shortening.

Balancing can partially eliminate vibrations of the machines' rotating parts and assemblies. However, in most cases the balancing requires rotor removal. The rotor removal procedure often involves significant financial and time losses due to the machine idling. Many years' experience of the mining machines operation shows that even after rotor balancing, hazardous vibrations, which are caused by service wear of the rotating assemblies and parts, eventually occur in the machine again.

There is also an entire class of mining equipment for which the unbalance is the result of a processing procedure performed by the equipment, for example centrifugal driers. Maintenance of uniform rotation of such centrifuges' rotors within a wide range of operating velocities is an urgent practical task.

Unbalanced rotor's rotational dynamics is suggested to be reviewed in the following sequence: rotation of statically unbalanced rotor; rotation of rotor with moment unbalance; and then rotation of rotor with dynamic unbalance. Such a sequence of consideration gives a better understanding of a physical picture and effects occurring in a rotor system while rotating. Statically unbalanced rotor's rotational dynamics is focused on in this paper, for it reflects the authors' main ideas and approaches in the simplest way.

Analysis of earlier studies and publications. A course on theoretical mechanics clearly states that no rotational moment needs to be applied to maintain a uniform rotation velocity of a perfectly rigid rotor fixed in rigid supports, center of mass and geometric center of which coincide with rotational axis, with no external or internal friction. And that is something entirely different with the implementation of rotation of a rotor, elastic shaft of which is fixed in rigid supports and center of mass and geometric center are displaced relative to the rotational axis as well as mutually displaced. In this case even with no moments produced by dissipative forces, a rotational moment needs to be applied to a rotor to maintain its uniform rotation velocity [1]. There are also other equations to determine a rotational moment that maintains uniform rotation of a rotor with center of mass displaced relative to the rotational axis, which are presented in the works by F.M. Dimentberg and J. Kozheshnik.

Rotational moments, determined by these equations, shall consider the action of the rotor system's dissipative forces. However, the dissipative forces, either as independent terms or implicitly, are not included into rotational moment equations and the rotational moment itself in these equations is dependent on the location of center of mass. On the other hand, the dissipative forces in theoretical mechanics do not depend on the location of body's center of mass, and are introduced to the rotational moment equation as an external moment of a couple of forces impeding body's rotation. This couple action is compensated by increasing the rotational moment on engine's shaft. The above-mentioned controversial facts emphasize the existence of an issue of correct determination of the rotational moment for unbalanced rotor. Certain components of this moment, as well as their nature, are the subject of the research conducted. The importance to find a solution to the issue brought up raises no doubts, because the drive power losses due to rotor's considerable unbalance, as V.A. Schepetilnikov states in his work, may be up to 36%.

Basic material. *Rotational moment of balanced rotor.* It is generally known that a free rigid body rotates around the major axis of inertia. With no external and internal friction, this body can rotate for no matter how long, not requiring any power supply. The rotational moment needs to be applied to the body to change the rotation velocity.

Rotors of various machines and mechanisms are not free bodies, since they are mounted in supports. Nevertheless, taking certain design measures may help achieve a qualitative alignment of the rotor's major axis of inertia with the rotational axis. At low rotation velocities, the alignment of the major axis of inertia with the rotor's rotational axis allows achieving the conditions similar to those of the free body rotation. Therefore, a well-balanced rotor uniformly rotates around the rotational axis and needs no rotational moment to be applied if it is not affected by any other forces and moments. Thus, it is considered that if a rotor is affected by a couple of forces, it rotates around its rotational axis uniformly accelerated or uniformly decelerated.

Such a rotor's rotation is described by a differential equation of body's rotational motion, with no external and internal resistance forces considered

$$I\frac{d\omega}{dt} = M_{kp}, \qquad (1)$$

where I is axial moment of inertia; ω is angular velocity of rotor's rotation; M_{kp} is rotational moment.

Equation (1) can also be called an equation to determine the main rotational moment of balanced rotor with a rigid shaft. External and internal friction forces, which are often referred to as dissipative or damping forces, in balanced rotor dynamics are considered by supplementing equation (1) with an external moment of a couple of forces, impeding body's rotation

$$I\frac{d\omega}{dt} = M_{kp} - (c+\mu)\omega^2, \qquad (2)$$

where c is coefficient of external friction; μ is coefficient of internal friction.

The action of this moment of the couple is compensated by increasing a rotational moment on rotor's shaft. Equation (2) clearly shows that the external moment of the couple does not depend on the location of rotor's center of mass relative to the rotational axis. Let's emphasize this important feature of all damping forces.

However, the practice shows that even a well balanced rotor has residual unbalance and its major axis of inertia does not coincide with the rotational axis. So it is reasonable to consider the actual rotor's rotational dynamics as rotation of a rigid body with unsymmetrical distribution of mass relative to its geometrical axis or rotational axis.

If the unbalanced rotor's shaft is rigid and mounted in rigid supports, then such a "rotor – supports" system is a deformable system with constant unsymmetrical location of mass relative to the rotational axis. If the shaft is elastic, then the same "rotor – supports" system is considered a non-deformable system with variable unsymmetrical location of mass relative to the rotational axis. Let's review the features of these systems' rotation for the main cases of misalignment of the major axis of inertia with the rotational axis, with no external resistance forces and friction forces occurring in bearing units.

Rotational moment for statically unbalanced rotor with rigid (non-deformable) shaft. Let's consider the rotation of a rotor with mass m, a rigid and weightless shaft of which is fixed in rigid supports. Such a rotor rotates around geometrical axis Z that is the shaft's middle line coinciding with the rotational axis specified by the supports. Rotor's static unbalance is defined by displacement of rotor's center of mass C relative to shaft's middle line by distance e. In this case, the major axis of inertia is parallel to the shaft's middle line and the rotational axis. Fig. 1 demonstrates the rotation pattern of the rotor system described.

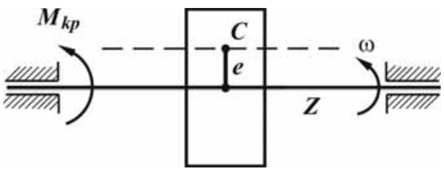

Fig. 1. Rotation pattern of rotor with rigid (non-deformable) shaft in rigid supports: M_{kp} – rotational moment; e – eccentricity; C – center of mass; Z – geometrical axis; ω – angular velocity of rotor's rotation

If the major axis of inertia coincides with the rotor's rotational axis, then equation (1) fully determines the rotational moment applied to change the rotor system's rotation velocity, with no damping forces acting. However in our case, rotor's center of mass C is displaced from the rotational axis by distance e, which changes the rotor's axial moment of inertia I by value me^2, i.e. $I_e = I + me^2$. So the balanced rotor's axial moment of inertia I is symmetrical relative to the shaft's middle line. As opposed to it, the statically unbalanced rotor's axial moment of inertia I_e relative to the shaft's middle line contains additional asymmetrical part me^2 that is produced by unbalanced mass. Therefore, changing the rotation velocity requires overcoming the statically unbalanced rotor's axial moment of inertia I_e, which comprises the sum of the balanced rotor's axial moment of inertia I and its additional asymmetrical part me^2.

Similarly to equation (2), we can assume that when changing the statically unbalanced rotor's rotation velocity, the presence of additional asymmetrical part me^2 in its axial moment of inertia will change the rotational moment by value $me^2\omega^2$. In this case, the equation of the statically unbalanced rotor's rotation around the shaft's middle line, with no damping forces acting, will take on the following form

$$(I + me^2)\frac{d\omega}{dt} = M_{kp} - me^2\omega^2 . \qquad (3)$$

At statically unbalanced rotor's uniform rotation, we have $M_{kp} - me^2\omega^2 = 0$ or

$$M_{kp} = me^2\omega^2 . \qquad (4)$$

Equation (4) shows that, with the rotational moment specified, the rotor's angular velocity first increases and later on achieves the maximum value. Then the rotational moment is used up to maintain the rotor's uniform rotation velocity. The maximum angular velocity of rotor's uniform rotation, with the rotational moment specified, can be determined using the following dependence

$$\omega_{max} = \sqrt{\frac{M_{kp}}{me^2}} . \qquad (5)$$

Rotational moment for statically unbalanced rotor with elastic shaft. With no damping forces acting, let's consider rotation of a rotor with mass m, an elastic and weightless shaft of which is fixed in rigid supports. The rotor rotates around geometrical axis Z that is the shaft's middle line. The rotor's static unbalance is specified by displacement of the rotor's center of mass C from the shaft's middle line by distance e. Under the action of static unbalance, the shaft is bent and the rotor is displaced by distance a from the rotational axis specified by the supports. In this case, the major axis of inertia is parallel to the shaft's middle line and the rotational axis. The shaft's middle line and the rotational axis do not coincide. Fig. 2 shows the pattern of rotation of a rotor with a bent shaft.

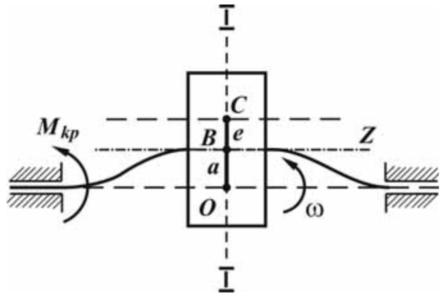

Fig. 2. Pattern of rotation of rotor with elastic shaft in rigid supports: a is shaft's bend; B is track of shaft's middle line in monitoring plane I – I; O is track of rotational axis in monitoring plane I – I

Dynamics of changing the locations of the major axis of inertia, the shaft's middle line, and the rotor's rotational axis can be easily studied by their tracks in the monitoring plane. In our case, one monitoring plane, which goes through the rotor's center of mass perpendicular to the rotational axis, is sufficient for using. Fig. 3 presents monitoring plane I – I, where point C is a track of the major axis of inertia, point B is a track of the shaft's middle line, point O is a track of the rotational axis. In addition to axes' tracks, Fig. 3 shows centrifugal force action direction F_c, shaft's elastic force F_y, and rotor's moment M_{kp} in a rotating coordinate system XOY originating on the rotational axis.

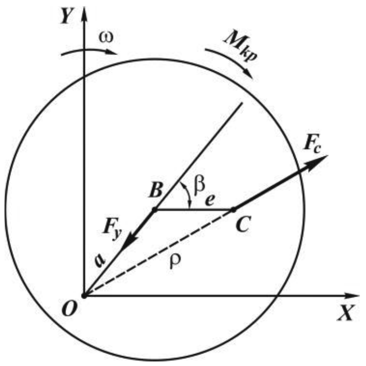

Fig. 3. Monitoring plan I – I and XOY coordinate system of rotor: F_c is centrifugal force; F_y is shaft's elastic force; β is rotor's turn angle around shaft's middle line; ρ is center of mass displacement relative to rotational axis

Dynamics of a rotor with an elastic shaft is drastically different from that of a rotor with a rigid shaft fixed in rigid supports. Any unbalance of the rotor makes the shaft bent, starting from certain velocities. The shaft's bend makes the "rotor – supports" system become deformed, and its center of mass rotates around the rotor shaft's middle line, synchronously with the shaft's middle line rotation around the rotational axis, at uniform velocity specified. Moreover, the rotor location relative to the rotational axis changes only if the "rotor – supports" system's rotation velocity changes. The "rotor – supports" system's location is changed by forces and moments affecting the rotor.

Also, due to the rotor shaft's bend, the "rotor – supports" deformed system becomes a system with variable moments of inertia. Besides, this system's moments of inertia change not only due to the shaft's bend but also due to the rotor's turn around the shaft's middle line on certain angle β. As a result, the rotor's center of mass is located at distance ρ from the rotational axis (Fig. 3). Value ρ is determined using the following dependence

$$\rho = \sqrt{a^2 + e^2 + 2ae\cos\beta}\,. \qquad (6)$$

Angle β achieves the maximum value of 180° at supercritical velocities. A certain effect of rotor's self-centering occurs at that.

Rotor's center of mass displacement by distance ρ relative to the rotational axis changes axial moment of inertia I^s of the "rotor – supports" entire system by value $m\rho^2$, i.e. $I^s = I + m\rho^2$ (Huygens – Steiner theorem). The "rotor – supports" system's axial moment of inertia can finally be presented as the following expression

$$I^s = I + ma^2 + me^2 + 2mae\cos\beta\,. \qquad (7)$$

Expression (7) shows that the shaft's middle line displacement by value a relative to the rotational axis causes the occurrence of additional asymmetrical part ma^2 of rotor's axial moment of inertia I. This asymmetrical part of the rotor's axial moment of inertia depends on the shaft's bend value at relevant rotation velocity. It exists as long as the shaft' bend does. Addition of $2mae\cos\beta$ to the rotor's axial moment of inertia is responsible for the rotor's turn around the shaft's middle line at relevant rotation velocity.

Value $ma^2 + me^2$ determines a general asymmetrical part of the "rotor – supports" system's moment of inertia, conditioned that the rotor rotates around the shaft's middle line and the shaft's middle line with the rotor rotate around the rotational axis.

If changing the rotation velocity of the statically unbalanced rotor with bent elastic shaft, the presence of additional asymmetrical part ma^2 and additional part $2mae\cos\beta$ in rotor's axial moment of inertia will change the rotational moment of the "rotor – shaft" entire system by values $ma^2\omega^2$ and $2mae\omega^2\cos\beta$, respectively. In this case, the equation of rotation of the statically unbalanced rotor with bent elastic shaft around the shaft's middle line and the shaft's middle line rotation around the rotational axis, with no damping forces acting, will take on the following form

$$\left(I + ma^2 + me^2 + 2mae\cos\beta\right)\frac{d\omega}{dt} = \\ = M_{kp} - m\omega^2\left(a^2 + e^2 + 2ae\cos\beta\right). \qquad (8)$$

At uniform rotation of the statically unbalanced rotor with bent elastic shaft, we have $M_{kp} - ma^2\omega^2 - me^2\omega^2 - 2mae\omega^2\cos\beta = 0$, or, taking into account damping forces, we have

$$M_{kp} = ma^2\omega^2 + me^2\omega^2 + 2mae\omega^2\cos\beta + (c+\mu)\omega^2. \qquad (9)$$

At supercritical velocities, with rotor's uniform rotation, angle $\beta = 180°$. Then equation (8), with no damping forces acting, is as follows

$$M_{kp} = m\omega^2(a-e)^2\,. \qquad (10)$$

Equation (10) means that if, with unlimited increase of velocity, difference $a - e$ tends to zero, then the rotational moment tends to zero as well.

Components of rotational moment for statically unbalanced rotor with elastic shaft. Let's analyze equation (9) obtained in the previous section for the case of uniform rotation of statically unbalanced rotor with bent elastic shaft.

Rotational moment M_{kp}, applied to rotor's shaft, is used up first of all to maintain rotor's center of mass rotation around shaft's middle line and compensate the action of damping forces applied to rotor's shaft as an external moment of a couple of forces. Let's note that this moment of a couple cannot be attributed to a moment of force acting relative to certain point. I.e. such a moment of a couple of forces cannot be presented as a force applied to rotor shaft's middle line (geometrical axis). It means that the presence of damping forces does not introduce to the general force pattern shown in Figure 3 an additional third force, balancing the action of centrifugal force F_c and shaft's elastic force F_y, which are not on the same line and are applied to rotor's different points. Absence of an additional third force in the force pattern (Fig. 3) allows us to assume that the damping forces action is not the cause of rotor's turn on angle β around shaft's middle line. The assumption stated is confirmed in paper [1] and in the works by J.P. Den Hartog, showing that, even if not considering the damping components, rotor's center of mass turns around its geometrical axis on angle 180° at supercritical velocities. It is also well-known from the works by V.A. Schepetilnikov – the rotor dynamics expert – that damping forces do not impede the shaft's bend value change and the rotor's center of mass radial displacement relative to the rotational axis. In conclusion let's point out the fact that F.M. Dimentberg in his works considers rotation of statically unbalanced rotor, with no damping forces acting. His researches are considerably different from the researches presented in the works by J. Kozheshnik, describing the unbalanced rotor's dynamics, with damping forces taken into account. However, the results of the both researches are the same: there are no damping forces in the equations of rotation moments.

Part $2mae\omega^2 \cos\beta$ of rotation moment equation (9) is of interest for further studying. It is related to rotor elastic shaft's bend a, initial eccentricity e, and responsible for rotor's turn around shaft's middle line on angle β. If there is shaft's bend caused by the action of centrifugal force applied to center of mass, shaft's middle line does not coincide with the rotational axis in the rotor attachment point. Thus, part $ma^2\omega^2$ equation of rotational moment (9) applied to the rotor shall be used up to ensure rotation of shaft's middle line relative to the rotational axis. However, no external forces and moments, which would impede shaft's middle line rotation relative to the rotational axis, were detected during more detailed theoretical and experimental researches of rotation process of statically unbalanced rotor with elastic shaft's bend. The moment, impeding the rotor shaft's middle line rotation relative to the rotational axis, is obviously of internal nature and related to the change of the "rotor – supports" deformed system's axial moment of inertia. This moment can be presented as force F_a applied to the rotor shaft's middle line (geometrical axis) and acting on arm a around the rotational axis.

The above-stated assumption allows introducing to the general pattern of forces shown in Fig. 4 an additional third force balancing the action of centrifugal force F_c and shaft's elastic force F_y.

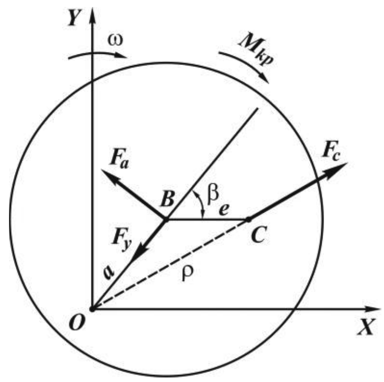

Fig. 4. General pattern of forces acting on statically unbalanced rotor with bent elastic shaft: F_a is additional force; F_c is centrifugal force; F_y is shaft's elastic force; β is rotor's turn angle around shaft's middle line; ρ is center of mass displacement relative to rotational axis

The force F_a prevents rotation of shaft's middle line together with rotor around rotational axis.

Conclusions. The researches conducted show that the "rotor – supports" deformed system with an elastic shaft is a system with variable moments of inertia depending on rotor's rotation velocity. Therefore, moments impeding the rotor's rotation occur in the system. The equation obtained allows finding total rotational moment of the drive in each particular case. Total rotational moment is defined as the sum of rotational moments needed to ensure the rotor's rotation and the production cycle implementation. The research results can be used to design machines and mechanisms.

Comparative analysis with V.A. Schepetilnikov's experimental data, showing significant losses of the drive's power, confirms the correctness of the research conducted. The equation obtained confirms and explains why the rotational moment M_{kp}, with rotor shaft's bend, exceeds the rotational moment sufficient for rotor rotation with no shaft's bend.

The equation, which considers the action of friction forces in bearing units and aerodynamic drag forces, can be used as a master equation in computer software to calculate required rotational moment of the drive or to process experimental results. When using Volterra principle and a concept of complex form of non-elastic resistances accounting [2], the equation (9) obtained is applicable for solution of issues in the area of elastic systems' oscillations with considerable asymmetry.

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Мета. Дослідження складових частин повного обертаючого моменту для статично неврівноваженої системи типу „ротор – опори“ з пружним вигнутим валом.

Методика. У роботі застосовувалися теоретичні підходи класичної механіки щодо дослідження процесу обертання неврівноваженого твердого тіла. В основі цих підходів лежить методика визначення осьових моментів інерції статично неврівноважених обертових систем типу „ротор–опори“ з твердим (недеформованим) валом і пружним (деформованим) валом.

Результати. Проведені дослідження показують, що статично неврівноважена обертова система типу „ротор – опори“, котра має пружний вал, є системою зі змінними моментами інерції, що залежать від швидкості обертання ротора. Відповідно, у цій системі виникають моменти, перешкоджаючі обертанню ротора. Отримане загальне рівняння моментів дозволяє визначити повний обертаючий момент приводу в кожному конкретному випадку. Повний обертаючий момент визначається як сума обертаючих моментів, необхідних для забезпечення обертання системи „ротор – опори“ та виконання виробничого процесу.

Наукова новизна. Наведене в роботі рівняння обертання для деформованої системи „ротор–опори“ відображає особливості поведінки статично неврівноваженого ротора з вигнутим пружним валом на різних швидкостях обертання, а саме положення середньої лінії вигнутого вала щодо осі обертання, положення центру мас ротора щодо середньої лінії вигнутого вала та осі обертання, кут розвороту ротора навколо середньої лінії вала. Рівняння повного обертаючого моменту, отримане в даній роботі, підтверджує та пояснює перевищення обертаючого моменту при вигині вала ротора над обертаючим моментом, достатнім для обертання ротора при відсутності прогину вала.

Практична значимість. Рівняння повного обертаючого моменту можна використовувати в якості керуючого рівняння в комп'ютерних програмах для розрахунків потрібного обертаючого моменту приводу або обробки результатів експериментів.

Ключові слова: *момент інерції, система „ротор–опори“, сили демпфірування, деформована система, повний обертаючий момент*

Цель. Исследование составляющих частей полного вращающего момента для статически неуравновешенной системы типа „ротор – опоры“ с упругим изогнутым валом.

Методика. В работе применялись теоретические подходы классической механики для изучения процесса вращения неуравновешенного твердого тела. В основе этих подходов лежит методика определения осевых моментов инерции статически неуравновешенных вращающихся систем типа „ротор–опоры“ с жестким (недеформируемым) валом и упругим (деформируемым) валом.

Результаты. Проведенные исследования показывают, что статически неуравновешенная вращающаяся система типа „ротор–опоры“, имеющая упругий вал, является системой с переменными моментами инерции, которые зависят от скорости вращения ротора. Соответственно, в этой системе возникают моменты, препятствующие вращению ротора. Полученное общее уравнение моментов позволяет определить полный вращающий момент привода в каждом конкретном случае. Полный вращающий момент определяется как сумма вращающих моментов, необходимых для обеспечения вращения системы „ротор–опоры“ и выполнения производственного процесса.

Научная новизна. Приведенное в работе уравнение вращения для деформированной системы „ротор–опоры“ отражает особенности поведения статически неуравновешенного ротора с изогнутым упругим валом на различных скоростях вращения, а именно: положение средней линии изогнутого вала относительно оси вращения, положение центра масс ротора относительно средней линии изогнутого вала и оси вращения, угол разворота ротора вокруг средней линии вала. Уравнение полного вращающего момента, полученное в данной работе, подтверждает и объясняет превышение вращающего момента при изгибе вала ротора над вращающим моментом, достаточным для вращения ротора при отсутствии прогиба вала.

Практическая значимость. Уравнение полного вращающего момента можно использовать в качестве управляющего уравнения в компьютерных программах для расчета необходимого вращающего момента привода или обработки результатов экспериментов.

Ключевые слова: *момент инерции, система „ротор–опоры“, силы демпфирования, деформированная система, полный вращающий момент*

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