на одній вісі дозволить зменшити крок навивання каната, збільшити канатоємність підіймальної машини.

Ключові слова: *шахтна підіймальна установка, канат, поздовжньо-поперечні коливання струни каната, кут девіації*

Цель. Исследовать влияние способов крепления канатов к барабану однобарабанной шахтной подъемной машины с расположением копровых шкивов на одной оси на минимальное расстояние между струнами каната подъемной установки в статическом и динамическом режимах.

Методика. Разработана математическая модель для определения расстояния между произвольными точками струн канатов в пространстве в статике. Для исследования продольно-поперечных колебаний струны каната составлена конечно-элементная модель подъемной установки.

Результаты. Показано, что при перекрестном способе крепления канатов в случае, если внутренний угол девиации меньше наружного, исключается трение навиваемого каната о соседний виток. Проведен анализ влияния колебаний канатов на минимальное расстояние между ними при перекрестном креплении канатов.

Научная новизна. При перекрестном способе расположения канатов увеличение расстояния между копровыми шкивами или количества витков между навивающейся и свивающейся ветвями на барабане приводит к увеличению расстояния между струнами канатов. Введен критерий отсутствия касания струн канатов при перекрестном креплении.

Практическая значимость. Перекрестный способ крепления канатов к барабану однобарабанной шахтной подъемной машины с расположением копровых шкивов на одной оси позволит уменьшить шаг навивки каната, увеличить канатоемкость подъемной машины.

Ключевые слова: *шахтная подъемная установка, канат, продольно-поперечные колебания струны каната, угол девиации*

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RESONANCE PHENOMENA IN QUASI-ZERO STIFFNESS VIBRATION ISOLATION SYSTEMS

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РЕЗОНАНСНІ ЯВИЩА У ВІБРОЗАХИСНИХ СИСТЕМАХ КВАЗІНУЛЬОВОЇ ЖОРСТКОСТІ

Purpose. To study dynamic processes in nonlinear oscillatory systems with quasi-zero stiffness and with one or many degrees of freedom, which are widely used in industry for cargo and personnel vibration isolation during transportation. The previous studies of such systems were based only on the numerical approaches. In this paper, we propose to investigate thoroughly the dynamics of the above mentioned systems and the conditions for the occurrence of resonance phenomena in them using the asymptotic methods of nonlinear mechanics and applying the apparatus of special periodic functions.

Methodology. The methods of studying resonance oscillations of vibration isolation equipment are based on the asymptotic methods of nonlinear mechanics, wave theory of motion and the use of special Ateb-functions.

Findings. In this work, for the nonlinear quasi-zero stiffness vibration isolation systems with one and two degrees of freedom, we analytically obtained the conditions of resonance oscillations, threshold values of resonance amplitudes depending on the system parameters.

Originality. For the first time, the dynamic processes in systems with concentrated masses and quasi-zero stiffness were analyzed based on analytical approaches. In contrast to numerical approaches, the analytical approaches allow investigating the features of the dynamics of such systems more precisely.

Practical value. The proposed method may solve the problems of analysis, and the problems oscillatory systems synthesis at the design stage, as they allow us to choose such elastic properties of dynamical systems that prevent resonance phenomena. These modes of equipment operation may assure efficient and safe transportation.

Keywords: *mathematical model, nonlinear oscillations, quasi-zero stiffness, vibration isolation system, resonance, special functions*

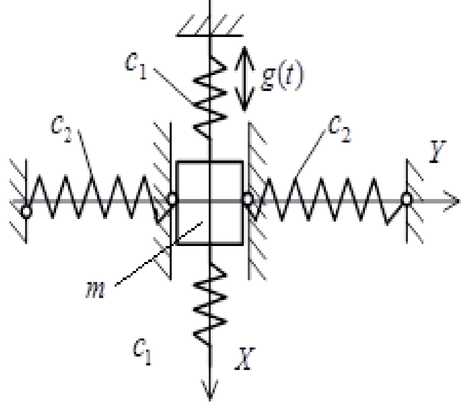
Introduction. Background and literature review. Further increase in machine productivity, intensification of technological processes and the application of new technologies based on the theory of oscillations are closely connected with the condition that the modern machine devices should reliably operate in a wide range of loadings, amplitudes and forced oscillations. In particular, rotary power tools are widely used in industrial production. However, a significant drawback of this class machines is the high level of vibration loads. Therefore, the problem of reducing the harmful effects of vibration on the operator body to the level of public health standards is relevant technical problem. Various vibration isolation devices are used in order to prevent negative consequences of vibrations and to reduce their level in modern technology. Scientific researches of vibration isolation efficiency problem in transport, agriculture, printing machine-building, as well as the latest developments of vibro-protection devices constructions indicate that the protection of machine units from vibration is one of the required elements of technological progress. The analytical method for dynamic processes investigations for a certain class of nonlinear discrete vibration isolation systems is developed in this paper.

The problem of the development of the effective analytical methods that allow optimal engineering solutions by choosing parameters of an oscillation system is closely connected with the problem of constructing and investigating ordinary differential equations solutions describing motions of mechanical systems. Classical analytical methods for nonlinear systems with one degree of freedom are generalized to systems with finite number of freedom degrees [1]. However, the increasing number of freedom degrees of a system leads to significant complications in analytical calculations and does not lead to more accurate results. The use of numerical methods in many cases makes it impossible to make general conclusions about the important issues of dynamics: stability of movement, prediction of resonance phenomena, selection of rational parameters at the design stage in order to provide the desired laws of motion, certain amplitude-frequency characteristics (AFC), etc. Effective general results about characteristics of dynamic processes can be obtained only by using adequate mathematical models and performing the detailed analysis of the solutions of corresponding differential equations.

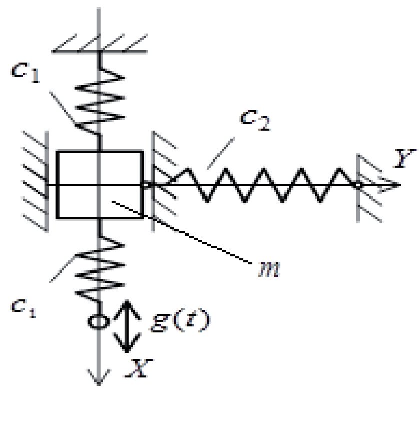
The most coherent and complete structure for investigating nonlinear oscillatory systems with a small parameter is obtained in [2], where the so-called asymptotic Krylov-Bogolyubov-Mitropolski method (KBM) is generalized to the case of non-autonomous systems and systems with many degrees of freedom.The asymptotic KBM method was developed in case of more complex systems as well [3]. The dynamic processes that occur in systems with finite number of freedom degrees are described by systems of second order ordinary differential equations [4, 5]. The number of equations in the system depends on the freedom degrees number. To describe the dynamic processes in systems with distributed parameters, partial differential equations are used. The presence of even “small nonlinearities” in these systems causes significant difficulties for analytical research. First, this is due to the complexity of the construction and analysis of the solutions of nonlinear differential equations systems or nonlinear partial differential equations systems. However, the presence of dissipative and external disturbing forces in real mechanical systems leads in many cases to the rapid damping of oscillations with higher frequencies and setting of dynamic processes with a frequency close to one of the range of fundamental frequeces (in most cases, the first fundamental frequency or frequency of forced disturbance). Taking into account the aforementioned property of nonlinear dynamical systems while constructing the approximate solutions of differential equations, which describe oscillatory processes in systems, simplifies the use of mathematical tools (including asymptotic methods of nonlinear mechanics). Single-frequency method of constructing two-parametric set of solutions is effective in the study of complex oscillatory systems. For some classes of mechanical systems (strongly non-linear with n degrees of freedom, nonlinear systems with distributed parameters), this method is at present the only possible analytical method of investigation.

Resonance phenomena in vibration isolation systems with one degree of freedom close to “zero” stiffness systems. Vibration isolation technique based on the use of systems known as vibration isolation zero stiffness systems has become widespread in recent decades. Examples of such systems are shown in fig. 1, *a*, 1, *b*. Despite the wide range, they are described mathematically by one-type ordinary differential equations with strong nonlinearity, namely

$$m\frac{d^2x}{dt^2}+\beta\frac{dx}{dt}+cx+c_0x^3=f(t). \qquad (1)$$



a



b

Fig. 1. Zero stiffness mechanical systems with one degree of freedom

In equation (1):

- m is mass of the body which is exposed to vibration load, that arises due to the basis movement in accordance with the law *g (t)*. The function $f(t)$ in this case takes the following form $f(t) = -m\dfrac{d^2 g(t)}{dt^2}$;
- β is the proportionality coefficient in the power of resistance, which, for simplicity, is assumed to be proportional to the velocity of the body $\vec{R} = -\beta\vec{V}, V = \dfrac{dx}{dt}$;
- c, c_0 are fixed values, determined through stiffness and equal $c = 2c_1 - 2\dfrac{c_2\Delta}{l}$ for ***a*** and $c = c_1 - \dfrac{c_2\Delta}{l}$ for scheme ***b***; $c_0 = \dfrac{2c_2(\Delta + l)}{l^3}$ for scheme ***a*** and $c_0 = \dfrac{c_2(\Delta + l)}{l^3}$ for scheme ***b***;
- function $g(t)$, and therefore $f(t)$ is periodic in time and the maximum periodic perturbation is small compared to the restoring force, i.e. $\max f(t) << \max\left(cx + c_0 x^3\right)$.

It should be noted:

a) in the above mentioned "reduced" stiffness, Δ is the previous deformation of auxiliary springs, the length of which in unstrained state is equal to l;
b) depending on the ratio between the parameters c_1, c_2, Δ, l, the coefficient c can be positive, negative or equal to zero;
c) numerical integration and appropriate results analysis on its basis for particular cases of the specified equation were conducted before (see, for example, [5]).

Note. Start of reference systems in the above mentioned vibration isolation systems is in the static equilibrium position or in the position, where the horizontally placed elastic elements are unstrained.

Due to above mentioned facts, further, we will only consider the case of small values of c in comparison with the parameter c_0. This allows to investigate the dynamic processes of the considered vibration isolation system using dependence [4] at first approximation for nonresonance case $x(t) = aca\left(3{,}1,\sqrt{\dfrac{2c_0}{m}}at + \psi\right)$, where the parameters a and ψ are defined from the system

$$\frac{da}{dt} = -\frac{2\beta}{m\Pi(1.3)}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{7}{4}\right)}a = -1.50489\frac{\beta}{m}a;$$

$$\frac{d\psi}{dt} = \frac{c}{\sqrt{2mc_0}}\frac{\Gamma\left(\frac{3}{4}\right)}{\sqrt{\pi}\Gamma\left(\frac{1}{4}\right)}\frac{\sqrt{\pi}\Gamma\left(\frac{3}{4}\right)}{\Gamma(\frac{5}{4})}a =$$

$$= \Omega = \sqrt{\frac{2c_0}{m}}a + 0{,}32272\frac{c}{\sqrt{mc_0}}.$$

As to resonance oscillations at the frequency μ, they:

1) take place when the amplitude is close to

$$a^* = 0.425\mu\left(\frac{m}{c_0}\right)^{\frac{1}{2}};$$

2) are described by differential equations

$$\frac{da}{dt} = -\frac{8\beta}{3\sqrt{\pi m}}a + \frac{H}{\pi a}\sqrt{\frac{m}{2c_0}}(\alpha_1 \cos\vartheta + \beta_1 \sin\vartheta);$$

$$\frac{d\vartheta}{dt} = 2.354238\sqrt{\frac{c_0}{m}}\left(a - a^*\right) -$$

$$-\frac{H}{\pi a^2}\sqrt{\frac{m}{2c_0}}(\alpha_2 \cos\vartheta + \beta_2 \sin\vartheta).$$

Normal oscillations of a system with two degrees of freedom. Analogues of the above vibration isolation systems for the case of two degrees of freedom are systems, the models of which are shown in fig. 2, *a*; 2, *b*. Taking into account that the horizontal elastic elements satisfy the linear law of elasticity $F_i = c_1\Delta_i$ $(i = 1,2,3)$ and vertical elastic elements satisfy the nonlinear law of elasticity $F_{j+3} = c_{j+3}\Delta^3_{j+3}$ $(j = 1,2)$, equations that describe the motion of the specified mechanical system are reduced to the form

$$m_1\frac{d^2x_1}{dt^2} + (c_1 + \frac{c_4(\overline{\Delta}_4 + l_4)}{l_4^3})x_1^3 + c_2(x_1 - x_2)^3 -$$
$$-\frac{c_4\overline{\Delta}_1}{l_4}x_1 + \beta\frac{dx_1}{dt} = 0;$$

$$m_2\frac{d^2x_2}{dt^2} + (c_3 + \frac{c_5(\overline{\Delta}_5 + l_5)}{l_5^3})x_2^3 + c_2(x_2 - x_1)^3 -$$
$$-\frac{c_5(\overline{\Delta}_5)}{l_5}x_2 + \beta\frac{dx_2}{dt} = 0. \qquad (2)$$

In relations (2):

m_1, m_2 are the masses of the first and second bodies respectively ;

x_1, x_2 are coordinates of the mentioned bodies at an arbitrary point of time;

c_i is the proportionality factor in a restoring force of i-th elastic element;

l_i and $\overline{\Delta}_i$ are the length and the initial deformation in the equilibrium position of the system of i-th elastic element respectively.

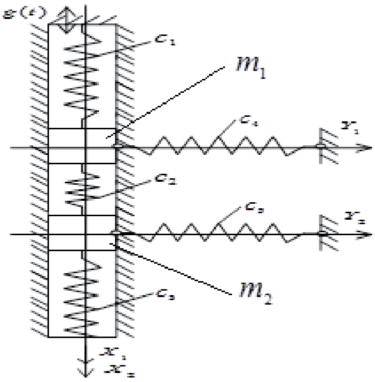
We denote

$$\overline{c}_1 = \frac{1}{m_1}\left(c_1 + \frac{2c_4\left(\overline{\Delta}_4 + l_4\right)}{l_4^3}\right);\ \overline{c}_2 = \frac{c_2}{m_1};$$

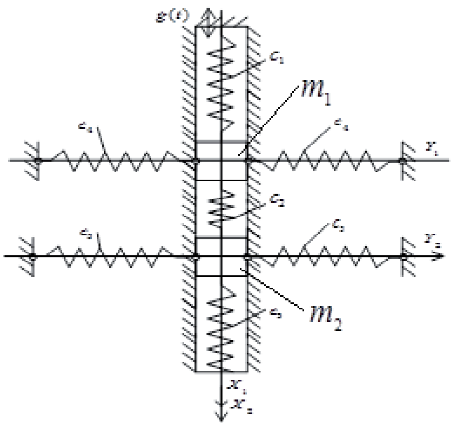
$$\overline{c}_3 = \frac{1}{m_2}\left(c_3 + \frac{c_5\left(\overline{\Delta}_5 + l_5\right)}{l_5^3}\right);\ \widetilde{c}_2 = \frac{c_2}{m_2};\ \overline{c}_4 = \frac{c_4\overline{\Delta}_4}{m_1 l_4};$$

$\tilde{c}_4 = \frac{c_5 \bar{\Delta}_5}{m_2 l_5}$ – in the case of a mechanical system that meets fig. 2, *a* and $\bar{c}_1 = \frac{1}{m_1}\left(c_1 + \frac{c_4\left(\bar{\Delta}_4 + l_4\right)}{l_4^3} \right)$;

$\bar{c}_3 = \frac{1}{m_2}\left(c_3 + \frac{2c_5\left(\bar{\Delta}_5 + l_5\right)}{l_5^3} \right)$ – in the case of fig. 2, *b*, $\bar{\beta}_1 = \frac{\beta}{m_1}, \bar{\beta}_2 = \frac{\beta}{m_2}$.



a



b

Fig. 2. Scheme of the mechanical system with two degrees of freedom

Then the differential equations can be represented in a more compact form

$$\begin{aligned} &\frac{d^2 x_1}{dt^2} + \bar{c}_1 x_1^3 + \bar{c}_2 (x_1 - x_2)^3 = \bar{c}_4 x_1 - \bar{\beta}_1 \frac{dx_1}{dt}; \\ &\frac{d^2 x_2}{dt^2} + \bar{c}_3 x_1^3 - \tilde{c}_2 (x_1 - x_2)^3 = \tilde{c}_4 x_2 - \bar{\beta}_2 \frac{dx_1}{dt} \quad . \end{aligned} \tag{3}$$

Below we consider the case when the masses of both bodies are equal, and the resistance and the previous deformations of elastic elements are small. This suggests that the maximum values of the right parts of equations (3) are small, and therefore one can use common pertrurbations methods for their research. According to them, first of all, we will describe the dynamic process of the generating system

$$\begin{aligned} &\frac{d^2 x_1}{dt^2} + \bar{c}_1 x_1^3 + \bar{c}_2 (x_1 - x_2)^3 = 0; \\ &\frac{d^2 x_2}{dt^2} + \bar{c}_3 x_2^3 - \bar{c}_2 (x_1 - x_2)^3 = 0. \end{aligned} \tag{4}$$

Solutions to (4) will be sought in the form

$$\begin{aligned} x_1(t) &= aca(3{,}1, \omega(a)t + \psi), \\ x_2(t) &= abca(3{,}1, \omega(a)t + \psi). \end{aligned} \tag{5}$$

To find the unknown parameters of (4), taking into account (5), we obtain the algebraic equation

$$b^4 + \left(\frac{\bar{c}_3}{\bar{c}_2} - 2 \right) b^3 - \left(\frac{\bar{c}_1}{\bar{c}_2} - 2 \right) b - 1 = 0 \ . \tag{6}$$

In the case where extreme horizontal springs are the same, and the previous vertical deformation of springs and their lengths are equal, that is, $\bar{c}_1 = \bar{c}_3 = \bar{c}$, real roots of an algebraic equation (6) are equal to

$$b_1 = 1; \ b_2 = -1; \ b_3 = 1 - \frac{\bar{c}}{\bar{c}_2} - \sqrt{\frac{\bar{c}}{\bar{c}_2}\left(\frac{\bar{c}}{4\bar{c}_2} - 1 \right)} \ ;$$

$$b_4 = 1 - \frac{\bar{c}}{\bar{c}_2} + \sqrt{\frac{\bar{c}}{\bar{c}_2}\left(\frac{\bar{c}}{4\bar{c}_2} - 1 \right)} \ . \tag{7}$$

Using (5), we can write the solutions to (4) as:

a) $x_1 = aca\left(3{,}1, \sqrt{c} \cdot at + \theta\right); \quad x_2 = aca\left(3{,}1, \sqrt{c} \cdot at + \theta\right)$;

b) $x_1 = aca\left(3{,}1, \sqrt{2c_2 + c} \cdot at + \theta\right)$;

$x_2 = -aca\left(3{,}1, \sqrt{2c_2 + c} \cdot at + \theta\right)$;

c) $x_1 = aca(3{,}1, \sqrt{c + c_2\left(\frac{c}{2c_2} + \sqrt{\frac{c^2 - 4cc_2}{4c_2^2}} \right)^3} \cdot at + \theta)$;

$$x_2 = \left(1 - \frac{c}{2c_2} - \sqrt{\frac{c^2 - 4cc_2}{4c_2^2}} \right) a \times$$

$$\times ca(3{,}1, \sqrt{c + c_2\left(\frac{c}{2c_2} + \sqrt{\frac{c^2 - 4cc_2}{4c_2^2}} \right)^3} at + \theta) \ ; \tag{8}$$

d) $x_1 = aca(3{,}1, \sqrt{c + c_2\left(\frac{c}{2c_2} - \sqrt{\frac{c^2 - 4cc_2}{4c_2^2}} \right)^3} \cdot at + \theta)$;

$$x_2 = \left(1 - \frac{c}{2c_2} + \sqrt{\frac{c - 4cc_2}{4c_2^2}} \right) \times$$

$$\times aca(3{,}1, \sqrt{c + c_2\left(\frac{c}{2c_2} - \sqrt{\frac{c^2 - 4cc_2}{4c_2^2}} \right)^3} \cdot at + \theta).$$

From the obtained results, it follows that:

1) in the case of *a*) normal forms of oscillations x_1 and x_2 occur in one phase, and for the case *b*) they occur in opposite phases;

2) normal oscillations in forms *c*) and *d*) occur if the stiffnesses of springs are connected by the relationship $c > 4c_2$.

Having determined the normal oscillations forms of the unperturbed system (8), we proceed to consider perturbed equations in the first approximation. Due to the general methods of perturbation theory for nonlinear oscillations systems, the ratio of (8) can be also considered as a solution to equations (4) with the condition that they will have parameters a and θ as functions of time. In order to find these parameters, we obtain a system of differential equations

$$\dot{a} = \frac{\varepsilon a}{2\pi\omega(a)} \int_0^{2\pi} k(1+b) ca(3,1,l\psi) sa(1,3,l\psi) d\psi = 0;$$

$$\dot{\theta} = \frac{\varepsilon}{2\pi\omega(a)} \int_0^{2\pi} k(1+b) ca^2(3,1,l\psi) d\psi = \frac{\varepsilon 0.4571}{\omega(a)} k(1+b),$$

where $k = k_1 = k_2$; $l = \frac{\Gamma(0.25)}{\sqrt{\pi}\Gamma(0.75)} = 1.6692$.

As expected, the amplitude of the normal modes of oscillations of the considered system remains constant in the first approximation of asymptotic distribution, as a system is conservative. Regarding to frequencies of perturbed oscillations Ω_s, $s = 1,2,3,4$, they depend on the amplitude and are determined by the ratio

$$\Omega_s = \omega(a) + \frac{\varepsilon 0{,}4571}{\omega(a)} k(1+b_s),$$

where $\omega(a) = \sqrt{\frac{\pi}{2}\left[c + c_2(1+b_s)^3\right]}a$ and b_s are determined in accordance with (7).

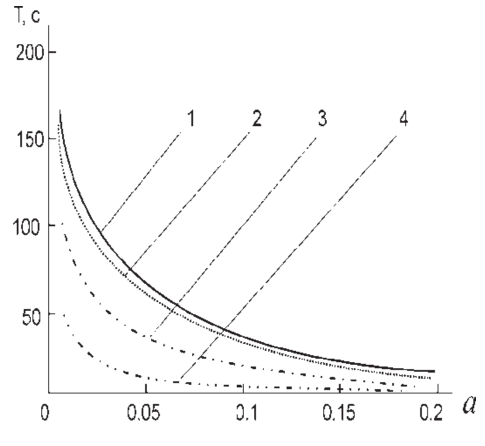


Fig. 3. Graphics of periods dependence of normal oscillations modes on the amplitude 1 – a); 2 – b); 3 – c); 4 – d)

Fig. 3 shows the dependence of the period of oscillation normal modes on the amplitude for the four above described cases (a – d) of the solution form to the equation (4).

From the received graphs, it follows that:

1) when the amplitude increases, the period for all normal oscillation modes of the conservative system under consideration falls;

2) for the equel values of the amplitude the oscillation period is the largest if the motion of bodies is in one phase.

Conclusions. The paper develops the research method for oscillations of the system with quazi-zero stiffness, which is used to protect equipment from vibration loads. In this paper, resonance conditions of the mentioned systems are obtained and the laws of non-resonant and resonant amplitudes changes are described. On basis of the presented method, we obtained the equations in a standard form, describing the laws of change of dynamic process influential parameters for both non-resonant and resonant cases. The developed method to study oscillatory processes of strongly nonlinear systems with kvazi-zero stiffness allows solving not only the problem of analysis, but also the equally important synthesis problem of technical oscillatory systems at the design stage, choosing such elastic properties of dynamical systems that prevent resonance phenomena.

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Мета. Дослідження динамічних процесів у нелінійних коливальних системах квазінульової жорсткості з одним та багатьма ступенями вільності, що широко використовуються у промисловості для віброзахисту вантажів та персоналу при транспортуванні. Такі системи раніше в літературі досліджувались виключно на базі чисельних підходів. У цій роботі пропонується за допомогою асимптотичних методів нелінійної механіки із застосуванням апарату спеціальних періодичних функцій грунтовно дослідити динаміку віброзахисних систем та умови виникнення резонансних явищ у них.

Методика. Вивчення резонансних коливань віброзахисного обладнання базується на асимптотичних методах нелінійної механіки, хвильовій теорії руху та використанні спеціальних Ateb-функцій.

Результати. У роботі для вказаних нелінійних віброзахисних систем квазінульової жорсткості з одним та двома ступенями вільності аналітично отримані умови резонансних коливань та порогові значення резонансних амплітуд залежно від параметрів системи.

Наукова новизна. Полягає в тому, що вперше аналіз динамічних процесів у системах квазінульової жорсткості із зосередженими масами здійснене на базі аналітичних підходів, що дозволяють, на відміну від чисельних підходів, точніше дослідити особливості динаміки таких систем.

Практична значимість. Запропонована методика дозволяє розв`язати не тільки задачі аналізу, але й не менш важливі задачі синтезу технічних коливальних систем ще на стадії проектування, вибрати такі пружні характеристики динамічних систем, що унеможливлюють у них резонансні явища. Такі режими роботи обладнання дозволяють здійснювати ефективні та безпечні перевезення.

Ключові слова: *математична модель, нелінійні коливання, квазінульова жорсткість, віброзахисна система, резонанс, спеціальні функції*

Цель. Исследование динамических процессов в нелинейных колебательных системах квазинулевой жесткости с одним или многими степенями свободы, которые широко используются в промышленности для виброзащиты грузов и персонала при транспортировке. Такие системы ранее в литературе исследовались исключительно на базе численных подходов. В этой работе предлагается с помощью асимптотических методов нелинейной механики с применением аппарата специальных периодических функций основательно исследовать динамику указанных систем и условия возникновения резонансных явлений в них.

Методика. Изучение резонансных колебаний виброзащитного оборудования базируется на асимптотических методах нелинейной механики, волновой теории движения и использовании специальных Ateb-функций.

Результаты. В работе для указанных нелинейных виброзащитных систем квазинулевой жесткости с одним и двумя степенями свободы аналитически получены условия резонансных колебаний и пороговые значения резонасных амплитуд в зависимости от параметров системы.

Научная новизна. Заключается в том, что впервые анализ динамических процессов в системах квазинулевой жесткости с сосредоточенными массами осуществлен на базе аналитических подходов, которые позволяют, в отличие от численных подходов, точнее исследовать особенности динамики таких систем.

Практическая значимость. Предложенная методика позволяет решать не только задачи анализа, но и не менее важные задачи синтеза технических колебательных систем еще на стадии проектирования, выбрать такие упругие характеристики динамических систем, которые делают невозможными в них резонансные явления. Такие режимы работы оборудования позволяют осуществлять эффективные и безопасные перевозки.

Ключевые слова: *математическая модель, нелинейные колебания, квазинулевая жесткость, виброзащитная система, резонанс, специальные функции*

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