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MATHEMATICAL MODELING AND ANALYSIS OF HEAT TRANSFER IN STRUCTURES WITH FOREIGN ELEMENTS

Purpose. The study aims to develop linear and nonlinear mathematical models for determining temperature fields in isotropic three-dimensional media containing foreign thermoactive semi-through elements. This approach enhances the accuracy of temperature regime analysis under thermal loads and contributes to the improvement of design methods for devices whose individual units include foreign thermoactive elements.

Methodology. To obtain analytical-numerical solutions of linear and nonlinear boundary value problems of heat conduction, asymmetric unit functions were used. As a result, the thermal conductivity coefficient for a structure with a foreign semi-through cylindrical element is represented as a unified whole. This ensures the fulfillment of ideal thermal contact conditions at the interfaces of dissimilar materials in the structure, reducing the problem to solving a single heat conduction equation with discontinuous and singular coefficients. For the nonlinear boundary value problem, a linearizing function was introduced, allowing the transformation into a second-order linear partial differential equation with discontinuous and singular coefficients, and a quasi-linear boundary condition. By approximating the temperature as a function of spatial coordinates on the inclusion surface and the layer's boundary surface using piecewise-constant functions, the nonlinear boundary value problem was fully linearized.

Findings. Linear and nonlinear mathematical models were developed to determine the temperature field and analyze thermal regimes in devices containing a foreign thermoactive semi-through inclusion. A linearizing function was proposed to simplify the nonlinear boundary value problem. Analytical-numerical solutions for both the linear and nonlinear heat conduction problems were obtained, allowing the determination of the temperature distribution as a function of spatial coordinates. Comparative analysis revealed a 7 % difference between results for constant and linearly varying thermal conductivity coefficients, explained by the small values of the thermal conductivity temperature coefficient for the selected construction materials.

Originality. A method for linearizing the nonlinear mathematical model of heat conduction was proposed. Analytical-numerical solutions to the corresponding linear and nonlinear boundary value problems were obtained in closed form. The use of asymmetric unit functions allowed for a correct mathematical description of heat transfer processes in media containing foreign thermoactive semi-through elements.

Practical value. The developed heat transfer mathematical models enable the assessment of media in terms of their thermal resistance, contributing to the improved performance of devices containing foreign thermoactive semi-through elements. This prevents overheating and extends their operational life. The results can be applied to practical problems of heat exchange and thermal insulation in industrial structures, including predicting temperature fields in mining equipment mechanisms, ventilation systems, and compressor stations. Implementing the proposed models improves the efficiency of ore extraction and processing, as well as reduces heat loss in industrial systems.

Keywords: *temperature field, material thermal conductivity, structural thermal resistance, thermo-sensitive material, convective heat transfer*

Introduction. The temperature field within mechanisms used in the mining industry is a critical factor influencing their performance, operational efficiency, and reliability. Elevated temperatures in equipment components can lead to reduced efficiency, increased energy consumption, and a shortened service life. Therefore, the study of thermal processes in mining machinery is a relevant task for ensuring uninterrupted operation and optimizing performance characteristics.

The formation of a temperature field in mining equipment occurs due to heat generation caused by friction between moving parts, high mechanical loads, environmental influences, and the operation of electric motors. Extremely high temperatures are observed in areas such as drill bit contacts, crushers, conveyor belts, and bearing assemblies. Heat generation imposes additional loads on mining machinery, affecting its reliability. Studies show that the relative impact of temperature on equipment failures is greater than that of other factors such as humidity, vibration, and dust. Without effective heat dissipation, mechanisms may overheat, and localized overheating can result in accelerated wear, loss of mechanical properties of materials, and even emergency situations.

Various experimental and numerical methods are used to analyze the temperature field in mining machinery. These include infrared thermography, thermal sensors, thermocouples, as well as computer simulations based on heat conduction equations. Each method has its own advantages and limitations and is chosen depending on the operating conditions of the equipment and the required measurement accuracy.

Temperature field analysis not only allows the determination of heating levels in specific machine units but also helps optimize their design for better heat dissipation. For example, assessing thermal loads in the bearing assemblies of excavators or drilling rigs enables accurate prediction of their service life and timely preventive maintenance. Furthermore, thermal distribution calculations assist in designing effective cooling systems, reducing the risk of overheating, and extending the operational life of equipment.

Extensive use of numerical modeling enables the prediction of temperature regimes in equipment and the determination of optimal design parameters to enhance heat dissipation efficiency. One of the important aspects of thermal analysis is the influence of the surrounding environment – such as elevated temperatures in underground mine spaces – on the operation of mining machinery. For instance, in underground conditions where ambient air temperatures are significantly higher than at the surface, overheating occurs more rapidly, requiring specialized cooling measures.

To ensure effective heat dissipation in mining machinery, various technologies are employed, including liquid and air cooling, heat-resistant lubricants, thermal insulation materials, and active temperature control systems. An equally important task is continuous monitoring of the thermal state of equipment, achieved through integrated sensors that allow real-time tracking of temperature changes and help prevent accidents.

In summary, the study of temperature fields in mining industry mechanisms is crucial for ensuring their uninterrupted operation, reducing wear, and increasing reliability. The application of modern methods for analyzing temperature regimes and effective heat dissipation technologies contributes to the development of more reliable and productive equipment for the mining sector.

Literature review. The analysis of the literature on heat transfer and thermomechanical processes reveals substantial scientific contributions in the field of modeling temperature fields, mechanical stresses, and temperature gradients in various structural materials and media. However, most research focuses on general engineering applications, while issues specific to the mining industry require deeper investigation.

In [1], a thermomechanical model suitable for industrial production applications was developed. The nonlinearity of the model is due to the dependence of material properties on temperature, while heterogeneity arises from the structure being composed of different materials. A numerical example for a simplified geometric structure of blast furnace hearth walls was provided to evaluate performance according to the developed model.

In [2], a numerical model considering thermal–fluid–solid interaction was developed for analyzing thermal stresses in the blast furnace hearth bottom with typical erosion wear. This model enables the assessment of the influence of various operating parameters – such as tapping temperature and cooling intensity – on temperature distribution and thermal stresses in the furnace lining.

The study in [3] presented a method for modeling heat transfer in porous materials with temperature-dependent properties. While this approach is intended for electronics applications, it can be adapted to predict the behavior of rocks and materials under mining conditions, especially for assessing thermal impacts on mining equipment or underground structures.

The analytical solutions presented in [4] address the distribution of temperature, displacements, and stresses in layered plates under thermomechanical loading. These results can be valuable for analyzing the stability of retaining structures in mines and quarries, where layered rocks are subject to thermomechanical effects caused by changes in temperature regimes.

The reconstruction of temperature fields from limited observations is important for thermal management of electronic equipment. In [5], a deep learning method combining an adaptive UNet and a shallow multilayer perceptron (MLP) was proposed to transform the problem into an image-to-image regression task. The adaptive UNet reconstructs the general temperature field, while the MLP specializes in accurate prediction of high-gradient temperature zones. A drawback of this approach is the need for a large dataset for model training, which is difficult to obtain under real operating conditions.

In [6], the thermomechanical loads of restrained columns under longitudinal heating with different boundary conditions were analyzed. The temperature distribution was determined using the differential quadrature method (DQM). The influence of temperature and material properties on deflection and critical load was investigated. The main limitation of this approach is the simplified temperature distribution model, which does not account for significant gradients that may arise under critical thermal loads.

Study [7] presented core equations and datasets for a thermal model predicting temperature fields and heating rates during localized laser treatment of Fe–C–Ni alloys. While the model can be applied to other materials undergoing solid-state transformations during laser processing, the use of averaged thermophysical parameters introduces errors in the obtained results.

The article [8] developed a model for predicting the temperature field of a steel plate during roller quenching to control its shape. The cooling mechanism was analyzed, and heat transfer coefficients for each surface were obtained; however, the results exhibit certain inaccuracies when modeling homogeneous media.

Research [9] examines the influence of boundary conditions on heat exchange rates, which is particularly critical in mining operations, where temperature gradients can affect the strength and stability of rock formations.

Thermal process modeling in electronic devices, as presented in [10 and 11], can be applied to optimize the operation of sensors and measuring equipment used in mines under extreme temperature conditions. In [10], a thermal model based on infrared camera measurements was developed, while in [11], a finite-difference model was used to study device thermal behavior under various operating conditions, though results exhibited significant errors.

The steady-state thermoelastic solution for thick cylinders subjected to pressure and external heat flux on the inner surface was presented in [12]. However, the influence of temperature gradients on material deformation was not considered, reducing the model's accuracy.

Mechanical defects can cause temperature rises and thermal stresses in thermoelectric materials, reducing device reliability. In [13], using the complex variable method, thermoelectric – elastic fields around an elliptical defect in a two-dimensional thermoelectric plate were studied. Results indicate that temperature at the defect tip increases with defect size, potentially exceeding the material's melting temperature, and stresses may surpass yield limits – important factors in material failure analysis.

Thermal analyses of cylinders with varying thicknesses made of functionally graded materials subjected to nonuniform heat fluxes concentrated on inner and outer layers were carried out in [14, 15]. However, these studies do not allow for the evaluation of the thermal state under localized heating.

Functionally graded materials with continuously varying properties are beneficial for thermal protection and biomedical applications. In the case of thin coatings on substrates, conventional mesh discretization is inefficient. The approximate transfer method proposed in [16] uses finite difference concepts to transfer boundary conditions from coating to substrate, enabling numerical modeling of only the substrate with convection boundary conditions via a hybrid finite element method.

In [17], a nonlinear three-dimensional heat conduction problem was simplified to the Laplace equation using an intermediate function. A generalized triple function was proposed, and a general solution to the Laplace equation was obtained. However, simplifying a nonlinear problem to a Laplace equation may lead to loss of accuracy in temperature field predictions, as the inherent nonlinearity may not be adequately captured.

The methods in [18] were improved, and new approaches were developed for creating mathematical models that analyze heat transfer in piecewise-homogeneous media. Two-dimensional and three-dimensional heat transfer models were presented, with differential equations containing coefficients dependent on the thermophysical properties of phases and geometric structure. Approaches for finding analytical and analytical-numerical solutions to boundary value problems were proposed in [19]. Heat transfer processes in homogeneous and layered structures with foreign inclusions of canonical shapes were analyzed in [20].

Linear and nonlinear mathematical models for determining the temperature field and analyzing temperature regimes in isotropic media with localized heating were developed in [21]. The obtained results allow the analysis of heat transfer processes and improvement of structural thermal resistance.

In [18, 19], little attention was given to models that account for localized heating, medium heterogeneity, and the thermal sensitivity of construction materials. The classical analytical and numerical methods applied in [20, 21] are insufficient for effectively accounting for these factors in individual elements and structural units used in underground mines and surface quarries. Therefore, this study proposes a method for developing heat conduction models that incorporate these aspects.

Overall, the reviewed publications provide valuable methodological approaches to modeling thermal processes; however, their application to mining industry conditions requires adaptation and further research, particularly to account for rock heterogeneity, localized thermal effects, and long-term temperature changes in underground structures. Consequently, it is important to consider the heterogeneity of such elements, the thermal sensitivity of construction materials, and the locality of thermal loading in subsequent studies.

Problem formulation. ***The object of research and its mathematical models.*** The object of this study is the heat conduction process in isotropic media containing a foreign semi-through thermoactive inclusion.

Hypothesis: The investigation considers heat conduction in an isotropic three-dimensional medium with a foreign semi-through cylindrical thermoactive inclusion, within the framework of the classical theory of heat conduction, where Fourier's law holds.

Assumptions and Simplifications: the medium is not anisotropic, meaning that thermophysical parameters are constant in all spatial directions; the heat conduction process is steady-state, as temperature variations are determined solely by spatial coordinates.

To develop linear and nonlinear mathematical models for determining the temperature field and analyzing thermal regimes in a medium with a foreign semi-through thermoactive inclusion, asymmetric unit functions are used. This approach makes it possible to represent the thermophysical parameters of the construction materials efficiently, reducing the boundary value problem to a single heat conduction equation with discontinuous and singular coefficients. For the nonlinear heat conduction model – arising due to the thermal sensitivity of heterogeneous material properties – a linearizing function is introduced.

Case description. An isotropic layer contains a foreign semi-through cylindrical inclusion with radius R, in a cylindrical coordinate system ($Or\varphi z$). In the area of inclusion $\Omega_0 = \{(R, \varphi, h) : 0 \le \varphi \le 2\pi\}$ uniformly distributed internal heat sources with specific power $q_0 = \text{const}$ are concentrated. On the outer boundary surface $L_+ = \{(r, \varphi, h) : 0 \le r < \propto, 0 \le \varphi \le 2\pi\}$ convective heat exchange with the surrounding medium at temperature $t_c = \text{const}$ occurs according to Newton's law. The opposite surface $L_- = \{(r, \varphi, -l) : 0 \le r < \propto, 0 \le \varphi \le 2\pi\}$ is thermally insulated. On the inclusion surface $K_R = \{(R, \varphi, z) : 0 \le \varphi \le 2\pi, 0 \le z \le h\}$ ideal thermal contact is assumed $t_0(R, z) = t{<}_1(R, z)$, $\ddot{e}_0 \dfrac{\partial t_0(r,z)}{\partial r} = \ddot{e}_1 \dfrac{\partial t_1(r,z)}{\partial r}$ for $r = R$, $0 \le \varphi \le 2\pi$, $0 \le z \le h$ and $t_0(r, 0) = t_1(r, 0)$, $\ddot{e}_0 \dfrac{\partial t_0(r,z)}{\partial z} = \ddot{e}_1 \dfrac{\partial t_1(r,z)}{\partial z}$ for $z = 0$, $0 \le r < \infty$, $0 \le \varphi \le 2\pi$.

Here λ_1, λ_0 are the thermal conductivity coefficients of the layer and inclusion materials, respectively; $t_0(r, 0)$,

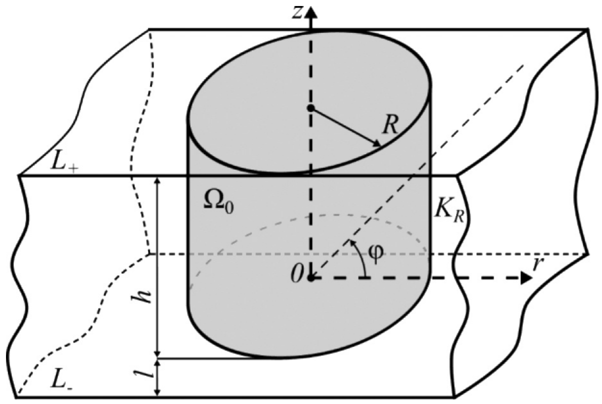

Fig. 1. Isotropic layer with a thermally active semi-through inclusion

$t_1(r, 0)$ are the temperatures on the inclusion surface $r = 0$ (Fig. 1).

In the above structure, the temperature distribution $t(r, z)$ in spatial coordinates r, z is determined by solving the heat conduction equation

$$\frac{1}{r}\operatorname{div}\left[r\lambda(r,z)\operatorname{grad}\theta(r,z)\right] = -q_0 S_-(R-r)S_-(z), \quad (1)$$

with boundary conditions

$$\begin{gathered}\theta(r,z)\big|_{r\to\infty} = 0; \quad \left.\frac{\partial\theta(r,z)}{\partial r}\right|_{r\to\infty} = 0;\\ \lambda_1\left.\frac{\partial\theta(r,z)}{\partial z}\right|_{z=h} = \alpha\,\theta(r,z)\big|_{z=h}; \quad \left.\frac{\partial\theta(r,z)}{\partial z}\right|_{z=-l} = 0,\end{gathered} \quad (2)$$

where $\lambda(r, z)$ is the coefficient of thermal conductivity of the inhomogeneous layer; α is the coefficient of heat transfer from the boundary surface of the layer L_+; $S_\pm(\zeta)$ are asymmetric unit functions

$$S_\pm(\zeta) = \begin{cases} 1, & \zeta > 0 \\ 0.5 \mp 0.5, & \zeta = 0. \\ 0, & \zeta < 0 \end{cases}$$

An isotropic thermosensitive (thermophysical parameters of structural materials depend on temperature) layer containing an alien semi-through thermally active cylindrical inclusion is presented. Taking into account the thermosensitivity of the materials of the medium, the conditions of ideal thermal contact on the surface of the inclusion K_R will have the form $t_0(R, z) = t_1(R, z)$; $\lambda_0(t)\dfrac{\partial t_0(r,z)}{\partial r} = \lambda_1(t)\dfrac{\partial t_1(r,z)}{\partial r}$, for $r = R$ and $t_0(r, 0) = t_1(r, 0)$, $\lambda_0(t)\dfrac{\partial t_0(r,z)}{\partial z} = \lambda_1(t)\dfrac{\partial t_1(r,z)}{\partial z}$, for $z = 0$.

Here, $\lambda_1(t)$, $\lambda_0(t)$ are the thermal conductivity coefficients of the layer and inclusion materials, respectively (Fig. 1).

The temperature distribution $t(r, z)$ in the spatial coordinates r, z in the above construction is determined by solving the nonlinear heat conduction equation

$$\frac{1}{r}\operatorname{div}\left[r\lambda(r,z,t)\operatorname{grad}t(r,z)\right] = -q_0 S_-(R-r)S_-(z), \quad (3)$$

with boundary conditions

$$\begin{gathered}t(r,z)\big|_{r\to\infty} = t_c; \quad \left.\frac{\partial t(r,z)}{\partial r}\right|_{r\to\infty} = 0;\\ \left.\frac{\partial t(r,z)}{\partial z}\right|_{z=-l} = 0; \quad \lambda_1(t)\left.\frac{\partial t(r,z)}{\partial z}\right|_{z=h} = \alpha\left(t(r,z)\big|_{z=h} - t_c\right),\end{gathered} \quad (4)$$

where $\lambda(r, z, t)$ is the thermal conductivity coefficient of the inhomogeneous thermally sensitive layer.

Linear mathematical model for determining the temperature field in a layer with a semi-through cylindrical inclusion heated by a heat flux. The thermal conductivity coefficient of the layer with a foreign thermoactive inclusion is expressed as

$$\lambda(r, z) = \lambda_1 + (\lambda_0 - \lambda_1)S_-(R - r)S_-(z). \quad (5)$$

A function is introduced

$$T(r, z) = \lambda(r, \zeta)\theta(r, z), \quad (6)$$

and differentiating it with respect to r and z and taking into account the expression for $\lambda(r, z)$ (5). As a result, the ratio

$$\begin{gathered}\lambda(r,z)\frac{\partial\theta(r,z)}{\partial r} = \\ = \frac{\partial T(r,z)}{\partial r} + (\lambda_0 - \lambda_1)\theta(r,z)\big|_{r=R}\,\delta_+(r-R)S_-(z);\\ \lambda(r,z)\frac{\partial\theta(r,z)}{\partial z} = \\ = \frac{\partial T(r,z)}{\partial z} - (\lambda_0 - \lambda_1)\theta(r,z)\big|_{z=0}\,S_-(R-r)\delta_-(z),\end{gathered} \quad (7)$$

is obtained.

Here $\delta_\pm(\zeta) = \dfrac{dS_\pm(\zeta)}{d\zeta}$ denotes asymmetric Dirac delta functions.

As a result of substituting expressions (7) into relation (1), we obtain a second-order partial differential equation with discontinuous and singular coefficients

$$\Delta T + (\lambda_0 - \lambda_1)F(r, z) = -q_0 S_-(R - r)S_-(z), \quad (8)$$

where

$$\begin{gathered}F(r,z) = \frac{R}{r}\theta(r,z)\big|_{r=R}\,\delta'_+(r-R)S_-(z) - \\ - \theta(r,z)\big|_{z=0}\,S_-(R-r)\delta'_-(z);\end{gathered} \quad (9)$$

$\Delta = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial}{\partial r}\right) + \dfrac{\partial^2}{\partial z^2}$, is the Laplacian in cylindrical coordinates.

Unknown functions $\theta(R, z)$, $\theta(r, 0)$ are approximated by piecewise-constant functions with respect to z and r

$$\begin{gathered}\theta(R,z) = \theta_1 + \sum_{i=1}^{n-1}(\theta_{i+1} - \theta_i)S_-(z - z_i);\\ \theta(r,0) = \theta_1 + \sum_{j=1}^{k-1}(\theta_{j+1} - \theta_j)S_-(r - r_j).\end{gathered} \quad (10)$$

Here

$$z_i \in (0; h); \quad z_1 \le z_2 \le \ldots \le z_{n-1};$$
$$r_j \in (0; R); \quad r_1 \le r_2 \le \ldots \le r_{k-1},$$

where n, k are the numbers of interval subdivisions $(0; h)$, $(0; R)$; $\theta_i\,(i = \overline{1,n})$, $\theta_j\,(j = \overline{1,k})$ are unknown approximation values $\theta(R, z)$, $\theta(r, 0)$.

After substituting expressions (10) into equation (8), we obtain a second-order partial derivative equation with a discontinuous and singular right-hand side

$$\Delta T = (\lambda_0 - \lambda_1)F_1(r, z) - q_0 S\_(R - r)S\_(z), \qquad (11)$$

where

$$F_1(r,z) = \theta(r,0)S\_(R-r)\delta'\_(z) -$$
$$-\frac{R}{r}\theta(R,z)\delta'_+(r-R)S\_(z).$$

The Henkel integral transformation in the coordinate r is applied to equation (11) and boundary conditions (2) taking into account relation (6). As a result, an ordinary second-order differential equation with constant coefficients and a discontinuous and singular right-hand side is obtained.

$$\frac{d^2\overline{T}}{dz^2} - \xi^2\overline{T} = (\lambda_0 - \lambda_1)\left[A\frac{\delta'\_(z)}{\xi} - A(z)S\_(z)\right] - \qquad (12)$$
$$-\frac{Rq_0}{\xi}J_1(R\xi)S\_(z),$$

and boundary conditions

$$\left.\frac{d\overline{T}(z)}{dz}\right|_{z=-l} = 0; \quad \left.\frac{d\overline{T}(z)}{dz}\right|_{z=h} = -\frac{\alpha}{\lambda_1}\overline{T}(z)\Big|_{z=h}. \qquad (13)$$

Here

$$A = R\theta_k J_1(R\xi) - \sum_{j=1}^{k-1} r_j(\theta_{j+1} - \theta_j)J_1(r_j\xi);$$

$$A(z) = R\xi J_1(R\xi)\left[\theta_1 + \sum_{i=1}^{n-1}(\theta_{i+1} - \theta_i)S\_(z - z_i)\right];$$

$\overline{T}(\xi,z) = \int_0^\infty rJ_0(r\xi)T(r,z)dr$ is the Hankel transform of $T(r, z)$; ξ is the Hankel transform parameter; $J_\nu(x) = \sum_{n=0}^\infty (-1)^n \frac{(x/2)^{\nu+2n}}{n!(\nu+n)!}$ is the Bessel function of the first kind of order ν.

The general solution of equation (12) is determined by the method of variation of constants

$$\overline{T}(z) = c_1e^{\xi z} + c_2e^{-\xi z} + (\lambda_0 - \lambda_1)A_3(z) - \frac{Rq_0}{\xi^3}J_1(R\xi)A_1(z),$$

where $A_3(z) = A\frac{\text{ch}\,\xi z}{\xi}S\_(z) - \frac{R}{\xi}J_1(R\xi)\left[\theta_1 A_1(z) + A_2(z)\right];$

$A_1(z) = (\text{ch}\,\xi z - 1)\{\text{S}\_(z).$

Boundary conditions (13) were used and, as a result, the solution of problem (12, 13) was obtained in the form

$$\overline{T}(z) = \frac{1}{\xi}\left\{(\lambda_0 - \lambda_1)\left[AB_1(z) - RJ_1(R\xi)(B_2(z) + B_3(z))\right] - \frac{Rq_0}{\xi^2}J_1(R\xi)B_4(z)\right\}. \qquad (14)$$

Here

$$B_1(z) = \text{ch}\,\xi z S\_(z) + B_1E(z);$$
$$B_2(z) = \theta_1[(\text{ch}\,\xi z - 1)S\_(z) + B_2E(z)];$$
$$B_3(z) = \sum_{i=1}^{n-1}(\theta_{i+1} - \theta_i)\left[(\text{ch}\,\xi(z - z_i) - 1)S\_(z - z_i) + D_iE(z)\right];$$
$$B_4(z) = (\text{ch}\,\xi z - 1)S\_(z) + B_1E(z);$$
$$B_1 = \frac{\lambda_1\xi\,\text{sh}\,\xi h - \alpha\,\text{ch}\,\xi h}{(\lambda_1\xi - \alpha)e^{\xi(2l+h)} - (\lambda_1\xi + \alpha)e^{-\xi h}};$$
$$B_2 = \frac{\lambda_1\xi\,\text{sh}\,\xi h - \alpha(\text{ch}\,\xi h - 1)}{(\lambda_1\xi - \alpha)e^{\xi(2l+h)} - (\lambda_1\xi + \alpha)e^{-\xi h}};$$
$$E(z) = e^{\xi(z+2l)} - e^{-\xi z};$$
$$D_i = \frac{\lambda_1\xi\,\text{sh}\,\xi(h - z_i) - \alpha(\text{ch}\,\xi(h - z_i) - 1)}{(\lambda_1\xi - \alpha)e^{\xi(2l+h)} - (\lambda_1\xi + \alpha)e^{-\xi h}}.$$

The inverse Henkel integral transformation is applied to relation (14) and, as a result, we obtain

$$T(r,z) = \int_0^\infty \xi J_0(r\xi)\overline{T}(\xi,z)d\xi. \qquad (15)$$

Unknown approximation values $\theta_i\,(i = \overline{1,n})$ and $\theta_j\,(j = \overline{1,k})$ and temperature $\theta(R, z)$ and $\theta(r, 0)$ determined by solving a system of $n + k$ linear algebraic equations are obtained after some mathematical transformations from expression (15).

As a result, the desired temperature field in a layer with a semi-through thermally active cylindrical inclusion is expressed by formula (15), from which the temperature value at an arbitrary point of the “layer-inclusion” structure is obtained.

Nonlinear mathematical model for determining the temperature field in a thermosensitive layer with a semi-through cylindrical thermoactive inclusion. The thermal conductivity coefficient for a thermosensitive layer with a foreign cylindrical thermoactive inclusion is expressed as

$$\lambda(r, z, t) = \lambda_1(t) + [\lambda_0(t) - \lambda_1(t)]S\_(R - r)S\_(z). \qquad (16)$$

A linearizing function is introduced

$$\vartheta(r,z) = \int_0^{t(r,z)}\lambda_1(\zeta)d\zeta + \left\{\int_{t(R,z)}^{t(r,z)}\left[\lambda_0(\zeta) - \lambda_1(\zeta)\right]d\zeta - \right. \qquad (17)$$
$$\left. - \int_{t(R,0)}^{t(r,0)}\left[\lambda_0(\zeta) - \lambda_1(\zeta)\right]d\zeta\right\}S\_(R - r)S\_(z),$$

after differentiating which with respect to the variables r and z, the relation is obtained

$$\lambda(r,z,t)\frac{\partial t(r,z)}{\partial r} = \frac{\partial\vartheta(r,z)}{\partial r} +$$
$$+\left.\left\{\left[\lambda_0(t) - \lambda_1(t)\right]\frac{\partial t(r,z)}{\partial r}\right\}\right|_{z=0} S\_(R - r)S\_(z);$$
$$\lambda(r,z,t)\frac{\partial t(r,z)}{\partial z} = \frac{\partial\vartheta(r,z)}{\partial z} + \qquad (18)$$
$$+\left.\left\{\left[\lambda_0(t) - \lambda_1(t)\right]\frac{\partial t(r,z)}{\partial z}\right\}\right|_{r=R} S\_(R - r)S\_(z).$$

Taking into account expressions (18), the original equation (3) is transformed to the following form

$$\Delta\vartheta+\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\lambda_0(t)-\lambda_1(t)\right]\frac{\partial t(r,z)}{\partial r}S_-(R-r)\right\}\Bigg|_{z=0}S_-(z)+\frac{\partial}{\partial z}\left\{\left[\lambda_0(t)-\lambda_1(t)\right]\frac{\partial t}{\partial z}S_-(z)\right\}\Bigg|_{r=R}S_-(R-r)= \qquad (19)$$

$$=-q_0S_-(R-r)S_-(z).$$

The linearizing function (17) made it possible to transform the boundary conditions (4), which took the following form

$$\vartheta(r,z)\big|_{r\to\infty}=0;\quad \frac{\partial\vartheta(r,z)}{\partial r}\bigg|_{r\to\infty}=0;\quad \frac{\partial\vartheta(r,z)}{\partial z}\bigg|_{z=-l}=0; \qquad (20)$$

$$\frac{\partial\vartheta(r,z)}{\partial z}\bigg|_{z=h}=\alpha(t|_{z=h}-t_c)-$$
$$-\left\{\left[\lambda_0(t)-\lambda_1(t)\right]\frac{\partial t(r,z)}{\partial z}\right\}\Bigg|_{\substack{r=R\\z=h}}S_-(R-r). \qquad (21)$$

As a consequence, the nonlinear boundary value problem (3, 4) is reduced to a partially linearized second-order partial differential equation with discontinuous coefficients (19), linearized boundary conditions (20), and partially linearized boundary condition (21).

Unknown functions $t(R, z)$, $t(r, 0)$ and $t(r, h)$ are approximated by piecewise-constant functions along the z and r coordinates.

$$t(R,z)=t_1+\sum_{i=1}^{n-1}(t_{i+1}-t_i)S_-(z-z_i);$$
$$t(r,0)=t_1+\sum_{j=1}^{k-1}(t_{j+1}-t_j)S_-(r-r_j); \qquad (22)$$

$$t(r,h)=t_1+\sum_{l=1}^{p-1}(t_{l+1}-t_l)S_-(r-r_l), \qquad (23)$$

where $z_i \in (0;\, h)$; $z_1 \le z_2 \le \ldots \le z_{n-1}$; $r_j \in (0;\, R)$; $r_l \in (0;\, \infty)$; $r_1 \le r_2 \le \ldots \le r_{k-1}$; n, k, p are the division numbers of the intervals $(0;\, h)$, $(0;\, R)$, $(0;\, r^*)$; r^* is the value of the radial coordinate at which the temperature at the boundary surface of the L_+ layer is practically zero; $t_i\,(i=\overline{1,n})$, $t_j\,(j=\overline{1,k})$, $t_l\,(j=\overline{1,m})$ are unknown approximate values of temperature $t(R, z)$, $t(r, 0)$, $t(r, h)$.

The functions $t(R, z)$, $t(r, 0)$, described by relations (22), are differentiated with respect to the variables z and r respectively. As a result, we obtain

$$\frac{\partial t(R,z)}{\partial z}=\sum_{i=1}^{n-1}(t_{i+1}-t_i)\delta_-(z-z_i);$$
$$\frac{\partial t(r,0)}{\partial r}=\sum_{j=1}^{k-1}(t_{j+1}-t_j)\delta_-(r-r_j). \qquad (24)$$

As a result of substituting expressions (23, 24) into relations (19 and 21), a second-order linear differential equation with partial derivatives and a discontinuous and singular right-hand side with respect to the linearizing function $\vartheta(r, z)$ is obtained

$$\Delta\vartheta=-\frac{1}{r}\sum_{j=1}^{k-1}r_j\xi(t_{j+1}-t_j)\left[\lambda_0(t_{i+1})-{}_1\;t_{i+1}\right]\times$$
$$\times S_-(z)\delta'_-(r-r_j)-\sum_{i=1}^{n-1}\left(t_{i+1}-t_i\right)\left[\lambda_0(t_{i+1})-\lambda_1(t_{i+1})\right]\times \qquad (25)$$
$$\times S_-(R-r)\delta'_-\left(z-z_i\right)-q_0S_-(R-r)S_-(z),$$

with linear boundary condition

$$\frac{\partial\vartheta(r,z)}{\partial z}\bigg|_{z=h}=\alpha t(r,h). \qquad (26)$$

The Henkel integral transformation in the coordinate r is applied to equation (25) and boundary conditions (20, 26), resulting in an ordinary second-order differential equation with constant coefficients and a singular right-hand side

$$\frac{d^2\overline{\vartheta}}{dz^2}-\xi^2\overline{\vartheta}=$$
$$=-\frac{R}{\xi}J_1(R\xi)\left[\sum_{i=1}^{n-1}A_i\delta'_-(z-z_i)+q_0S_-(z)\right]-\xi AS_-(z), \qquad (27)$$

and boundary conditions

$$\frac{d\overline{\vartheta}(z)}{dz}\bigg|_{z=-l}=0,\quad \frac{d\overline{\vartheta}(z)}{dz}\bigg|_{z=h}=\frac{B}{\xi}, \qquad (28)$$

where

$$A_i=(t_{i+1}-t_i)[\lambda_0(t_{i+1})-\lambda_1(t_{i+1})];$$

$$A=\sum_{j=1}^{k-1}r_jJ_1(r_j\xi)(t_{j+1}-t_j)\left[\lambda_0(t_{j+1})-\lambda_1(t_{j+1})\right];$$

$$B=(t_p-t_c)\delta_+(\xi)-\sum_{l=1}^{p-1}r_lJ_1(r_l\xi)(t_{l+1}-t_l).$$

The general solution of equation (27) is defined as

$$\overline{\vartheta}(z)=c_1e^{\xi z}+c_2e^{-\xi z}-$$
$$-\frac{1}{\xi}\left\{RJ_1(R\xi)\left[\sum_{i=1}^{n-1}A_ich\xi(z-z_i)S_-(z-z_i)-\right.\right.$$
$$\left.\left.-\frac{q_0}{\xi^2}(ch\xi z-1)S_-(z)\right]+A(ch\xi z-1)S_-(z)\right\},$$

and using boundary conditions (28) we find the integration constants c_1 and c_2. As a result, we obtain the solution to problem (27, 28)

$$\overline{\vartheta}(z)=\frac{1}{\xi}\left\{\begin{matrix}AB(z)+RJ_1(R\xi)\left[\sum\limits_{i=1}^{n-1}A_iB_i(z)-\frac{q_0}{\xi^2}B(z)\right]-\\-\alpha BP(z)\end{matrix}\right\}. \qquad (29)$$

Here

$$B(z)=P(z)\,\mathrm{sh}\,\xi h-(\mathrm{ch}\,\xi z-1)S_-(z);$$

$$B_i(z)=P(z)\,\mathrm{sh}\,\xi(h-z_i)-\mathrm{ch}\,\xi(z-z_i)\{S_-(z-z_i);$$

$$P(z)=\frac{ch\xi(z+l)}{sh\xi(h+l)}.$$

The inverse Henkel integral transformation is applied to relation (29) and the expression for the linearizing function $J(r, z)$ is defined in the following form

$$\vartheta(r,z)=\int_0^{\infty}\xi J_0(r\xi)\overline{\vartheta}(z)d\xi. \qquad (30)$$

To determine the unknown approximate values t_i $(i=\overline{1,n})$, $t_j(j=\overline{1,k})$, $t_l(j=\overline{1,m})$ of temperatures $t(R, z)$, $t(r, 0)$, $t(r, h)$ after certain mathematical transformations, a system of nonlinear algebraic equations is obtained as a result of substituting the expression of the temperature dependence of the thermal conductivity coefficient of the layer materials and including it in the relations (17, 30).

The sought temperature field $t(r, z)$ for the given structure is determined using the obtained nonlinear algebraic equation with the help of relationships (17 and 30) by substituting into them the specific expression for the temperature dependence of the thermal conductivity coefficient of the materials in the structure.

Partial example. In many practical problems, for certain construction materials, a linear temperature dependence of the thermal conductivity coefficient is used in the form of the following relation

$$\lambda=\lambda_m^0(1-k_m t), \qquad (31)$$

where λ_m^0, k_m are the reference and temperature coefficients of thermal conductivity of the materials for the inclusion ($m = 0$) and the layer ($m = 1$).

Using relation (17) for the linearizing function, from expressions (30, 31) we obtain formulas for determining the temperature $t(r, z)$ in the inclusion region

$$\{(r, \varphi, z) : r \le R, 0 \le \varphi \le 2\pi, z \le h\};$$

$$t(r,z)=\frac{1}{k_0}\left(1-\sqrt{1-k_0\left(\frac{2\vartheta(r,z)}{\lambda_0^0}+\vartheta+\vartheta(r)+\vartheta(z)\right)}\right), \qquad (32)$$

and in the area outside the inclusion $\{(r, \varphi, z) : r > R, 0 \le \varphi \le 2\pi i, -l \le z \le h\}$;

$$t(r,z)=\frac{1}{k_1}\left(1-\sqrt{1-\frac{2k_1\vartheta(r,z)}{\lambda_1^0}}\right), \qquad (33)$$

where $\vartheta = t_n(\lambda_j - t_n k\lambda)$; $\vartheta(r) = t(r, 0)[(t(r, 0)k\lambda - \lambda j]$; $\vartheta(z) = t(R, z)[t(R, z)k\lambda - \lambda j]$; $\lambda j=2\left(\frac{\lambda_1^0}{\lambda_0^0}-1\right)$; $k\lambda=\frac{\lambda_1^0}{\lambda_0^0}k_1-k_0$.

Formulas (32 and 33) describe the temperature field in a thermally sensitive “layer-inclusion” structure.

Analysis of numerical results. A numerical analysis of the dimensionless temperature $t^* = \lambda_0 t/(q_0 R)$ was performed for the following initial data: layer material is VK94-I ceramics, inclusion material is silver, $h = R = 2$ mm, $l = 0$ mm, $q_0 = 200$ W. In the temperature range [20 °C; 1,230 °C], the temperature dependences of the thermal conductivity coefficient for the above materials are performed by interpolation and will be as follows

$$\lambda_1(t) = 13.67(1 - 0.00064t);$$
$$\lambda_0(t) = 422.54(1 - 0.00031t), \qquad (34)$$

which is a partial case of relation (31).

Numerical calculations of the dimensionless temperature t^* as a function of the dimensionless radial coordinate $r^* = r/R$ and axial coordinate $z^* = z/R$ were carried out for both the linearly varying and constant thermal conductivity coefficients (λ_1 = 13.4 W/(deg · m), λ_0 = 419 W/(deg · m)). The results indicate that maximum temperatures occur in the region affected by concentrated internal heat sources, consistent with the physical model of the heat transfer process.

The number of divisions $n = k = p = 10$ intervals $(0; h)$, $(0; R)$, $(0; r^*)$ respectively, for the given values of thermophysical (reference λ_m^0 and temperature k_m coefficients of thermal conductivity for materials of the layer ($m = 1$) and inclusion ($m = 0$)) and geometric (radius R and inclusion height h, layer thickness $l + h$)) parameters of the structure made it possible to perform calculations with an accuracy of $\varepsilon = 10^{-6}$.

The results obtained for the selected materials with a linear temperature dependence of the thermal conductivity coefficient differ from the results obtained for a constant thermal conductivity coefficient by 7 %. Their slight difference is explained by the fact that the values of the temperature coefficient of thermal conductivity for the given materials, as shown by the relation (34), are small.

Discussion of results. The boundary value problems of thermal conductivity are formulated in accordance with the real physical process that occurs in the considered media. As a result, the differential equations of heat conduction and boundary conditions strictly describe mathematical models of the stationary heat conduction process, which correspond to the physical process.

In the above studies, asymmetric unit functions were used, which made it possible to effectively describe the thermophysical parameters of a medium with an alien semi-through inclusion and local temperature perturbations, as a result of which the obtained partial differential equations contain discontinuous and singular coefficients. To linearize the nonlinear boundary value problem (2, 3), a linearizing function (17) was introduced, which made it possible to effectively obtain the corresponding quasilinear boundary value problem (19–21). A segment-constant approximation of the temperature as a function of spatial coordinates on the inclusion surfaces and the boundary surface of the layer was performed, which subsequently made it possible to apply the Henkel integral transformation and, as a result, obtain analytical-numerical solutions (15, 30) of the boundary value problems. The temperature distribution is determined by relations (19, 32, 33).

It should be noted that the above-mentioned analyzed works did not consider an approach for linearizing boundary value problems of thermal conductivity for thermally sensitive media by an analytical method. Unlike [3], where a homogeneous medium was considered and the use of the Kirchhoff transformation made it possible to linearize the boundary value problem. In the above studies, a new linearizing function was introduced, the application of which to nonlinear problems made it possible to effectively obtain their analytical and numerical solutions. And this, in turn, leads to a minimal error in the obtained results, in contrast to the use of numerical methods, which was not achieved in [4, 5]. The use of asymmetric unit functions makes it possible to effectively describe thermophysical parameters for media with foreign semi-through inclusions, which leads to the solution of a single differential equation of heat conduction with partial derivatives with discontinuous and singular coefficients, which was not done in [6].

The presented studies concern only the stationary process of thermal conductivity and these studies were performed for media with foreign semi-through inclusions. In the future, such studies can be continued for layered media with foreign semi-through inclusions, for non-stationary heat conduction processes, as well as for anisotropic layered media with foreign semi-through inclusions.

Since the architecture of modern electronic devices concentrates individual thermally active nodes and their elements in the form of structures with foreign semi-through inclusions, there is a need to develop mathematical models of the thermal conductivity process. These models can be linear or nonlinear for isotropic layered media containing foreign semi-through inclusions. The mathematical models of thermal conductivity presented in the conclusion are simplified, but they allow for the development of more complex mathematical models of the thermal conductivity process for composite media.

Based on the obtained analytical and numerical solutions for both linear and nonlinear boundary value problems of heat transfer, it is proposed to develop computational algorithms and software tools for their numerical implementation. This will allow for research into the impact of thermal sensitivity on temperature distribution for a range of materials used in the design of digital electronic devices.

It is proposed to take into account the presence of semi-through foreign inclusions in isotropic media and the thermal sensitivity of structural materials for the analysis of thermal regimes, which significantly complicates the process of solving the corresponding linear and nonlinear boundary value problems of thermal conductivity. The sought solutions to these problems describe the behavior of temperature as a function of spatial coordinates somewhat more adequately to the real physical process.

The study was performed for a stationary heat conduction process, as a result of which the developed models are limited, since they allow determining the temperature change only by spatial coordinates. Heat conduction problems contain boundary conditions of the first, second, and third kind on the boundary surfaces of the media, which is a disadvantage, although this does not reduce the generality of the research.

Further research may concern the development of mathematical models for determining temperature fields in layered media with foreign semi-through inclusions for the unsteady process of thermal conductivity and for more complex boundary conditions.

Conclusions. A linear mathematical model has been developed for determining the temperature field and subsequently analyzing the thermal regimes in the structures of electronic devices containing a foreign thermoactive semi-through inclusion. An analytical–numerical solution to the corresponding boundary value problem has been obtained, and based on it, the temperature behavior as a function of spatial coordinates has been determined and represented graphically.

A nonlinear mathematical model has also been developed for determining the temperature field and analyzing the thermal regimes in thermosensitive structures of electronic devices containing a foreign thermoactive semi-through inclusion. A linearizing function was introduced, enabling the nonlinear boundary value problem to be transformed into a quasi-linear one. For the case of a linear temperature dependence of the thermal conductivity coefficients of the structure's materials, an analytical–numerical solution to the corresponding boundary value problem has been obtained. This solution makes it possible to determine the temperature both inside the inclusion and in the surrounding base material.

The temperature behavior as a function of spatial coordinates has been visualized for both constant thermal conductivity coefficients and those linearly dependent on temperature. The results for the selected materials show that, under a linear temperature dependence of thermal conductivity, the deviation from the results obtained for constant thermal conductivity is 7 %.

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Математичне моделювання й аналіз теплообміну в конструкціях із чужорідними елементами

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Мета. Розроблення лінійних і нелінійних математичних моделей визначення температурних полів в ізотропних просторових середовищах із чужорідними теплоактивними напівнаскрізними елементами. Це дозволить підвищити точність аналізу температурних режимів унаслідок теплових навантажень і сприятиме вдосконаленню методів проєктування пристроїв, окремі вузли яких містять чужорідні теплоактивні елементи.

Методика. Для визначення аналітично-числових розв'язків лінійної й нелінійної крайових задач теплопровідності використані асиметричні одиничні функції. У наслідку коефіцієнт теплопровідності для конструкції із чужорідним напівнаскрізним циліндричним елементом представлений єдиним цілим. Такий підхід забезпечує виконання умов ідеального теплового контакту на поверхнях спряження неоднорідних матеріалів структури, що приводить до розв'язування одного рівняння теплопровідності з розривними й сингулярними коефіцієнтами. Для розв'язування нелінійної крайової задачі запроваджена лінеаризуюча функція, що дає змогу отримати лінійне диференціальне ріняння другого порядку із частковими похідними й розривними та сингулярними коефіцієнтами й квазілійну крайову на умову. Унаслідок виконаної апроксимації температури, як функції просторових координат на поверхні включення та граничній поверхні шару сегментно-сталими функціями, нелінійна крайова задача цілком лінеаризована.

Результати. Розроблені лінійна й нелінійна математичні моделі визначення температурного поля й аналізу теплових режимів у пристроях, окремі вузли яких містять чужорідне теплоактивне напівнаскрізне включення. Запроваджена лінеаризуюча функція для спрощення нелінійної крайової задачі. Отримані аналітично-числові розв'язки як лінійної, так і нелінійної крайових задач теплопровідності, на основі яких визначена графічна поведінка температури як функції просторових координат. Порівняльний аналіз показав відмінність у 7 % між результатами для сталого й лінійно змінного коефіцієнта теплопровідності, що пояснюється невеликими значеннями температурного коефіцієнта теплопровідності для вибраних конструкційних матеріалів.

Наукова новизна. Запроваджено метод лінеаризації нелінійної математичної моделі процесу теплопровідності. Запропоновано спосіб визначення у замкнутому вигляді аналітично-числових розв'язків відповідних лінійної та нелінійної крайових задач. Використання асиметричних одиничних функцій дало змогу забезпечити коректний математичний опис теплообмінних процесів у середовищах із чужорідним теплоактивним напівнаскрізним елементом.

Практична значимість. Із використанням розроблених математичних моделей теплообміну є можливість аналізувати середовища щодо їх термостійкості, що сприяє підвищенню ефективності роботи пристроїв, окремі вузли яких містять чужорідні теплоактивні напівнаскрізні елементи. Це запобігає перегріванню та продовжує термін їх експлуатації. Результати дослідження можуть бути застосовані у практичних задачах теплообміну й теплоізоляції промислових конструкцій, у тому числі для прогнозування температурних полів у механізмах гірничого обладнання, вентиляційних системах і компресорних станціях. Використання запропонованих моделей сприяє підвищенню ефективності процесів видобування й обробки руди, а також зменшенню втрат теплової енергії у промислових системах.

Ключові слова: *температурне поле, теплопровідність матеріалу, термостійкість конструкцій, термочутливий матеріал, конвективний теплообмін*

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