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CLUSTERING OF GROUP EXPERT ESTIMATES BASED ON MEASURES IN THE THEORY OF EVIDENCE

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КЛАСТЕРИЗАЦІЯ ГРУПОВИХ ЕКСПЕРТНИХ ОЦІНОК З ВИКОРИСТАННЯМ МЕТРИК ТЕОРІЇ СВІДОЦТВ

Purpose. The main purpose of the article is a study of new approaches and development of mathematical models of group expert estimate structuring (clustering) based on mathematical apparatus of modern theories.

Methodology. The study methodology is based on the mathematical apparatus of the theory of evidence, cluster analysis. Jousselme measure was used to determine the similarities and differences of clusters.

Findings. The proposed methodology of expert information structuring allows assessing the degree of consistency of expert assessments within the expert group; in the case of its absence it is possible to receive a partition of the expert committee into the groups with similar expert estimates. The expert assessments in these groups are characterized by uniformity and consistency. A measure of consistency is characterized by the degree of proximity of expert assessments.

Originality. Methods of mathematical theory of evidence were used to identify and analyse the expert information. Unlike existing approaches, this theory allows considering specific forms of un-factors, such as a combination of uncertainty and fuzzy arising from the process of interaction between the expert judgments. The structure of such interactions may be different in nature - they can be consistent, compatible, or arbitrary; they can be arbitrarily nested and overlap. This allows getting more “subtle” analysis of expert assessments. To split a commission of experts into groups with similar views, we proposed to use Jousselme measure for characterizing the degree of difference between the generated groups of expert evidence. Expert evidence belongs to one group, if the value of Jousselme measure for all evidence of this group does not exceed a predetermined threshold. A measure, reflecting the degree of conflict between the analysed evidence and formed plurality of expert evidence, was used to select the order of consideration of expert evidence.

Practical value. The proposed method of structuring of group expert estimates generated under uncertainty and conflicting expert evidence constitutes the theoretical basis for the construction of information technologies of the analysis of the expert information using methods of un-factors modelling. This information technology can be used as the tools of decision support systems to advise the person making a decision according to the “Situation-Variant solutions” model.

Keywords: *theory of evidence, metrics, expert evaluation, clustering, uncertainty*

Introduction. One of the main purposes of the expertise is to obtain a consensus between members of the expert committee that is the basis for making recommendations for the decision-maker (DM). However, it is not always possible to achieve this consistency.

The real situations show that in the expert committee there are experts whose judgments stand out from the majority.

In [1] it is noted that the results of the expertise can lead to one of the three main cases: 1) experts' estimates are "close" (related) to each other. It could indicate a fairly high degree of consistency in the expert group; 2) the majority of experts is divided into a small number of groups (clusters), in which the experts' assessments are "close" to each other to a greater or lesser degree; 3) a set of experts are divided into a large number of small groups.

In [1] it is stated that case 2 indicates a small number of expert groups in the set of experts who reflect different points of view.

Case 3 characterizes the inconsistency of the expert committee and may indicate both unfounded choice of the method of getting the expert information and the presence the sub-groups of experts (usually minor ones) or individual experts whose assessments differ vastly from the majority's assessments.

Cases 2 and 3 are characterized by the lack of consistency in the expert assessments that leads to some difficulties in the aggregation of estimates. This, in turn, raises the general problem of finding and developing approaches for structuring (clustering, ranking) of expert group assessments.

Analysis of the recent research and publications. After analysing the classical methods for clustering of expert group judgments into consistent (in some sense) sub-groups, we defined, that it is not always possible to apply them efficiently.

There are two important conditions that we should take into account to solve problems of structuring (clustering) of expert group evaluations and to select appropriate methods [2]: the diversity of measurement scales of expert assessments (nominal, ordinal, absolute, interval, and others), which allow obtaining the expert data suitable for decision making support in the form of labels, rankings, numbers, intervals, binary relations, etc.; limited number of experts n ($n \le 30$) in groups.

For example, to analyse the expert information formed in the numerical scales (both absolute and ball ones) clustering algorithms are widely used, which can be roughly divided into three categories: distance-based methods such as the Euclidean distance, the Mahalanobis distance, the Kolmogorov-Smirnov Test, the Bhattacharyya distance measure, etc.; clustering algorithms dealing with mathematical programming methods (dynamic and integer); density-based clustering, which provides an estimate of probability density function (modal analysis, decomposition of mixtures of probability distributions, method of histogram and others).

Methods for non-numerical data clustering are used to analyse the expert assessments made, for example, in the scales of the order or relation. Nowadays, the Kemeny method, based on metric with the same name (the distance $d(B_i, B_j)$) between two binary relations B_i and B_j defined on the space of non-numeric expert opinions, is the most efficient [3].

This distance characterizes the measure of similarity (proximity) of objects to each other. The computation of the Kemeny median is the integer programming problem, whose computational complexity is arbitrarily high.

Unsolved aspects of the problem. The effectiveness of these methods for clustering of expert group assessments depends upon the correct considering of various un-factors (incompleteness, uncertainty, fuzziness, inaccuracy, ambiguity, and others), which appear while receiving and processing the expert information [4].

The specific forms of un-factors may exist in actual practice, for example, uncertainty and fuzziness or a mix of both, which are formed in the process of interaction between the experts' judgments (evidence). The structure of such judgments can be varied: compatible evidence, consistent evidence or arbitrary evidence; they can be potentially nested and overlap.

In such a context, there is a problem of expert information structuring coming from different sources and generated under uncertainty and conflict (contradictory, not coinciding expert judgments), and identification of experts or groups of experts characterized by consistent judgments.

Different aspects of the imperfection (uncertainty, conflict, impression) of the information can be modelled within the Dempster-Shafer theory (DST, Evidence theory), which is a mathematical tool able to characterize the imperfect information [5, 6].

In this paper, we propose using metrics in evidence theory to solve this problem [7].

Objectives of the article. Let us consider a set of alternatives $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$ and group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$, expressing their opinions and generating a set of individual choice rankings (orderings) $\mathrm{B} = \left\{B_j \mid j = \overline{1,t}\right\}$, where B_j has a weak ordering.

The problem is to determine groups of experts $\mathrm{E} \Rightarrow \{Gr_1\}, \{Gr_2\}, \ldots, \{Gr_j\}, \ldots, \{Gr_{[t/2]}\}$ ($Gr_p \subseteq \mathrm{E}$, $\{Gr_p\} = \{E_1, \ldots, E_r\}$, $t \ge r \ge 2$), with similar judgments. So it is required to construct a rule that allows determining uniquely the identity of the expert E_j to the group Gr_p.

Presentation of the main research. Let $\Omega = \{\omega_i \mid i = \overline{1,n}\}$ be a finite set (frame of discernment) of n exclusive and exhaustive elements (hypotheses) [5, 6]. The power set 2^Ω of Ω gives a set of focal elements $\mathrm{B} = \{B_j \mid j = \overline{1,s}\}$, $s = 2^\Omega$, each of which appears to be a focal element, based on which the level of confidence, that the best choice is in the selected subsets, is determined.

Any subset $B_j \subseteq \Omega$ can be constructed from elements of Ω with operator $\cap$ so that:

$$\begin{aligned}
&1.\ B_j = \{\varnothing\};\\
&2.\ B_j = \{\omega_j\};\\
&3.\ B_j = \{\omega_i \mid i = \overline{1,p}\},\ p < n;\\
&4.\ B_j = \Omega = \{\omega_i \mid i = \overline{1,n}\}.
\end{aligned} \qquad (1)$$

The 1st statement corresponds to a situation for which none of the alternatives ($\omega_i \in \Omega$) satisfies an expert choice, i.e. their choice is empty; the 2nd is to support that an expert selected one alternative ($\omega_i \in \Omega$); the 3rd is to support that an expert selected p alternatives and the fourth statement is to show that an expert finds it difficult to choose any of the proposed alternatives ($\omega_i \in \Omega$), i.e. all alternatives are equal.

Three important functions are defined in DST: the basic probability assignment (or mass) function, the Belief function, and the Plausibility function ($\forall B \subseteq \Omega$) [5, 6]:

- *basic probability assignment m*: $2^\Omega \to [0,1]$

$$0 \le m(B_j) \le 1, \forall(B_j \in 2^\Omega), m(\varnothing) = 0, \sum_{B_j \in 2^\Omega} m(B_j) = 1; \quad (2)$$

- *belief function Bel*: $2^\Omega \to [0,1]$

$$Bel(A) = \sum_{B_j \subseteq A,\, B_j \in 2^\Omega} m(B_j); \quad (3)$$

- *plausibility function Pl*: $2^\Omega \to [0,1]$

$$Pl(A) = \sum_{B_j \cap A \ne \varnothing,\, B_j \in 2^\Omega} m(B_j). \quad (4)$$

A method for measuring the distance between two BPAs is proposed in [7, 8] and is called *Jousselme et. al.'s* distance

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2}(m_1 - m_2)^T \underset{=}{D}(m_1 - m_2)}, \quad (5)$$

where m*i* is a 2^Ω-dimensional column vector with basic probability assignment of focal elements generated on independent group *i* of evidence as its coordinates; $(m_i)^T$ is the transpose of vector m*i* (row vector); $(m_1 - m_2)$ stands for vector subtraction; $\underset{=}{D}$ is a $2^\Omega \times 2^\Omega$ matrix whose elements are

$$D(B_i, B_j) = \begin{cases} 1, & if B_i = B_j \\ S(B_i, B_j), & \forall B_i, B_j \in \Omega \end{cases}. \quad (6)$$

As the similarity function $S(B_i, B_j)$ between two focal elements B_i and B_j, the Jaccard's coefficient $S(B_i, B_j) = \left|B_i \cap B_j\right| / \left|B_i \cup B_j\right|$ was used, where $|\cdot|$ means the cardinality of the corresponding subsets.

Jousselme distance measure satisfies the following properties:

1. $d_J(m_1, m_2) \ge 0$;
2. $d_J(m_1, m_2) = 0 \Leftrightarrow m_1 = m_2$;
3. $d_J(m_1, m_2) = d_J(m_2, m_1)$;
4. $d_J(m_1, m_2) \le d_J(m_1, m_3) + d_J(m_3, m_2)$.

Jousselme measure $d_J(m_1, m_2)$ can be used to assign the conflict measure between 2 experts in a set of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$ [9]

$$\mathrm{Conf}(1, 2) = d_J(m_1, m_2). \quad (7)$$

The conflict measure between one expert *i* and the other $t - 1$ experts is defined by [9]

$$\mathrm{Conf}(i, \mathrm{E}) = \frac{1}{t-1} \sum_{j=1, i \ne j}^{t} \mathrm{Conf}(i, j). \quad (8)$$

As a general rule, expert survey leads to a situation in which the majority of experts are divided into a small number of groups (clusters). The expert judgments in one cluster are similar to each other whereas judgments of different clusters are dissimilar. This indicates insufficiently high consistency in an expert group and allows assuming that the opinions of experts are far from each other.

Therefore, the problem definition is to divide a given group of experts into several sub-groups (clusters) with similar and consistent expert estimates, and characterize each sub-group.

Let $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$ be a set of decision makers (experts) who present their opinions on a set of alternatives $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$, where t is the number of experts and n is the number of alternatives in a set. Then a set of focal elements given by each expert $\mathrm{X} = \{X_j \mid j = \overline{1,t}\}$ will be formed where X_j is a 2^{A}-dimensional vector with opinion of expert E_j (focal elements) as its coordinates. All elements of the set X are satisfying the conditions (1).

Then a basic probability assignment as a vector $\mathrm{m}_j = \{m_i \mid i = \overline{1,s}\}$, $s = 2^{\mathrm{A}}$, associated with a given subsets X_j, $j = \overline{1,t}$, is defined. All elements of m_j satisfy the conditions (2).

We have to divide the original set of experts E ⇒ ⇒ $\{Gr_1\}, \{Gr_2\}, \ldots, \{Gr_j\}, \ldots, \{Gr_{[t/2]}\}$ ($Gr_p \subseteq \mathrm{E}$, $\{Gr_p\} = \{E_1, \ldots, E_r\}$, $t \ge r \ge 2$), and identify a sub-groups of experts with similar opinions (expert evidence).

To solve this problem we propose the method that consists of the following.

Step 1: Calculate measure (5) for every pairs of $\langle \mathrm{m}_i, \mathrm{m}_j \rangle$, $i, j = \overline{1,t}$ $i \ne j$.

The results are stored in the form of a matrix having a property of symmetry of the main diagonal as follows

$$\begin{pmatrix} - & d(m_1, m_2) & \ldots & d(m_1, m_t) \\ d(m_2, m_1) & - & \ldots & d(m_2, m_t) \\ \ldots\ldots\ldots & \ldots\ldots\ldots & - & \ldots\ldots\ldots \\ d(m_t, m_1) & d(m_t, m_2) & \ldots & - \end{pmatrix}, \quad (9)$$

where $d(m_i, m_j) = d(m_j, m_i)$, $\forall i, j = \overline{1,t}$, $i \ne j$; t is the number of compared elements (objects); $d(m_i, m_j)$ stands for values of *Jousselme* measure (5).

Finally, for each expert E_j, a graph could be constructed (Fig. 1), which reflects the dissimilarity between expert E_j and other $t - 1$ experts.

Step 2: Identify sub-groups of experts $Gr_p \subseteq \mathrm{E}$, $p = \overline{1,[t/2]}$, , where $[x]$ is the greatest integer $< x$.

Decision rule: $\forall E_j \in Gr_p$, $j = \overline{1,r}$, , $t \ge r \ge 2$ must meet the conditions $\forall (i,j) = \overline{1,r}$, $i \ne j$: $l_{p-1} < d(m_i, m_j) \le l_p$, where l_{p-1}, l_p are some values which are responsible for belonging of expert E_j to the group Gr_p.

Let us consider a set of examples of the proposed method.

Example 1. Let us consider the frame of discernment $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$, $n = 4$ with Shafer's model and the set of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$, $t = 10$, expressing their opinions.

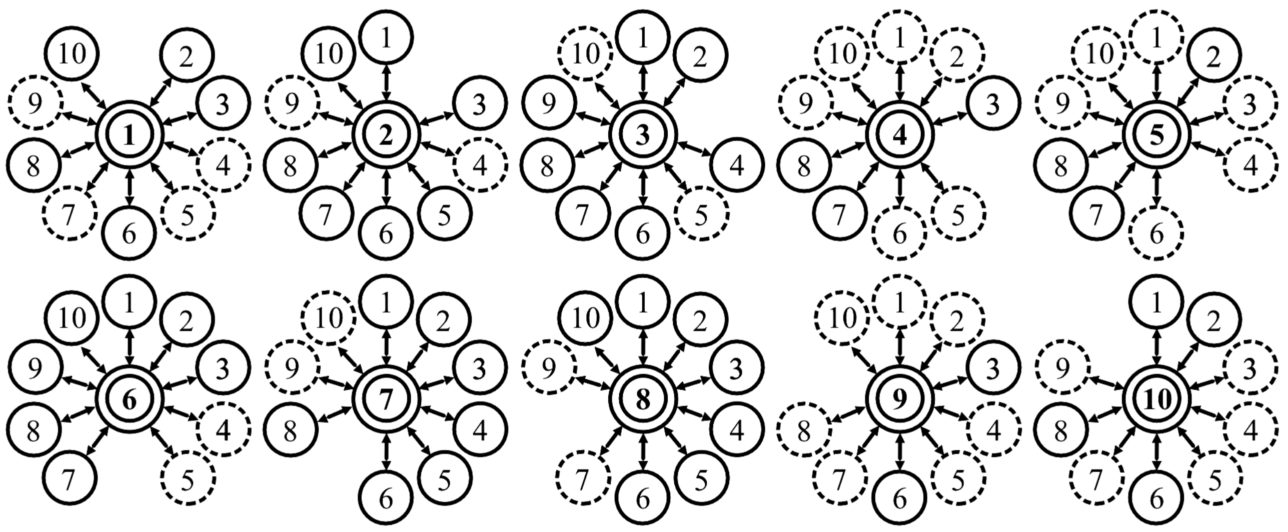

Fig. Schematic demonstration of measure $d_J(m_k, m_j)$

The results of the expertise are reported in Table 1.

To evaluate the dependency between two sources of an expert's evidence, we propose to compute a measure of discrepancy (5) between their outputs.

A square matrix of order 10 contains pairwise distances (5), calculated for a system of subsets X defined by a group of experts on A set, is given in Table 2.

Figure considers a graphical representation of measure $d_J(m_k, m_j)$, $\forall k, j = \overline{1,t}$, $k \neq j$: the value of element k is given in the centre (the schematic representation of the group of evidence k, i.e. the group of evidence generated by expert E_k); solid lines marked element j (the group of evidence generated by expert E_j), satisfying: $0 < d_J(m_k, m_j) \le 0.3$, and illustrated a low degree of dissimilarity between m_k and m_j; dashed lines marked elements (the groups of evidence) satisfying: $0.3 < d_J(m_k, m_j) \le 0.6$, and characterized the medium degree of dissimilarity between m_k and m_j; the set of elements satisfying $0.6 < d_J(m_k, m_j) \le 1$ is empty, and characterized the high degree of dissimilarity between m_k and m_j.

Let us compute the Conf(j, t), with the Equation (8). This measure must quantify how much expert E_j is in conflict with the rest of the set $E \backslash E_j$ (Table 3).

The results of grouping the initial set of experts are given in Table 4.

We considered two different procedures of presentations of generated groups of evidence:

Table 1

Basic probability assignments of focal elements $m_j(A_i)$

Expert E_j	$m_j(A_1)$	$m_j(A_2)$	$m_j(A_3)$	$m_j(A_4)$
E_1	0.1	0.5	0.3	0.1
E_2	0.2	0.3	0.4	0.1
E_3	0.3	0.2	0.2	0.3
E_4	0.5	0.1	0.1	0.3
E_5	0.1	0.1	0.6	0.2
E_6	0.1	0.3	0.2	0.4
E_7	0.3	0.1	0.4	0.2
E_8	0.3	0.3	0.3	0.1
E_9	0.1	0.2	0.1	0.6
E_{10}	0.1	0.6	0.2	0.1

Table 2

Representation of *Jousselme's* pairwise distances $d_J(m_k, m_j)$

	1	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	–	0.17	0.3	0.45	0.36	0.27	0.33	0.2	0.44	**0.1**
2	0.17	–	0.22	0.36	0.22	0.27	0.17	**0.1**	0.42	0.27
3	0.3	0.22	–	**0,17**	0.33	**0.17**	**0.17**	**0.17**	0.27	0.35
4	0.45	0.36	**0.17**	–	0.46	0.33	0.27	0.28	0.37	0.48
5	0.36	0.22	0.33	0.46	–	0.35	**0.2**	0.3	0.46	0.46
6	0.27	0.27	**0.17**	0.33	0.35	–	0.28	0.27	**0.17**	0.3
7	0.33	**0.17**	**0.17**	0.27	0.2	0.28	–	**0.17**	0.39	0.41
8	0.2	**0.1**	0.17	0.28	0.3	0.27	0.39	–	0.41	0.27
9	0.45	0.42	0.27	0.36	0.46	**0.17**	0.39	0.41	–	0.46
10	**0.1**	0.27	0.35	0.48	0.46	0.3	0.41	0.27	0.46	–

Table 3

Values of measure Conf(j, t)

	Conf(j, t)	**Order**
Conf(E_1, t)	0.290	6
Conf(E_2, t)	0.245	3
Conf(E_3, t)	0.240	1
Conf(E_4, t)	0.350	9
Conf(E_5, t)	0.349	8
Conf(E_6, t)	0.267	5
Conf(E_7, t)	0.266	4
Conf(E_8, t)	0.241	2
Conf(E_9, t)	0.375	10
Conf(E_{10}, t)	0.343	7

Table 4

Groups of experts

Group number	Expert number	A degree of dissimilarity of evidence m_k and m_j
Procedure 1		
1	1, 2, 3, 6, 8	low : $0 < d_J(m_k, m_j) \le 0.3$
2	5, 7	low : $0 < d_J(m_k, m_j) \le 0.3$
3	4, 9, 10	medium : $0.3 < d_J(m_k, m_j) \le 0.6$
Procedure 2		
1	1, 2, 3, 6, 8	low : $0 < d_J(m_k, m_j) \le 0.3$
2	5, 7	low : $0 < d_J(m_k, m_j) \le 0{,}3$
3	4, 9, 10	medium : $0{,}3 < d_J(m_k, m_j) \le 0{,}6$

- in the first case, experts' evidence was considered in order (starting with group of evidences generated by expert 1);

- in the second case, experts' evidence was considered starting with group of evidence generated by expert E_k with **min**(Conf(E_k, t)), and so on in order of increasing Conf(E_j, t).

Both procedures of the decomposition of the expert group are the same. As a result of the partition of the original expert group $\text{E} = \{E_j \mid j = \overline{1,t}\}$, , we identified three sub-groups (clusters) of experts:

$G_1 = \{E_1, E_2, E_3, E_6, E_8\}$ makes the experts' evidence in this clusters which do not differ by more than $d_J(m_k, m_j) = 0.3$ (max$(d_J(m_i, m_j) = 0.27)$. This fact presupposes the existence of a low dissimilarity between all groups of evidence, and leads to the conclusion that there is a consistency between the corresponding groups of evidence;

$G_2 = \{E_5, E_7\}$ is the experts' evidence in this clusters are also consistency ($d_J(m_5, m_7) = 0.2$). But in relation to other experts, the consistency decreases while increasing the values $d_J(m_k, m_j)$. In this case the value $d_J(m_k, m_j)$ varies in $0 < d_J(m_k, m_j) \le 0.45$;

$G_3 = \{E_4, E_9, E_{10}\}$, the experts in this clusters, are characterized by the medium dissimilarity between all generated groups of evidence: $\forall(E_i, E_j) \in Gr_3$, $i \ne j$: $d_J(m_i, m_j) \le 0.6$. The maximum degree of the dissimilarity between evidence in this group is **max**$(d_J(m_4, m_{10})) = 0.48$.

Example 2. Let $\text{A} = \{A_i \mid i = \overline{1,n}\}$, $n = 4$ be the frame of discernment (with Shafer's model) to be analysed by a set of experts $\text{E} = \{E_j \mid j = \overline{1,t}\}$, $t = 10$. Table 5 gives the basic probability assignments (BPA) of generated focal elements, defined by Equation (2).

Table 5 shows that only experts E_4 and E_6 generated focal elements of A. This leads to an increase of the degree of inaccuracy and uncertainty. The experts are characterized by inconsistency (e.g. experts E_3 and E_4 are in conflict according to alternative A_2). The coefficient of conflict $\left(\sum_{C \cap D = \varnothing} m_i(C) m_j(D)\right)$ varies between 0.56 and 0.92 and achieves the maximum value for evidence E_3 and E_7.

Based on the data in Table 5 we can compute the values of the distance given by Equation (5), and the measure Conf(j, t) by the formula (8).

Both procedures tend to the same result – all the experts $\text{E} = \{E_j \mid j = \overline{1,t}\}$ were grouped into three clusters:

$G_1 = \{E_1, E_2, E_5, E_6, E_{10}\}$ – the experts' evidence in this cluster does not differ by more than $d_J(m_k, m_j) = 0.3$ (max$(d_J(m_2, m_{10}) = 0.3)$). This fact presupposes the existence of a low dissimilarity between all groups of evidence, and leads to the conclusion that there is a consistency between the corresponding groups of evidence;

$G_2 = \{E_3, E_9\}$ – the experts' evidence in this cluster is also consistency ($d_J(m_3, m_9) = 0.2$). But in relation to other experts the consistency decreases with increasing values $d_J(m_k, m_j)$. In this case the value $d_J(m_k, m_j)$ varies in $0 < d_J(m_k, m_j) \le 0.45$;

$G_3 = \{E_4, E_7, E_8\}$ – the experts in this cluster are characterized by the medium dissimilarity between all generated groups of evidence: $\forall(E_i, E_j) \in Gr_3$, $i \ne j$: $d_J(m_i, m_j) \le 0.6$. The maximum degree of the dissimilarity between evidence in this clusters is **max**$(d_J(m_4, m_7) = 0.33$.

Table 5

Basic probability assignments of focal elements $m_j(A_i)$

Expert E_j	$m_j(A_1)$	$m_j(A_2)$	$m_j(A_3)$	$m_j(A_4)$
E_1	–	0.5	0.3	0.2
E_2	0.2	0.3	0.4	0.1
E_3	–	0.6	–	0.4
E_4	0.5	0.1	0.1	0.3
E_5	–	0.3	0.4	0.3
E_6	0.1	0.3	0.2	0.4
E_7	0.3	–	0.5	0.2
E_8	0.4	0.3	0.3	–
E_9	–	0.4	–	0.6
E_{10}	–	0.6	0.2	0.2

Conclusions. In this paper, we propose a new algorithm for clustering expert group judgments based on the mathematical apparatus of measures in the theory of evidence. Applied measures are used to quantify a notion of dissimilarity (distance) and conflict of evidence between $m_1(\cdot)$ and $m_2(\cdot)$.

In contrast to the existing methods for clustering expert judgments, the proposed algorithm allows getting better results of group decision making under multialternative, specific forms of un-factors (uncertainty and fuzziness) and conflict (conflicting, dissenting) expert judgments.

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Мета. Дослідження нових підходів і розробка математичних моделей структуризації (кластеризації) групових експертних оцінок на основі математичного апарату сучасних теорій.

Методика. Базується на математичному апараті теорії свідоцтв, кластерному аналізі. В якості критерію визначення схожості та відмінності кластерів розглянута метрика Jousselme.

Результати. Запропонована методика структуризації експертної інформації дозволяє оцінювати ступінь узгодженості експертних оцінок усередині експертної групи; у разі її відсутності – отримувати розбиття експертної комісії на групи, усередині яких оцінки експертів близькі між собою та характеризуються однорідністю та узгодженістю. Міра узгодженості характеризується ступенем близькості експертних оцінок.

Наукова новизна. Для виявлення й аналізу експертної інформації були використані методи теорії свідоцтв. На відміну від існуючих підходів, дана теорія дозволяє враховувати специфічні форми НЕ-факторів, наприклад, комбінація невизначеності та нечіткості, що виникають у процесі взаємодії між судженнями експертів. Структура таких взаємодій може мати різний характер – вони можуть бути узгодженими, сумісними, довільними; можуть довільним чином об'єднуватися та перетинатися. Це дозволяє проводити більш „тонкий" аналіз експертних оцінок. Для розбиття експертної комісії на групи зі схожими думками, запропоновано використовувати метрики теорії свідоцтв, що характеризують ступінь відмінності між виділеними групами експертних свідоцтв. Експертні свідоцтва належать одній групі, якщо значення зазначеної метрики для всіх свідоцтв даної групи не перевищує заданого порогового значення. Для вибору порядку розгляду експертних свідоцтв застосована міра, що відображає ступінь конфлікту між розглянутим свідоцтвом і сформованою множиною експертних свідоцтв.

Практична значимість. Запропонована методика структуризації групових експертних оцінок, сформованих в умовах невизначеності та наявності конфліктуючих експертних свідчень, формує теоретичне підгрунтя для побудови інформаційних технологій аналізу експертної інформації з системним використанням методів моделювання НЕ-факторів як інструментальних засобів систем підтримки прийняття рішень для розробки рекомендацій особі, що приймає рішення за схемою „Ситуація – Варіанти рішення".

Ключові слова: *теорія свідоцтв, метрики, експертні оцінки, кластеризація, невизначеність*

Цель. Исследование новых подходов и разработка математических моделей структуризации (кластеризации) групповых экспертных оценок на основе математического аппарата современных теорий.

Методика. Базируется на математическом аппарате теории свидетельств, кластерном анализе. В качестве критерия определения схожести и различия кластеров рассмотрена метрика Jousselme.

Результаты. Предложенная методика структуризации экспертной информации позволяет оценивать степень согласованности экспертных оценок внутри экспертной группы; в случае ее отсутствия — получать разбиение экспертной комиссии на группы, внутри которых оценки экспертов близки между собой и характеризуются однородностью и согласованностью. Мера согласованности характеризуется степенью близости экспертных оценок.

Научная новизна. Для выявления и анализа экспертной информации были использованы методы теории свидетельств. В отличие от существующих подходов, данная теория позволяет учитывать специфические формы НЕ-факторов, например, комбинация неопределенности и нечеткости, возникающие в процессе взаимодействия между суждениями экспертов. Структура таких взаимодействий может иметь различный характер — они могут быть согласованными, совместимыми, произвольными; могут произвольным образом объединяться и пересекаться. Это позволяет проводить более „тонкий“ анализ экспертных оценок. Для разбиения экспертной комиссии на группы со схожими мнениями, предложено использовать метрики теории свидетельств, характеризующие степень различия между выделенными группами экспертных свидетельств. Экспертные свидетельства принадлежат одной группе, если значение указанной метрики для всех свидетельств данной группы не превышает заданного порогового значения. Для выбора порядка рассмотрения экспертных свидетельств использована мера, отражающая степень конфликта между анализируемым свидетельством и сформированным множеством экспертных свидетельств.

Практическая значимость. Предложенная методика структуризации групповых экспертных оценок, сформированных в условиях неопределенности и наличия конфликтующих экспертных свидетельств, составляет теоретическое основание для построения информационных технологий анализа экспертной информации с системным использованием методов моделирования НЕ-факторов как инструментальных средств систем поддержки принятия решений для выработки рекомендаций лицу принимающему решение по схеме „Ситуация — Вариант решения“.

Ключевые слова: *теория свидетельств, метрики, экспертные оценки, кластеризация, неопределенность*

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