UDC 534.1

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# ANALYTICAL METHODS FOR DETERMINING THE EFFECT OF THE DYNAMIC PROCESS ON THE NONLINEAR FLEXURAL VIBRATIONS AND THE STRENGTH OF COMPRESSED SHAFT

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# АНАЛІТИЧНІ МЕТОДИ ВИЗНАЧЕННЯ ВПЛИВУ ДИНАМІЧНОГО ПРОЦЕСУ НА НЕЛІНІЙНІ ЗГИНАЛЬНІ КОЛИВАННЯ ТА МІЦНІСТЬ СТИСНУТОГО ВАЛА

**Purpose.** Determining the dynamic factor of industrial equipment safety by studying the dynamic processes in a nonlinear compressed shaft type oscillation system, which is widely used in mining industry. Such systems have previously been studied in literature solely based on the numerical modelling approach. In this paper, it is proposed to use asymptotic methods of nonlinear mechanics and the method of special periodic functions for thorough investigation of dynamics of the above systems and the conditions of resonance phenomena occurrence. We also describe the method for determining the dynamic factor of safety for boring equipment.

**Methodology.** The methods for analysing resonant oscillation regimes and determining the factor of safety for industrial equipment elements are based on asymptotic methods of nonlinear mechanics, wave theory of motion and the use of special Ateb-functions.

**Findings.** In this paper, the conditions of resonant oscillations for given nonlinear compressed shaft type systems are analytically obtained depending on system parameters and the method for calculating the dynamic factor of safety for industrial equipment elements is described.

**Originality.** Scientific novelty lies in the fact that, for the first time, the calculation of dynamic processes in compressed shaft type systems is done based on analytical approaches that allow, in contrast to numerical and experimental approaches, investigating the dynamics features of such systems more precisely and avoiding the occurrence of unwanted resonant modes in mining equipment.

**Practical value.** The presented method allows not only solving the problems of analysis, but also solving important problems of oscillation system synthesis at the design stage as well as choosing the elastic characteristics of dynamic systems and calculating the dynamic factor of safety for drilling equipment, taking into account the possible resonance phenomena. These mining machine features allow performing mining operations efficiently and safely.

**Keywords:** *nonlinear system vibrations, compressed shaft, elastic characteristics, dynamic factor of safety, special functions*

**Introduction. The relevance of the topic, the state of the problem under consideration and analysis of the recent research.** Differential equations of flexural vibrations of a body, which rotates around its axis with a constant angular velocity inside a continuous flow of homogeneous medium, can be written as [1]

$$\left(m+m_1\right)\frac{\partial^2 u}{\partial t^2}+2m_1\left(V\frac{\partial^2 u}{\partial t\partial x}-\Omega\frac{\partial w}{\partial t}\frac{\partial w}{\partial x}\right)-2\left(m+m_1\right)\Omega\frac{\partial w}{\partial t}-\left(N-m_1V^2\right)\frac{\partial^2 u}{\partial x^2}-$$
$$-M\frac{\partial^3 w}{\partial x^3}+EI\frac{\partial^4 u}{\partial x^4}-\left(m+m_1\right)\Omega^2 u=\varepsilon f\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4}\right); \quad (1)$$

$$\left(m+m_1\right)\frac{\partial^2 w}{\partial t^2}+2m_1\left(V\frac{\partial^2 w}{\partial t\partial x}+\Omega\frac{\partial u}{\partial t}\frac{\partial u}{\partial x}\right)+$$
$$+2\left(m+m_1\right)\Omega\frac{\partial u}{\partial t}-\left(N-m_1V^2\right)\frac{\partial^2 w}{\partial x^2}-$$
$$-M\frac{\partial^3 u}{\partial x^3}+EI\frac{\partial^4 w}{\partial x^4}-\left(m+m_1\right)\Omega^2 w=$$
$$=\varepsilon g\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4}\right). \quad (2)$$

In the equations (1, 2) $u(x,t)$ and $w(x,t)$ are the displacement vector components of the observable body $x$ with $t$ coordinate for an arbitrary time moment $t$; $m$ is the mass corresponding to the length unit of an elastic body; $m_1$ is the mass corresponding to a length unit of the continuous flow of homogeneous medium (CFHM), which moves alongside the body; $E$ is the elastic modulus of the body material; $I$ is the moment of inertia of the body cross section about the axis which matches the neutral axis in undeformed position (the given axis is normal to the oscillation plane); $V$ is the constant velocity of the continuous medium alongside the elastic body; $\Omega$ is the angular velocity of the elastic body rotation; $N$ is the tension force; $M$ is the drive momentum. The right parts of equations (1, 2) are functions which describe nonlinear components of the restoring force, resistance force and other forces, the possible values of which are much smaller than the restoring force, which is pointed by the parameter $E$, and are explained in series by the small parameter $\varepsilon$ degrees. Without limiting the generality, throughout this paper, the functions $f$ and $g$ are considered as polynoms, and from their contents it follows

$$f\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4}\right)=$$
$$=g\left(w,u,\frac{\partial w}{\partial t},\frac{\partial u}{\partial t},\frac{\partial w}{\partial x},\frac{\partial u}{\partial x},\ldots,\frac{\partial^4 w}{\partial x^4},\frac{\partial^4 u}{\partial x^4}\right).$$

Henceforth, it is considered that the medium, which flows alongside the body, does not influence its bending rigidity, while the relative momentum and the rotating momentum are small values. The equation system (1, 2) is complemented with boundary conditions, which determine motion conditions of the elastic body at $x = 0$ (beginning) and $x = l$ (end). Assuming motion conditions as a fixed hinge, we have

$$u\left(t,x\right)\big|_{x=j}=\frac{\partial^2 u}{\partial x^2}\big|_{x=j}=0\,;$$
$$w\left(t,x\right)\big|_{x=j}=\frac{\partial^2 w}{\partial x^2}\big|_{x=j}=0; \qquad j=0,l. \quad (3)$$

Thereby, the task is to build an approximate solution of the boundary value problem (1–3) and, based on this solution, create convenient dependencies for engineering calculation purposes, which can estimate the influence of the compressed shaft type oscillation system parameters on the nonlinear vibration dynamics.

The research on dynamic processes in different technical vibration systems using special Ateb-functions and the wave motion theory has been intensively used in the past ten years [2]. In the paper [3] these functions are used for finding the effective parameters in vibration protecting systems, whose mathematical models are described with ordinary differential equations. In paper [4], a thorough research on shock vibration systems is conducted. The mathematical models of these systems are described by equations of mathematical physics.

The main ideas of the wave theory are widely used in solving applied problems, where classic Fourier or D'Alembert partial differential equation integrating methods are not applicable. This is mostly related to problems which describe dynamical processes in longitudinally moving mediums. Namely, these are the longitudinal and flexural vibrations of belt drives, water transporting pipelines, screw machines along which a viscous or loose medium moves [5], vibro-separations to some extent, etc. It should be noted that the longitudinal part of the medium velocity affects not only the quantitative characteristics of the above mentioned systems, but also significantly influence the quality characteristics – result in a vibration or stability failure [6]. The latter is the most important from the practical point of view.

It should be noted, that the dynamic processes in "elastic body – moving medium" systems, were studied using the analytic and numeric integration of respective mathematical models in [7]. Paper [1] is dedicated to the development of approximate asymptotic research methods of respective vibration systems. The methods developed in the paper showed that, in the case of medium motion with velocity that approaches the critical level, there is energy redistribution in the system, which leads to sufficient amplitude increase.

However, the numeric approach to integration of differential equations which are modelling the dynamical process, does not allow determining a large number of dynamic characteristics of the system, such as the relationship between the critical fluid velocity and the body angular velocity, the body rigidity, etc. According to this fact, the analytical solving of the given problem has both theoretical and practical interest.

In the paper [8], the qualitative research methods for nonlinear vibrations in such systems, and systems of similar type are presented.

**The method of investigating the matematical model of nonlinear vibrations of the elastic body.** For solving the boundary problem (1–3), let us first consider a non-perturbated analog of such a system, namely the following equations

$$\left(m+m_1\right)\frac{\partial^2 u_0}{\partial t^2}-2\left(m+m_1\right)\Omega\frac{\partial w_0}{\partial t}-N\frac{\partial^2 u_0}{\partial x^2}+$$
$$+EI\frac{\partial^4 u_0}{\partial x^4}-\left(m+m_1\right)\Omega^2 u_0=0\,; \quad (4)$$

$$\left(m+m_1\right)\frac{\partial^2 w_0}{\partial t^2}+2\left(m+m_1\right)\Omega\frac{\partial u_0}{\partial t}-N\frac{\partial^2 w_0}{\partial x^2}+$$
$$+EI\frac{\partial^4 w_0}{\partial x^4}-\left(m+m_1\right)\Omega^2 w_0=0\,, \qquad (5)$$

with boundary conditions

$$u_0\left(t,x\right)\Big|_{x=j}=\frac{\partial^2 u_0}{\partial x^2}\Bigg|_{x=j}=0\,;$$
$$w_0\left(t,x\right)\Big|_{x=j}=\frac{\partial^2 w_0}{\partial x^2}\Bigg|_{x=j}=0\,; \qquad j=0,l. \qquad (6)$$

Developing the main idea of the wave theory of motion, it is shown that the solutions of the above mentioned equations can be interpreted as the imposition of direct and reflected waves, and can be written as

$$u_0\left(t,x\right)=a\cos\left(\kappa x+\omega t+\psi\right)+b\cos\left(\kappa x-\omega t-\varphi\right);$$
$$w_0\left(t,x\right)=c\sin\left(\kappa x+\omega t+\psi\right)+d\sin\left(\kappa x-\omega t-\varphi\right), \quad (7)$$

where *a, b, c, d* are amplitudes of the direct and reflected waves; $\kappa$ and $\omega$ are their wave number and frequency respectively; $\varphi$ and $\psi$ are the initial phases.

Taking into account (7), from the differential equation system (4, 5), the following dispersion relations can be written

$$\left(m+m_1\right)\omega^2+2\frac{c}{a}\left(m_1+m_2\right)\Omega\omega-N\kappa^2+$$
$$+EI\kappa^4+\left(m_1+m_2\right)\Omega^2=0;$$
$$\left(m+m_1\right)\omega^2+2\frac{b}{d}\left(m_1+m_2\right)\Omega\omega-N\kappa^2+$$
$$+EI\kappa^4+\left(m_1+m_2\right)\Omega^2=0.$$

The non-perturbated differential equation system (4, 5) is a linear system with constant coefficients, thereby the wave number and the frequency of the dynamical process does not depend on the wave amplitude. It allows stating that the amplitude components of the direct and reflected waves are the same, namely $|a|=|c|$ and $|b|=|d|$.

Besides, the relations (7) must satisfy the boundary conditions (6). The boundary conditions must be satisfied for an arbitrary moment of time. This is true when

$$\varphi=\psi\,;\quad \kappa_k=\frac{k\pi}{l};\quad |a|=|d|.$$

Therefore, single-frequency solutions of non-perturbated equations are transformed into the following

$$u_{0k}\left(t,x\right)=a_k\left(\cos\left(\kappa_k x+\omega_k t+\psi_k\right)-\right.$$
$$\left.-\cos\left(\kappa_k x-\omega_k t-\varphi_k\right)\right);$$
$$w_{0k}\left(t,x\right)=a_k\left(\sin\left(\kappa_k x+\omega_k t+\psi_k\right)+\right.$$
$$\left.+\sin\left(\kappa_k x-\omega_k t-\varphi_k\right)\right).$$

These single-frequency solutions have a corresponding dispersion relation

$$\left(m_1+m_2\right)\omega_k^2+2\Omega\omega_k-N\kappa^2+EI\kappa^4=0.$$

The dispersion relation defines the natural bending vibration frequency as a function of angular velocity and other parameters of the body as

$$\omega_k=\kappa_k\sqrt{\frac{EI\kappa_k^2-N}{m_1+m_2}-\Omega^2}-\Omega\,. \qquad (8)$$

In Figs. 1, *a, b* the dependency of the natural vibration frequency of the elastic body from the linear mass of the CFHM and the angular velocity at $l=10\ m$, $\Omega=10\ s^{-1}$ and $l=15.5\ m$, $\Omega=5\ s^{-1}$ respectively is displayed.

From the physical sense of the relation (8) it also follows that in an elastic body there will be a vibration failure when the angular velocity is equal to

$$\overline{N}=EI\kappa_k^2-\frac{\Omega^2}{m_1+m_2}.$$

In Figs. 2, *a* ($N$ = 1000 $N$) and 2, *b* ($N$ = 0 $N$) the dependency of the critical angular velocity on the length of the body and the linear mass of the CFHM is displayed.

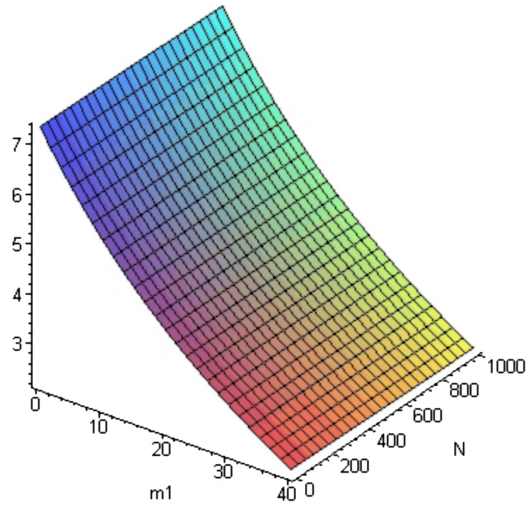
It is known [5] that the compression force acting on the elastic body, affects the frequency and stability of its bending vibrations. In the case under consiredation, the critical velocity of stable motion also depends on the linear mass of the CFHM and the angular velocity. The mentioned value of the compression force is defined as

$$\overline{N}=EI\kappa_k^2-\frac{\Omega^2}{m_1+m_2}.$$

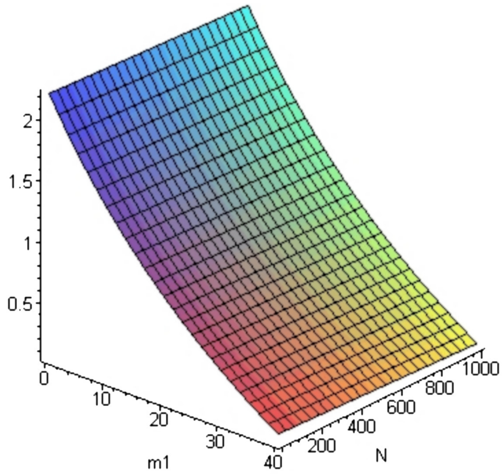
In Fig. 3, the dependency of the critical compression force on the body length and the angular velocity is displayed. The graphical dependencies in Fig. 3 are built at $N=700\ N$, $l=15.5$ m.

The graphical dependencies in Figs. 1–3 show that:

- for CHFMs of greater linear mass, the natural vibration frequency is smaller. In particular, for the linear mass 10 kg/m it is 21 percent greater than for the linear mass 20 kg/m (at the same values of all base parameters and $l=15.5$ m);
- the linear mass of the CFHM plays the dominant role for the critical angular velocity. Its increase from 20 to 30 kg/m leads to the decrease in the critical angular velocity by 7.6 percent;
- the influence of the longitudinal compression force on the critical angular velocity is greater for elastic bodies of significant length;




*a*



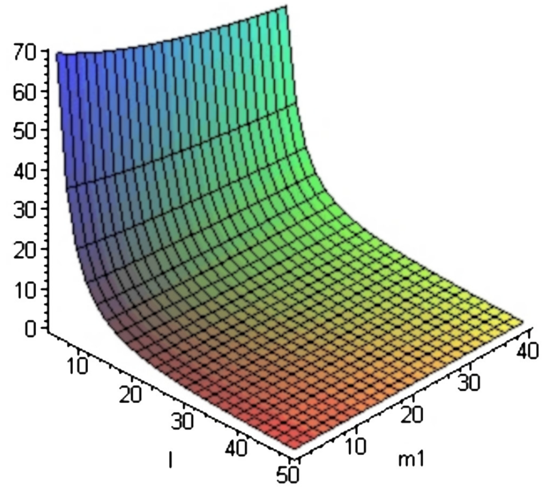

*b*

*Fig. 1. The dependency of the bending vibration frequency of an elastic body on the linear mass of the CFHM and the angular velocity:*
*a* – $l = 10$ *m*, $\Omega = 10\ s^{-1}$; *b* – $l = 15.5$ *m*, $\Omega = 5\ s^{-1}$

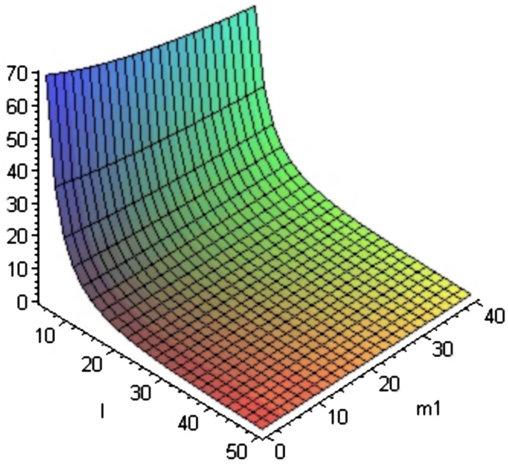
- the greater the angular velocity of the elastic body and the linear mass of the CFHM is, the smaller the critical value of the compression force is.

These results can also describe a multi-frequential process in a linear model of the "elastic body – moving medium" system.

**The analysis of the influence of periodic perturbation on flexural vibrations of the elastic fixed-axis-rotating body in CFHM.** A non-autonomic particular type of equations (1, 2), namely investigating the influence of external periodic forces on the vibration system is very complex for asymptotic research. This case is very important for practical operating of industrial equipment. As the first approximation, in the so-called non-resonance case, the effect of these forces results in a slight change of the vibration form, while in the resonance case it results in a considerable increase in the amplitude, namely the increase in dynamical tension of the body and pressure on the supporting bearings (bearing balls). Therefore, the resonance phenomenon negatively affects the operating of the equipment parts, reducing their exploitation resource. Below, the research on such elastic body vibrations is considered, and a comparative assessment of resonant and non-resonant vibration amplitudes under constant system parameters is conducted.




*a*




*b*

*Fig. 2. Dependency of the critical angular velocity on the linear mass of the CFHM and the body's length:*
*a* – $N = 1000$ *N*; *b* – $N = 0$ *N*

Hereby it is assumed that the external periodic perturbation frequency is close to flexural vibration frequency of the body. The functions in the right parts of the differential equation system, which describes the body's forced vibrations, are periodic by the phase of external periodic perturbation, i.e.

$$f\left(u, w, \frac{\partial u}{\partial t}, \frac{\partial w}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial w}{\partial x}, \ldots, \frac{\partial^3 u}{\partial x^3}, \frac{\partial^3 w}{\partial x^3}, \frac{\partial^4 u}{\partial x^4}, \frac{\partial^4 w}{\partial x^4}, \theta\right),$$

and

$$g\left(u, w, \frac{\partial u}{\partial t}, \frac{\partial w}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial w}{\partial x}, \ldots, \frac{\partial^3 u}{\partial x^3}, \frac{\partial^3 w}{\partial x^3}, \frac{\partial^4 u}{\partial x^4}, \frac{\partial^4 w}{\partial x^4}, \theta\right),$$

are $2\pi$ – periodic by $\theta = \mu t + \gamma_0$; $\mu$ is the frequency; $\gamma_0$ is the initial forced vibration phase,

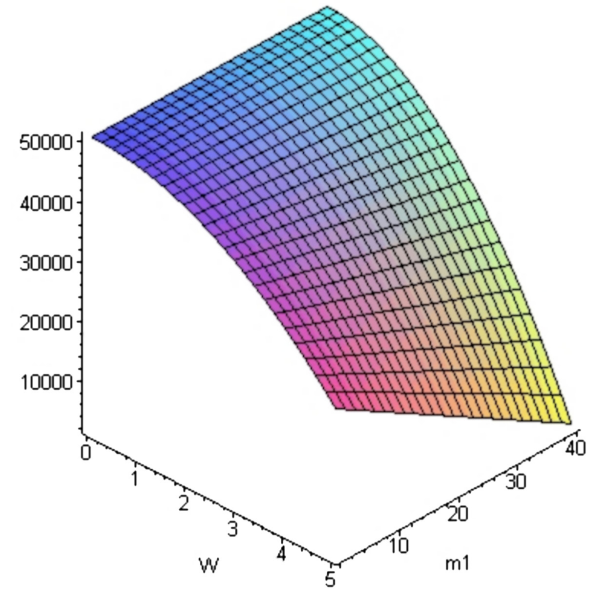



*Fig. 3. The dependency of the critical compression force on the base parameters*

$$f\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^3 u}{\partial x^3},\frac{\partial^3 w}{\partial x^3},\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4},\mu t\right)=$$
$$=f_1\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^3 u}{\partial x^3},\frac{\partial^3 w}{\partial x^3},\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4},\mu t\right)+$$
$$+h\cos\theta=k_1\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial x^2}\right)^3+h\cos\theta,$$

where $h$ is the amplitude.

Now, the resonance case in the elastic body vibrations is considered, so it is assumed that the periodic perturbation frequency is close to natural vibration of the "Elastic body, which rotates – CFHM" system, thus

$$p\mu\approx q\omega_k;$$

$$\omega=\Omega\left(Ik_k-1\right)\pm k_k\sqrt{\Omega^2 I\left(k_k^2 I-2\right)+\frac{N+EIk_k^2}{m+m_1}}.$$

Applying the wave motion theory [1] for describing resonance on the main frequency, results in the following

$$\frac{d\alpha}{dt}=\frac{\varepsilon}{4\pi l\omega}\left(\int_0^l\int_0^{2\pi}f\left(a,x,\varphi+\theta,\theta\right)\left(\cos\left(\kappa x+\varphi+\theta\right)-\right.\right.$$
$$\left.\left.-\cos\left(\kappa x-\varphi+\theta\right)\right)d\theta dx\right);$$
$$\frac{d\varphi}{dt}=\Delta-\frac{\varepsilon}{4\pi l\omega}\left(\int_0^l\int_0^{2\pi}f\left(a,x,\varphi+\theta,\theta\right)\left(\sin\left(\kappa x+\varphi+\theta\right)+\right.\right.$$
$$\left.\left.+\sin\left(\kappa x-\varphi+\theta\right)\right)d\theta dx\right),\qquad(9)$$

where $\Delta=\omega-\mu$ is the unbalance of natural and forced vibrations. In the case when the right parts of the equations (9) have the following properties

$$f_1\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^3 u}{\partial x^3},\frac{\partial^3 w}{\partial x^3},\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4},\mu t\right)=$$
$$=f\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots\right.$$
$$\left.\ldots,\frac{\partial^3 u}{\partial x^3},\frac{\partial^3 w}{\partial x^3},\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4}\right)+h\cos\theta;$$
$$g_1\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x},\ldots,\frac{\partial^3 u}{\partial x^3},\frac{\partial^3 w}{\partial x^3},\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4},\mu t\right)=$$
$$=g\left(u,w,\frac{\partial u}{\partial t},\frac{\partial w}{\partial t},\frac{\partial u}{\partial x},\frac{\partial w}{\partial x}\ldots,\right.$$
$$\left.\ldots,\frac{\partial^3 u}{\partial x^3},\frac{\partial^3 w}{\partial x^3},\frac{\partial^4 u}{\partial x^4},\frac{\partial^4 w}{\partial x^4}\right)+h\cos\theta,$$

then these relations can be written as

$$\frac{da}{dt}=\frac{\varepsilon}{4\pi l\omega}\left(\int_0^l\int_0^{2\pi}f\left(a,x,\varphi\right)\left(\cos\left(\kappa x+\psi\right)-\right.\right.$$
$$\left.\left.-\cos\left(\kappa x-\psi\right)\right)d\psi dx\right)+h_1\cos\varphi;$$
$$\frac{d\varphi}{dt}=\Delta-\frac{\varepsilon}{4\pi l\omega}\left(\int_0^l\int_0^{2\pi}f\left(a,x,\psi\right)\left(\sin\left(\kappa x+\psi\right)+\right.\right.$$
$$\left.\left.+\sin\left(\kappa x-\psi\right)\right)d\theta dx\right)-h_1\sin\varphi.$$

Tables 1–3 present the resonant amplitude values corresponding to different values of:

- angular velocities of the elastic body rotation $\Omega$, $s^{-1}$;
- CFHM linear velocity $V$, m/s;
- CFHM linear mass $m$ and the elastic body mass $m_1$, kg/m$^2$.

The main elastic body characteristics are considered as: $E$ = 2.06·10$^{11}$ N/m$^2$; $I$ = 6·10$^{-6}$m$^4$; $N$ = =7·10$^2$$N$; $k_1$=10; $l$= 10 m; $m$ = 40 kg/m$^2$; $m_1$ = 35 kg/m$^2$ (Table 1).

In Tables 2, 3 the resonance amplitude value calculations are made, using the same parameter values as in Table 1, excluding the linear masses, which are considered as equal: case a) $m$ = 40 kg/m$^2$, $m_1$= = 20 kg/m$^2$; case b) $m$ = 20 kg/m$^2$, $m_1$ = 20 kg/m$^2$.

The received theoretical data, and presented calculation relations allow concluding the following about the vibration system:

- firstly, on increasing the CFHM velocity along the elastic body, the resonant amplitude has several local maximums;
- secondly, local maximums of the resonant amplitude depend on the CFHM velocity as well as the body angular velocity;
- thirdly, lower values of CFHM linear mass correspond to lower values of resonant amplitude.

**The dynamic process influence on the body stress state.** Determining dynamic tensions caused by nonlinear vibrations, is very important from the practical point of view, along with determining the dynamics of rotating elastic bodies in a flow of homogeneous medium. In the case of vibrations caused by the effect of periodic forces, whose frequency is close to the natural frequency, the amplitude, and thus the dynamic tensions depend on the frequency of the external force. When the mentioned frequencies match, or when the frequency of the external force approaches the natural vibration frequency in a case of a weak damping, a resonance develops, i.e. the amplitude increases dramatically. This amplitude increase leads to considerable growth of dynamic tensions inside the body. In this case, the dynamic (resonance) tensions depend on the internal factors (physical and mechanical parameters of the body and CFHM, geometry, etc.) and the external factors (body angular velocity, perturbation forces, etc.), as shown in the above-considered relations. Therefore, for quantitative assessment of these factors, a convenient engineering mathematical tool that will allow describing the maximal values of the dynamical tensions in the case of nonlinear vibrations at fixed values of the angular velocity and the velocity of CFHM needs to be developed.

Thus, it is necessary not only to predict resonance phenomenon and calculate the resonance amplitudes, but also to estimate the dynamic tensions in case of resonance in observable elastic bodies. Summarizing the above aspects, the problem of estimating maximal dynamic tensions appearing in an elastic body, which has a constant angular velocity, in a CFHM, is not less important than the modelling of the dynamic process. In other words, the above-solved dynamics problems can be a basis for determining the strength characteristics of production equipment.

*Table 1*

The value of the resonance amplitudes

| $V$ / $\Omega$ | 0 | 3 | 5 | 8 | 10 | 15 | 18 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.035 | 0.022 | 0.018 | 0.022 | 0.024 | 0.025 | 0.021 | 0.055 | 0.028 | 0.012 |
| 3 | 0.038 | 0.039 | 0.065 | 0.041 | 0.031 | 0.048 | 0.05 | 0.06 | 0.038 | 0.35 |
| 5 | 0.11 | 0.14 | 0.03 | 0.035 | 0.03 | 0.04 | 0.049 | 0.055 | 0.052 | 0.041 |
| 8 | 0.05 | 0.2 | 0.13 | 0.05 | 0.07 | 0.08 | 0.05 | 0.08 | 0.06 | 0.04 |
| 10 | 0.05 | 0.12 | 0.09 | 0.08 | 0.11 | 0.15 | 0.14 | 0.08 | 0.05 | 0.04 |
| 12 | 0.14 | 0.18 | 0.24 | 0.28 | 0.25 | 0.15 | 0.38 | 0.32 | 0.31 | 0.2 |

*Table 2*

The value of the resonance amplitudes in case ***a***

| $V$ / $\Omega$ | 0 | 3 | 5 | 8 | 10 | 15 | 18 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.01 | 0.011 | 0.027 | 0.016 | 0.017 | 0.05 | 0.079 | 0.07 | 0.077 | 0.02 |
| 3 | 0.16 | 0.15 | 0.017 | 0.017 | 0.15 | 0.16 | 0.19 | 0.19 | 0.08 | 0.07 |
| 5 | 0.05 | 0.04 | 0.08 | 0.07 | 0.11 | 0.1 | 0.08 | 0.04 | 0.13 | 0.08 |
| 8 | 0.03 | 0.22 | 0.05 | 0.08 | 0.07 | 0.04 | 0.18 | 0.05 | 0.04 | 0.08 |
| 10 | 0.21 | 0.11 | 0.06 | 0.03 | 0.05 | 0.07 | 0.07 | 0.08 | 0.065 | 0.05 |
| 12 | 0.12 | 0.33 | 0.25 | 0.24 | 0.08 | 0.13 | 0.12 | 0.2 | 0.14 | 0.13 |

*Table 3*

The value of the resonance amplitudes in case ***b***

| $V$ / $\Omega$ | 0 | 3 | 5 | 8 | 10 | 15 | 18 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.03 | 0.04 | 0.05 | 0.02 | 0.02 | 0.03 | 0.05 | 0.03 | 0.06 | 0.04 |
| 3 | 0.03 | 0.11 | 0.03 | 0.02 | 0.09 | 0.08 | 0.04 | 0.08 | 0.05 | 0.03 |
| 5 | 0.02 | 0.04 | 0.01 | 0.12 | 0.05 | 0.11 | 0.03 | 0.03 | 0.03 | 0.08 |
| 8 | 0.02 | 0.02 | 0.03 | 0.03 | 0.05 | 0.07 | 0.04 | 0.07 | 0.02 | 0.15 |
| 10 | 0.09 | 0.04 | 0.09 | 0.04 | 0.03 | 0.07 | 0.15 | 0.06 | 0.03 | 0.04 |
| 12 | 0.12 | 0.15 | 0.17 | 0.16 | 0.15 | 0.05 | 0.03 | 0.08 | 0.14 | 0.22 |

For estimating the tensions, caused by the bending vibrations of machine elements (specifically the body rotating motion and CFHM motion), the following relation is used

$$\sigma_{max} = \max\left(\frac{EI}{W}\frac{\partial^2 u(x,t)}{\partial x^2}\right),$$

where $W$ is the resistance momentum, which is determined as $W = I/x_{\max}$.

Taking into account the base relations for describing the dynamic process, the tension can be calculated with the relations

$$\sigma_{max} = \frac{EI}{W}\max\Big(-ak^2\big(\cos(kx+\omega t+\phi) - \\ -\cos(kx-\omega t-\phi)\big)\Big),$$

where $a$ and $\phi$ parameters are determined by the differential equations (4, 5) [4].

**Conclusions.** The analytical relations and graphical dependencies in this paper show that:

1. The natural nonlinear vibration frequency of an elastic body decreases at:

- higher values of CFHM velocity;
- CFHM of higher specific weight;
- amplitude of transverse vibrations.

2. The prior tension force of the elastic body affects not only the main parameters of its nonlinear vibrations, but also their stability. At the prior tension force value that approaches $N = \left(\frac{\pi k}{l}\right)^2 EI$ a vibration failure occurs.

3. Resonant dynamic tensions of an elastic body, taking into account its angular velocity, are the higher for the lower "natural dynamic frequency" values (higher for the higher angular velocity values).

4. Resonant dynamic tensions at high values of angular velocity are several times greater than the resonant tensions of a "static auger" (which does not rotate). The latter should be taken into account while choosing the dynamic strength capacity.

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**Мета.** Знаходження динамічного коефіцієнта запасу міцності технологічного обладнання шляхом дослідження динамічних процесів у нелінійній коливальній системі типу стиснутого вала, що має широке використання в гірничорудній промисловості. Математичні моделі таких систем раніше в літературі досліджувалися, переважно, на базі чисельного моделювання. У цій роботі пропонується використати наближені методи нелінійної механіки й застосувати методику спеціальних періодичних функцій для ґрунтовного дослідження динаміки вказаних систем і умов виникнення явища резонансу в них, а також описати методику визначення динамічного коефіцієнта запасу міцності бурильного обладнання.

**Методика.** Методика аналізу резонансних режимів коливань і визначення характеристик міцності елементів машин базується на асимптотичних методах нелінійної механіки, хвильовій теорії руху й використанні спеціальних Ateb-функцій.

**Результати.** У роботі для вказаних нелінійних коливальних систем типу стиснутих валів аналітично отримані умови настання резонансу залежно від параметрів системи та описана методика обчислення динамічного коефіцієнта запасу міцності елементів машин.

**Наукова новизна.** Полягає в тому, що вперше обчислення динамічного коефіцієнта запасу міцності в системах типу стиснутих валів здійснено на базі аналітичного підходу, що дозволяє, на відміну від чисельних та експериментальних методів, точніше досліджувати особливості динаміки таких систем і уникати виникнення небажаних резонансів у роботі елементів гірничих машин.

**Практична значимість.** Представлена методика дозволяє вирішувати не тільки задачі аналізу, але й важливі задачі синтезу технічних коливальних систем на стадії проектування, здійснювати вибір пружних характеристик динамічних систем і обчислювати динамічний коефіцієнт запасу міцності бурильного обладнання, ураховуючи можливі резонансні явища у ньому. Такі характеристики елементів гірничих машин дозволяють ефективно здійснювати безпечні гірничі роботи.

**Ключові слова:** *коливання нелінійних систем, стиснутий вал, пружні характеристики, динамічний коефіцієнт запасу міцності, спеціальні функції*

**Цель.** Нахождение динамического коэффициента запаса прочности технологического оборудования путем исследования динамических процессов в нелинейной колебательной системе типа сжатого вала, которая имеет широкое использование в горнорудной промышленности. Математические модели таких систем ранее в литературе исследовались, преимущественно, на базе численных подходов. В этой работе предлагается использовать приближенные методы нелинейной механики и применить методику специальных периодических функций для основательного исследования динамики указанных систем и условия возникновения явления резонанса в них, а также описать методику определения динамического коэффициента запаса прочности шнекового оборудования.

**Методика.** Методика анализа резонансных режимов колебаний и определения характеристик прочности элементов машин базируется на асимптотических методах нелинейной механики, волновой теории движения и использовании специальных Ateb-функций.

**Результаты.** В работе для указанных нелинейных колебательных систем типа сжатых валов аналитически получены условия наступления резонанса в зависимости от параметров системы и описана методика вычисления динамического коэффициента запаса прочности элементов машин.

**Научная новизна.** Заключается в том, что впервые вычисление динамического коэффициента запаса прочности в системах типа сжатых валов осуществлено на базе аналитического подхода, который позволяет, в отличие от численных и экспериментальных методов, точнее исследовать особенности динамики таких систем и избегать возникновения нежелательных резонансов в работе элементов горных машин.

**Практическая значимость.** Представленная методика позволяет решать не только задачи анализа, но и важные задачи синтеза технических колебательных систем на стадии проектирования, осуществлять выбор упругих характеристик динамических систем и вычислять динамический коэффициент запаса прочности бурильного оборудования, учитывая возможные резонансные явления в нем. Такие характеристики элементов горных машин позволяют эффективно осуществлять безопасные горные работы.

**Ключевые слова:** *колебания нелинейных систем, сжатый вал, упругие характеристики, динамический коэффициент запаса прочности, специальные функции*

*Рекомендовано до публікації докт. техн. наук Є. В. Харченком. Дата надходження рукопису 28.04.16.*