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UNCERTAINTY ASSESSMENT IN MINERAL RESOURCE ESTIMATION USING GEOSTATISTICS AND MONTE CARLO SIMULATION

Purpose. The accurate estimation of mineral reserves is a critical step in the planning and development of mining operations, particularly for phosphate deposits. This study aims to enhance reserve estimation for the Bled El-Hadba phosphate deposit (eastern Algeria) by combining geostatistical and probabilistic approaches.

Methodology. The proposed method integrates ordinary kriging (OK) with Monte Carlo simulation (MCS) to better evaluate the reserves of the main mineralized layer. Ordinary kriging was applied on a 5 × 5 m grid to generate estimation maps of P_2O_5 grades and layer thicknesses, serving as a spatial model of the deposit. The kriging outputs (mean and variance) were then used as inputs for a Monte Carlo simulation involving 20 million iterations, preserving the statistical characteristics of the original data.

Findings. The combined use of OK and MCS reduced the fluctuations typically observed in kriging-based estimates and led to more stable and robust reserve values. The application of P90 and P50 probability categories contributed to more conservative and reliable classifications of mineral reserves, improving the assessment of proven and probable categories essential for economic decision-making.

Originality. This study introduces a novel integration of kriging with large-scale Monte Carlo simulation to manage uncertainty in phosphate reserve estimation. The approach enables a more precise characterization of spatial variability and supports probabilistic interpretations of resource quantities.

Practical value. The methodology offers a valuable tool for mine planners and decision-makers by improving confidence in reserve estimates and supporting sound economic evaluations. Its application to the Bled El-Hadba deposit demonstrates its effectiveness and potential for broader use in phosphate mining projects.

Keywords: *geostatistics, kriging, Monte Carlo simulation, phosphate, resource estimation, uncertainty*

Introduction. Mining exploration and the extractive industry are essential for international economic development [1, 2]. They provide the extremely important minerals required for the construction, energy production, and transportation sectors [2, 3]. A key phase in ensuring the success of a mining project is underground reconnaissance, which relies on field observations obtained from topographic surveys, geophysical measurements, and drilling (core drilling) [4, 5]. However, due to the high cost of drilling, the available geochemical data are often limited. This necessitates that geologists rely heavily on geological models based on their interpretation of existing data [5]. These interpretations employ various interpolation techniques, such as the polygon method, the inverse distance method, and geostatistical approaches [1, 3, 6].

Previous studies on the Bled El Hadba phosphate deposit (eastern Algeria) have revealed significant discrepancies in reserve estimates. The first study, conducted by the National Office of Geological and Mining Research in 2015 [7], estimated the probable reserves of the main layer at approximately 367 million tons, while the second study, carried out by DMT [8], evaluated the probable reserves at around 464 million tons. It should be noted that these reserves were estimated with cutoff grades of 16 % for ORGM and 20 % for DMT, respectively.

These differences constituted the main challenge that motivated our involvement in this in-depth study. Given the significant divergence between these two assessments, a thorough analysis of the methodologies employed, for the considered parameters, as well as potential variations in the estimation procedures, is deemed necessary [3].

This work primarily aims to model the spatial variability among samples taken during the reserve evaluation of the Bled El Hadba deposit. This modelling is based on an innovative approach that combines the theory of regionalized variables (geostatistics) and Monte Carlo simulation [1, 3, 9] to improve estimation accuracy. The combined use of ordinary kriging, performed by means of the SGeMS software [10], and the

Monte Carlo method represents a significant advancement, as it allows the generation of a wide range of probabilistic scenarios based on probability distributions derived from geostatistical results. This approach ensures better management of uncertainties, particularly those related to anisotropy observed in variograms.

Using a variographic model, we interpolated unsampled locations by kriging, in order to estimate the P_2O_5 grades, the thickness of the mineralized main layer, and to evaluate mineral reserves using the geostatistical method (ordinary kriging). In order to further minimize the uncertainty of the kriging results, primarily due to anisotropy observed in variograms [6], the Monte Carlo method was used. This simulation technique has been proven to be effective in processing and improving kriging results [11].

This work primarily focuses on modelling the spatial variability among samples collected during the reserve evaluation of the Bled El Hadba deposit. This modelling is based on an innovative approach that combines the theory of regionalized variables (geostatistics) and Monte Carlo simulation [6, 12, 13]. This robust statistical method is based on simulating a sequence of random variables (X_n) $n \geq 1$, which are independent and identically distributed according to a given distribution [6].

The combined use of ordinary kriging, performed with the SGeMS software [10], and the Monte Carlo method represents a significant advancement, as it enables the generation of a wide range of probabilistic scenarios based on probability distributions derived from geostatistical results. This approach ensures better management of uncertainties, particularly those related to the anisotropy observed in variograms.

A unique aspect of this research lies in the development of a Matlab-based code that uses the results of ordinary kriging for Monte Carlo simulation. This code automates this critical step, providing rigorous control over the quality of the simulations and reducing errors associated with manual manipulations. By applying this method to the Bled El Hadba deposit, we demonstrate its effectiveness in terms of both the accuracy of the estimates of the studied variables (P_2O_5 grades, thickness of the mineralized main layer) and the reduction of uncertainties, while also opening up new possibilities for its application in other mining contexts.

Material and methods. ***Geological setting.*** In order to apply the geostatistical method and the Monte Carlo simulation, the Bled El-Hadba deposit (mining field) was chosen to evaluate its reserves in the context of a technical and economic effectiveness study. The deposit comprises five distinct layers, with the median layer being the principal mineralized horizon. The geochemical data used has been collected from the exploration campaigns of the chosen site.

The Bled El Hadba deposit is located 14 km southeast of Bir El Ater and 6 km from the Algerian-Tunisian border. The area of the zone recognized by exploratory boreholes is approximately 2.9 km^2.

Structurally, the Bled El Hadba area constitutes the western flank of the antiform structure of Jebel Zrega, which crest line forms the Algerian-Tunisian border. This mining area is located symmetrically with respect to the southern flank of Jebel Onk (Djemi Djema and Kef Es Sennoun), which phosphate layer is on average about 40 m thick [7, 14, 15].

The wall and roof structures of the phosphate bundle (Fig. 1) illustrate well the monoclinal dip, towards the west of the phosphate series. Several horizontal setbacks, NW-SE, are cartographically visible, but they do not cause significant changes in the phosphate layer geometry [7, 14, 16].

The phosphate deposit geology is relatively simple and it was described by Dussert (1924). The geological

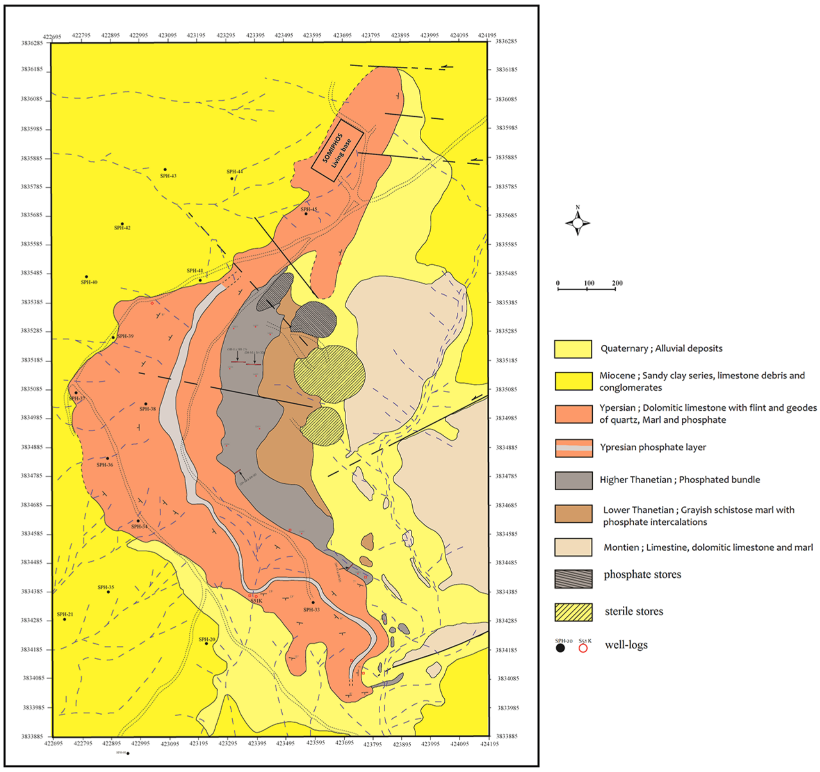

Fig. 1. Geological map of the structure of the Bled El Hadba phosphate deposit (Eastern Algeria) [14]

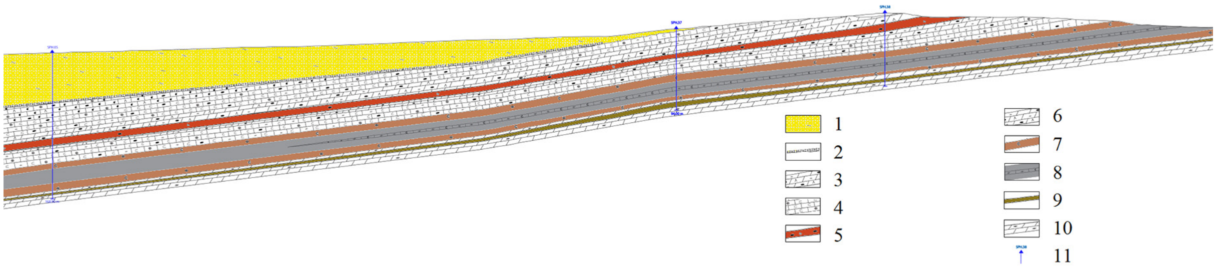

Fig. 2. Geological Profile oriented east–west (modified, after [14]*):*

1 – miocene detrial deposits (sands and clays); 2 – miocene basal conglomerate; 3 – whitish organic limestone, strongly gypsum; 4 – lumichellic limestone, residually phosphate, with flint nodules and quartz geodes; 5 – phosphate with nodules of flint, and debris of limestone and marl; 6 – gypsum marl, whitish, with flint nodules, residually phosphate; 7 – phosphated layer of the higher thanetian (CS + CB: lumachellic phosphate with quartz geodes); 8 – CM: phosphated layer of the higher thanetian; 9 – pelitic marl bedded, residually phosphate; 10 – coprolithic phosphate, marly cement; 11 – well-logs (exploratory boreholes)

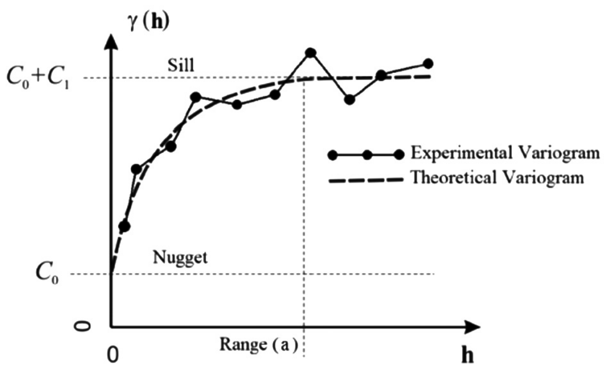

Fig. 3. Variogram parameters [17]:

a – range; C_0 – nugget effect; C_0 + C – Sill variance; γ(h) – semi-variance; h – distance (m)

map of the region shows the phosphate layers of the Thanetian age, under the Ypresian flint limestones and the Miocene sands (Fig. 1), plunge in a monocline fashion and under a gentle slope of 6 to 10° towards the West and the North-West, this dip becomes more accentuated towards the South of the deposit (Fig. 2). Towards the east, outcropping the phosphatic infraserial marls, then the limestones and marls of Montien [7, 14, 15].

Geostatistical study. In order to facilitate and improve the mineral reserves estimation, a geostatistical approach is used [4, 6, 14]. This approach involves several steps describing the kriging interpolation process.

Calculation and adjustment of variograms. The variogram is a fundamental tool in geostatistics that characterizes the spatial structure of regionalized variables. It underpins prediction and simulation algorithms and provides critical insights into the behaviour and properties of these variables. It is characterized by parameters such as the range, nugget effect, sill, and variograph (Fig. 3).

The determination of the necessary parameters of the experimental variograms must go through the adjustment of the theoretical variograms. The experimental variograms corresponding to the studied variables (thicknesses and contents of the phosphate main layer) are presented below.

Variograms of the thickness of the mineralized layer. The two-dimensional calculation of the experimental variograms along the four main directions is done in order to determine the spatial correlation between the mining soundings.

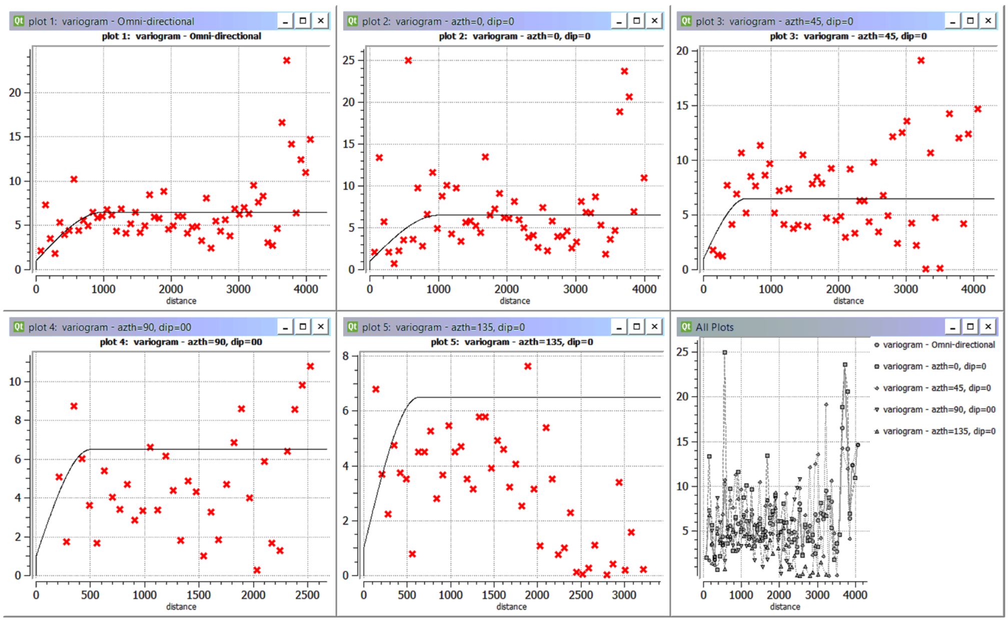

Fig. 4. Directional and omnidirectional variograms of thicknesses

Table 1

Experimental variogram parameters for the mineralized layer thickness

Directions	Range, *a*	Sill, *m*	Nugget effect
Omnidirectional	2,000	40	0

Note: Exponential model used for fitting

The experimental variograms corresponding to the different directions (0, 45, 90, 135° and the omnidirectional variogram) are calculated for multiple distances in steps of 75 meters. The calculation results are shown in Fig. 4.

The thickness variogram parameters (Table 1) show a range of a = 2,000 m, a plateau of $C(0)$ = 40. The adjustment of the theoretical variograms shows a continuity near the origin illustrated by a zero nugget effect ($C_0 = 0$). To make the adjustment, an exponential model was used.

The main parameters derived from the experimental thickness variograms have been summarized in Table1.

These parameters will be implemented (incorporated) in an exponential model to estimate, by kriging, the dispersion maps of the mineralized layer thicknesses.

Variograms of phosphate grades. In order to determine the theoretical model of the variograms expressing the spatial correlation between samples, the two-dimensional calculation of the corresponding experimental variograms has been carried out (Fig. 5).

The results of spatial correlation of the contents reach a range of $C(0) = 6.5$ for a span of a = 1,000 m. However, the adjustment of the theoretical variograms shows a discontinuity near the origin expressed by a nugget effect equal to ($C_0 = 1$). The establishment of these variograms is carried out by means of the spherical model.

The experimental variograms of the area of study show spatial continuity in the 0° direction with respect to the azimuthal direction. This continuity decreases (reduced span) in the other directions. Moreover, it should be also noticed the occurrence of different range in the 0° direction and the azimuth direction.

It has been observed that if one moves in another direction, spatial continuity will decrease (reduced span), which explains the existence of a geometric anisotropy [12, 18].

It can be noticed as well that the existence of a zonal anisotropy is emphasized by the existence of different ranges in the directions (90°). There is no practical adjustment model to treat this type of anisotropy. But if we consider that the regionalized variable is stationary of order 2, the range is approximately equal to the variance.

The main parameters characterizing the experimental variograms of the phosphate grades are presented in Table 2.

For the estimation by kriging of the dispersion maps of the mineralized layer contents, a spherical model is applied using these parameters.

Kriging grid. After carrying out the variographic modelling, the next step is to determine the geometric support on which the kriging will be performed. To do this, we considered a regular horizontal grid (2D) covering the entire area of the study with a mesh size of (5×5) m^2. Therefore, 578 × 810 points to be mapped have been recorded.

The discretization step was chosen in order to obtain better accuracy and, among other things, to facilitate the processing of the results.

Thickness estimation maps of the middle layer. The map in Fig. 6 shows the thickness estimations of the

Table 2

Experimental variogram parameters for P_2O_5 grades

Directions	Range, *a*	Sill, *m*	Nugget effect
Omnidirectional	1,000	6.5	1

Note: Spherical model used for fitting

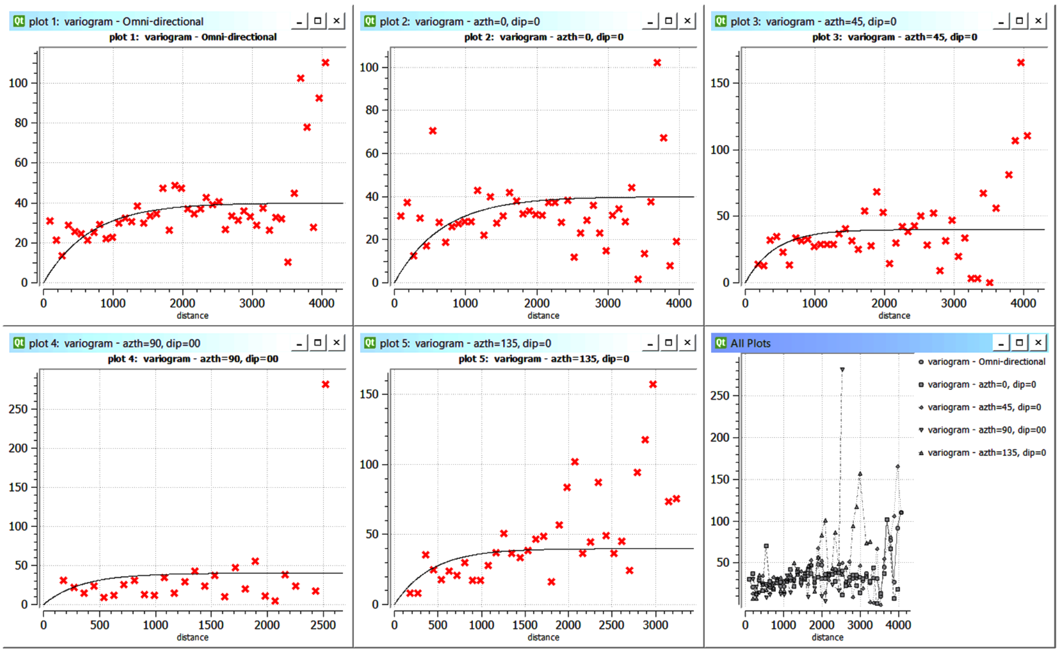

Fig. 5. Directional and omnidirectional variograms of grade

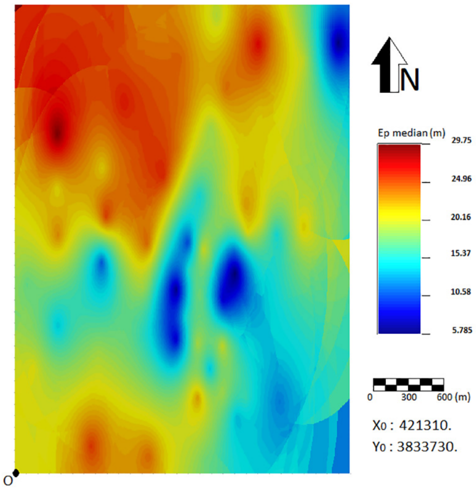

Fig. 6. Representative map of the kriging of the middle layer thicknesses

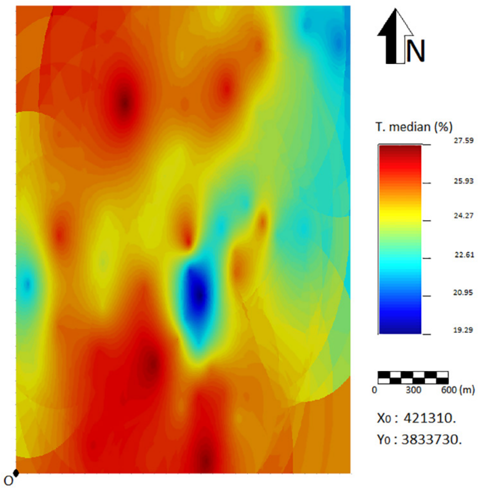

Fig. 7. Map of the kriging variance of the mineralized layer thickness in the middle layer (main layer)

mineralized layer, with a clear trend of thicknesses to the north-west direction. The power of this layer has varied from 6 to 29 meters.

Kriging variance of the middle layer thickness. The kriging variance map (Fig. 7) shows a good estimation quality around the soundings, with a maximum kriging variance of up to 64. There is a variance of 1 to 12 in the sampled sounding area. This makes it possible to say that the whole of the studied area is well explored, since the maximum variance of the thickness variogram is of the order of 40 m.

It should be noted that the highest variances of the studied layer thickness are up to 40, and that the thickness estimation varies from 6 to 29 m. These variances are explained by an uncertainty in the calculation and the variogram adjustment which are linked to a zonal anisotropy when calculating the experimental variograms.

***Phosphate Content Level Distribution Maps* (P_2O_5).** After establishing the experimental models of variograms, their properties (span *a*, range $C(0)$ and nugget effect) have been used, in order to perform ordinary kriging on the field data, and to obtain thereafter an overall site estimation accompanied by the variance estimation map.

The map of the ordinary kriging (Fig. 8) allows identifying all the places where the contents of P_2O_5 are of good quality (which can allow the phosphate to be exploited). This map shows the distribution of the mineralization in the middle layer.

The actual map (Fig. 8) shows that P_2O_5 contents ranged from 19 to 23 % in the central zone of the deposit towards the northeast with a slight increase in contents. The remaining contents are ranging from 23 to 27 %.

Kriging variance of* P_2O_5 *contents. The map in Fig. 9 shows the kriging variance of the study area, allowing us to judge the quality of the data used in the calculation procedure and to locate the places where the variance is high (perimeters surrounds).

This map (Fig. 9) shows a variance from 0 to 6% in the location explored by soundings, which reflects the good quality of estimation obtained by the kriging of the P_2O_5 contents.

The estimate of the contents contrary to the thicknesses presents a satisfactory variation with respect to the contents present in the deposit.

A complementary evaluation study of the uncertainty in the estimation is necessary because of the estimation quality of the thicknesses maps and the uncertainty presented by the high variances of the phosphate layer thicknesses.

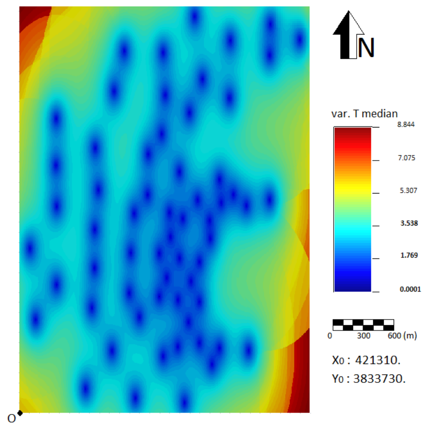

Fig. 8. Representative map of the final kriging of phosphate contents (P_2O_5), showing the distribution of the mineralization in the middle layer (main layer)

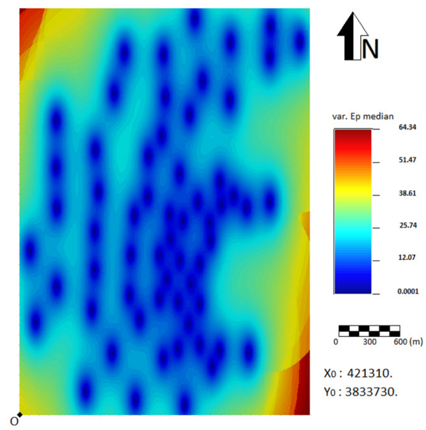

Fig. 9. Representative map of the kriging variance of the phosphate contents of the middle layer (main layer)

Table 3

Descriptive statistics of input variables for Monte Carlo simulation

Stochastic variables	Mean	Standard Deviation, σ	Coefficient of Variation, %	Distribution
Thicknesses, m	19.04	4.41	23.17	Normal
Grades, %	25.22	0.0147	5.85	Normal

Note: *Data used to generate virtual samples in MCS*

Monte Carlo simulation (MCS). Monte Carlo simulation is a mathematical technique used to generate random variables for modelling risks or uncertainties within a system [19–21]. It is widely applied in geotechnical and geological engineering research. In our study, this method is employed to model the uncertainty of output variables, particularly the classification of reserves (grade) and the volume of ore (P_2O_5), based on probabilistic models developed to analyse input variables such as thicknesses and grades (Table 3).

The Monte Carlo simulation process begins with the estimation of the mean response of the virtual samples randomly proposed by MCS. By taking the volume of ore as a variable, the convergence of the method with an iteration number (virtual samples) of about 20 million has been achieved. This number of iterations is largely justified by the high variance in the estimation of thicknesses by ordinary kriging and their variogram (of the order of 40) and is further supported by the input variable distributions (Table 3), which justify the use of 20 million iterations.

Once the convergence is reached, the next step is the Monte Carlo simulation (Fig. 13), considering 20 million as the number of iterations (Figs. 10 and 11). It should be noted that the fluctuation of the standard deviation and the mean response have stabilized with regards to the standard deviation of the order of $3 \cdot 10^6$ and has stabilized at about $0.25 \cdot 10^6$ (Fig. 11). The final calculation step consists of estimating the ore volume based on the inverse CDF graph (Figs. 12 and 13).

The inverse curve of the cumulative density shows estimations with a probability (degree of accuracy) of 10, 50 and 90 %. The results obtained are significantly better thanks to the quality of the input data. The different categories of phosphate reserves in the middle layer are presented in Table 4.

Reserves obtained by the Monte Carlo simulation method (MCS) are significantly improved, including the proven category (P90) reserves, which helps to engage the project with reduced uncertainty.

Discussion. The results of this study, with an estimate of 370 million tons, are close to the 367 million tons reported by ORGM in 2015, while deviating from the 463 million tons estimated by DMT in 2016. Table 4 confirms this reserve convergence at 370 Mt, strongly aligning with ORGM (2015) estimates. This discrepan-

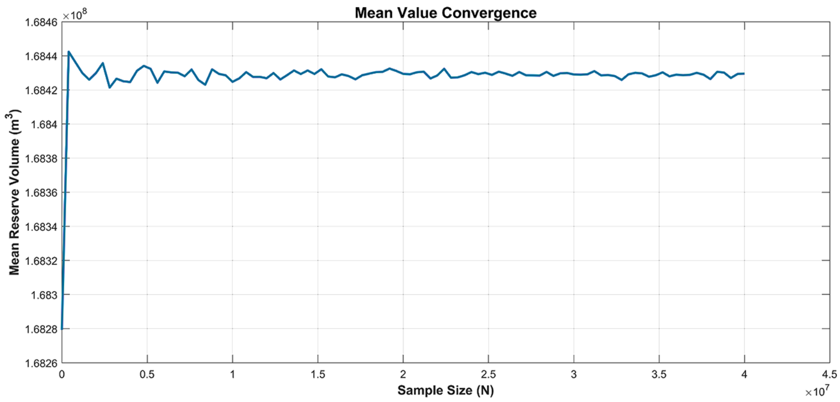

Fig. 10. Convergence of the estimated mean response as a function of the number of samples N

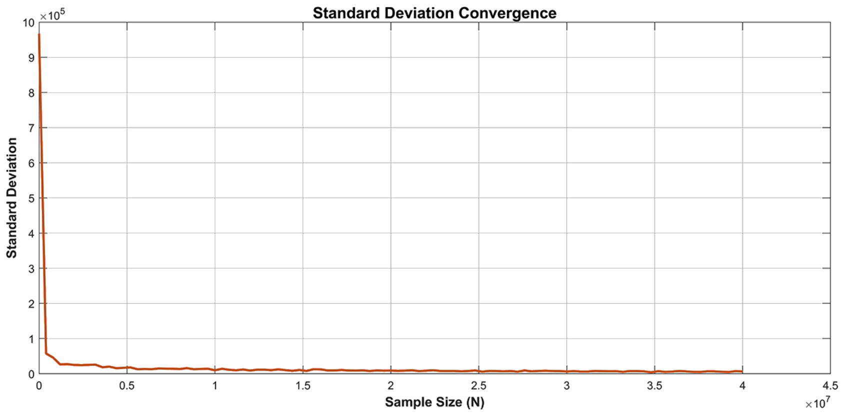

Fig. 11. Convergence of the estimated mean response with increasing sample size N

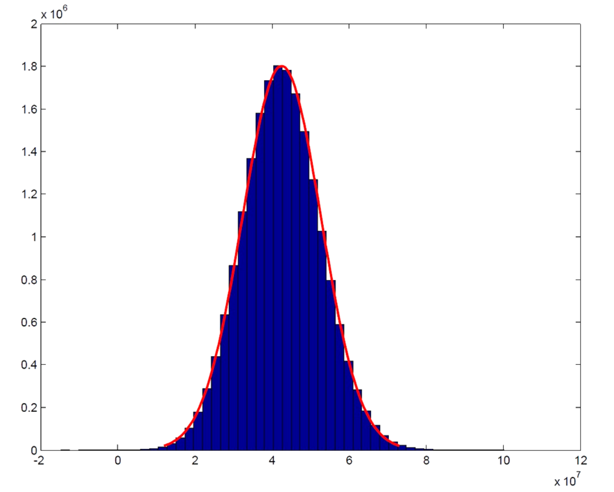

Fig. 12. Histogram and PDF of the distribution of reserves according to the normal distribution

cy highlights the importance of the employed methodology. By incorporating uncertainties through Monte Carlo simulation, our evaluation proves to be more cautious and realistic, providing a resolution to the previous divergences. This approach ensures greater precision and reliability, which are essential for strategic decision-making regarding the Bled El Hadba deposit.

The combination of ordinary kriging and Monte Carlo simulation proved to be a strategic decision for the evaluation of the Bled El-Hadba deposit. This approach not only allowed for the generation of reliable reserve estimates but also provided detailed dispersion maps of the thickness and content (P_2O_5) within the studied area.

By using ordinary kriging, spatial variability in the distribution of thickness and content was captured, while the integration of Monte Carlo simulation addressed the uncertainty in these estimations. This dual approach enhances confidence in the results and supports better decision-making for the implementation of mining operations.

The dispersion maps generated from this method are particularly valuable for planning and optimizing mining activities. These maps may serve as critical tools for identifying areas with the highest reserves and ensuring efficient resource allocation during mine exploitation.

The integration of probabilistic methods significantly reduced the uncertainty in reserve estimations, leading to more robust and realistic predictions. This ensures that mining strategies are based on solid data, minimizing risks and maximizing operational efficiency.

Monte Carlo simulation was applied to the kriging results, rather than to the field samples, further improving the reliability of the estimates. This approach leverages the accuracy of kriging at the sampling points while integrating uncertainty for unsampled areas. With over 2.5 million scenarios generated by the Monte Carlo simulation, the P_2O_5 reserve estimates were optimized, reducing uncertainties identified in the variance maps produced by kriging.

Extreme values were truncated at 10 % to eliminate outliers caused by excessive parameters, thus ensuring a more cautious and accurate reserve assessment.

The combination of kriging and Monte Carlo simulation provides detailed and reliable dispersion maps of

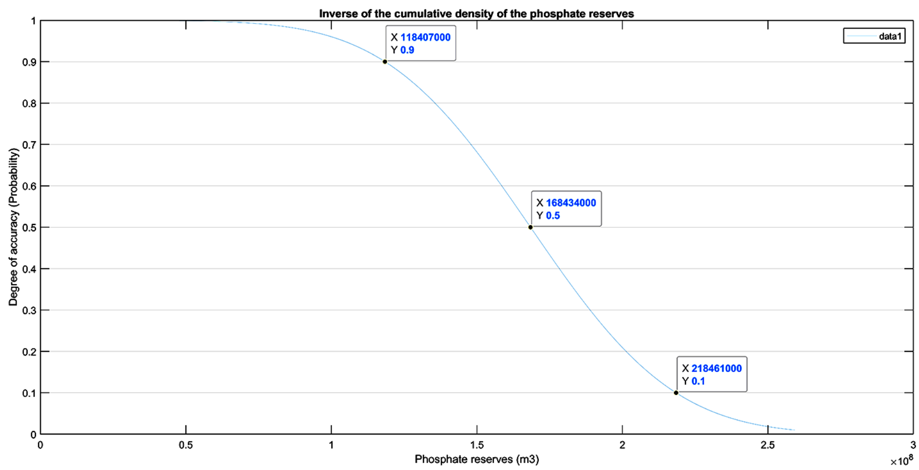

Fig. 13. Curve of the inverse of the cumulative density of the phosphate reserves (P_2O_5) *according to the different categories* (*Proved where P90* = 118,407,000 *m*^{3}, *Proved + Probable where P50* = 168,434,000 *m*^{3} *and Proved + Probable + Possible where P10 V* = 218,461,000 *m*^{3})

Table 4

Reserve categories estimated by the Monte Carlo Simulation (MCS)

Reserves in P_2O_5	Ore Reserve Categories		Cumulative (Total reserves)
	Proved	Probable	
Volume, m^3	118,410,000	50,034,000	168,444,000
Quantity, t	260,502,000	110,074,800	370,576,800
Mean-Grade, %	24.886		

Note: *Results based on 20 million iterations. "Proved" reserves have a 90 % probability (P90)* grades and thicknesses, which are essential for planning and optimizing mining operations at the Bled El-Hadba deposit.

Limitations and directions of research development. Although the combination of ordinary kriging and Monte Carlo simulation (MCS) offers significant advantages in terms of accuracy and uncertainty management, several limitations must be considered, particularly in the case of deposits with high spatial variability or complex geological structures.

First, the convergence of the probabilistic model relies on a large number of iterations, especially when geological variability is high. This can reduce the efficiency of the method in terms of computation time and required resources, which becomes a constraint in situations where rapid estimation is needed.

Moreover, applying Monte Carlo simulation alone, without geostatistical support, yields purely numerical results without spatial continuity. This approach limits the precision of the estimates and does not allow for the generation of dispersion maps that are useful for mine planning.

Integrating ordinary kriging helps mitigate this weakness by densifying the estimation points in space while preserving the original sample values, as kriging is an exact estimator. However, this interpolation introduces modelled values that may reduce the perceived variability if the variogram model or anisotropy directions are not properly defined. This potential bias can impact the results of the probabilistic simulation, thereby affecting the estimation of uncertainty.

Conclusion. The present work demonstrates the feasibility of combining one of the methods of geostatistical estimation (ordinary kriging) with the Monte Carlo simulation method (MCS) for evaluating the main layer of the Bled El-Hadba deposit, located in eastern Algeria (Wilaya of Tebessa). This evaluation was based on discrete sampling, where ordinary kriging enabled the creation of dispersion maps for the contents, as well as thickness maps of the phosphate layer (median), to assess the P_2O_5 reserves in this main layer of the deposit.

However, one of the main limitations of the Monte Carlo simulation, when applied solely to field samples, is that the results are numerical and do not allow for the generation of dispersion maps of grades. This limits the ability to obtain a complete spatial view of the reserves. Therefore, the necessity of using a probabilistic method such as Monte Carlo simulation becomes evident, both to enhance precision and quantify uncertainty in the estimation. The Monte Carlo method has proven effective in assessing uncertainty and refining the results by using virtual samples generated around the mean.

The results obtained through 2D kriging revealed a high variance in the estimation, particularly in the kriging variance map of the thicknesses. This underscores the importance of using 2D estimation, which is preferable to 3D estimation for reserve calculations. 2D estimation allows for better capturing the spatial variability of thicknesses. In contrast, 3D estimation can be influenced by anisotropy, leading to erroneous results, as demonstrated by DMT's estimations. Despite this, 3D visualization offers an advantage in mine planning, providing a useful representation for optimizing operations.

Finally, the combination of ordinary kriging with Monte Carlo simulation offers significant improvements in both technical and economic terms. It enhances the precision of the estimated variables, which ultimately leads to better management and more effective exploitation of mining sites.

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Оцінка невизначеності при підрахунку запасів мінеральних ресурсів із використанням геостатистики й моделювання Монте-Карло

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Мета. Точна оцінка мінеральних запасів є ключовим етапом у плануванні й розвитку гірничих робіт, особливо для фосфатних родовищ. Метою цього дослідження є підвищення точності оцінки запасів на фосфатному родовищі Блед Ель-Хадба (східний Алжир) шляхом поєднання геостатистичних і ймовірнісних підходів.

Методика. Запропонований метод інтегрує звичайне крігінгування (OK) із моделюванням Монте-Карло (MCS) для кращої оцінки основного мінералізованого шару. Звичайне крігінгування застосовувалося на сітці 5 × 5 м для створення карт оцінки вмісту P_2O_5 і товщини шару, що слугували просторовою моделлю родовища. Результати крігінгу (середнє значення й дисперсія) потім використовувались як вхідні дані для моделювання Монте-Карло з 20 мільйонами ітерацій, зберігаючи статистичні характеристики початкових даних.

Результати. Поєднання OK і MCS зменшило коливання, типові для оцінок, отриманих методом крігінгу, і призвело до більш стабільних і надійних значень підрахунку запасів. Використання ймовірнісних категорій P90 і P50 сприяло більш консервативній і достовірній класифікації мінеральних запасів, що покращило оцінку категорій «доведених» і «ймовірних», важливих для економічного обґрунтування.

Наукова новизна. Дослідження пропонує новий підхід до інтеграції крігінгу з великомасштабним моделюванням Монте-Карло для управління невизначеністю при оцінці фосфатних запасів. Метод дозволяє точніше охарактеризувати просторову мінливість і підтримує ймовірнісну інтерпретацію обсягів ресурсів.

Практична значимість. Методологія є цінним інструментом для планувальників шахт і осіб, які приймають рішення, покращуючи впевненість у оцінках запасів і підтримуючи обґрунтовану економічну оцінку. Її застосування до родовища Блед Ель-Хадба демонструє її ефективність і потенціал для ширшого використання у фосфатних проєктах.

Ключові слова: *геостатистика, крігінг, моделювання Монте-Карло, фосфат, оцінка ресурсів, невизначеність*

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