https://doi.org/10.33271/nvngu/2025-4/185

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ALGORITHM FOR SCHEDULING DRIVERS ON INTERCITY ROAD ROUTES: CASE STUDY OF THE SHIFT METHOD

Purpose. To develop a methodology for constructing an optimal work schedule for a group of trucks and drivers, which provides a guaranteed solution to the problem of insufficient productivity of the truck fleet, on the one hand, and a reduction in the shortage of drivers in the case of using a variable method of their work. At the same time, restrictions on the duration of work and rest of drivers are observed. The problem of low productivity is also manifested in the excessive duration of cargo delivery and too long downtime of trucks.

Methodology. The optimal schedule of trucks and drivers was obtained as a result of solving the problems of synchronous routing of several vehicles according to the criterion of minimum mileage and establishing the moments of start/completion of transport and technological operations. These two problems were solved using an improved two-stage algorithm for ordering mixed directed graphs containing cycles. The general ordering algorithm includes linear programming methods at the first stage, as well as "divide and conquer" methods and auxiliary graph colouring at the second stage.

Findings. A methodology for constructing an optimal work schedule for drivers and trucks has been developed, which allows for a variable work schedule for driver teams. A corresponding practical algorithm has been proposed. At the same time, the rules of the European Union regarding the work and rest regime of drivers have been observed. The total duration of cargo transportation is also reduced and the downtime of vehicles necessary for the rest of the drivers assigned to them is minimized. The effectiveness of the algorithm was tested on practical data from the activities of freight carriers. It was recorded that the duration of movement of trucks with cargo per day can be increased by 30.6 %. The overall productivity of trucks can be increased by 23.4 % without violating the European Agreement on the Work of Vehicle Crews.

Originality. For the first time, the problems of routing road freight transportation and optimizing truck and car schedules have been solved synchronously. The tightest schedules of drivers of several vehicles were obtained in accordance with the norms of the European Agreement on the Work of Vehicle Crews on Complex Routes.

Practical value. The developed methodology allows coordinating the operation of freight vehicles on the intercity transportation network, reducing their downtime, idling runs, and also reducing the time spent by driver teams when using the variable method.

Keywords: *freight transportation, truck routing, optimal schedule, mixed graph*

Introduction. The modern situation associated with intercity truck transportation is characterized by a growing shortage of labor resources. This is reflected in the state of road transport: productivity losses are increasing and the cost price is increasing due to idling runs of vehicles. Restrictions and controls under European Agreement concerning the Work of Crews of Vehicles Engaged in International Road Transport (AETR) are becoming increasingly strict. This state of the rules for the work of truck and road transport drivers restrains the development of road accidents, on the one hand, but causes a shortage of drivers, vehicle downtime and an increase in the duration of transport tasks.

The problem of shortage of driver working hours and truck working hours can be successfully solved by developing a driver work schedule in which they can be unattached to specific vehicles. Drivers work in shifts according to newly compiled optimal routes and at the same time the rules and restrictions of AETR are respected. However, combining the problems of routing and scheduling for multi-routes is a problem that is complex in a strong sense. The complexity of the task grows exponentially with the increase in the number of multi-route points [1]. Therefore, the goal of our research is to develop such a methodology that can help us gradually solve a complex problem and get a positive improvement effect.

Literature review. The complex problem of truck and bus routing has been known for almost a decade. Depending on its application and the specific conditions of transportation (long-distance, local, bulk delivery, or small-scale delivery), the problem can be classified in modern research. For example, the most famous review of the varieties of the complex problem of multi-routing and driver scheduling (MRDSP) is made in the paper [2]. However, studies that would enable the successful implementation of a variable work and rest method for drivers in intercity freight transportation remain unknown.

The most successful solution of the MRDSP is presented in relation to intercity bus passenger transportation [3]. This study assumes that bus drivers can transfer control to their partners, get to the desired resting place in a bus or taxi, or stay for a rest at the end of the shift. However, the presence of clear passenger flows makes it impossible to change the bus route scheme, so the problem has strong restrictions here and its solution is not very effective. In addition, the researchers showed that the

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shift work of drivers requires clear synchronization first of all. Most scientific works related to MRDSP cannot be solved by exact analytical methods. Authors usually use heuristic or metaheuristic methods to get the solution. The main goal of the mentioned article is to reduce the shortage of labor in bus transportation by more accurate scheduling. This was successfully achieved. The authors managed to show a result with a 56 % reduction in the need for drivers compared to approaches currently used in practice. However, intercity freight routes do not have a fixed configuration and are variable. This fact has not been taken into account in previous studies.

A common feature of most studies regarding the problem being solved is that the working time of drivers and the operating time of vehicles are separate independent objects of research [4, 5]. The results of the optimization of these objects are subsequently subject to synchronization. But drivers can be assigned to different trucks alternately. Truck drivers can also have arbitrary intermediate stops on the way, depending on the restrictions on the duration of driving. Therefore, it is the routes of trucks together with the schedules of their drivers that are subject to synchronization. Due to the lack of requirements for the synchronization of vehicle routing and truck driver schedules, we do not consider this issue in more detail, but refer to the review works [2, 6].

There are many studies where scheduling for multisystem of the machine-operator type is presented as combinatorial optimization problems [7]. Such problems are exponentially complex. Heuristics and multiheuristics are used. However, such scheduling problems do not take into account the AETR constraints, namely the constraints on the maximum continuous work time and the minimum rest time.

Discrete vehicle routing problems remain very relevant in the transportation industry. These problems consider intermediate points at which trucks need to be loaded [8, 9]. These are well-known problems of multitraveling salesmen, or delivery routes [10, 11]. Recently, such problems have become even more relevant due to the significant increase in the volume of small individual retail deliveries. However, methods for solving such problems remain unable to cover large-scale data with a large number of customers and orders.

Problems of cyclic routing are of particular interest. They occur when similar transport processes are repeated periodically, for example, when servicing regular customers. In such problems, the history of the execution of previous transport cycles is stored, which allows the carrier to improve and execute the driver schedule and vehicle routes many times in the future. Unfortunately, there are known only studies in which cyclic schedules are used only for a single vehicle and a single driver [12]. This type of problem greatly limits its practical value.

The main reason for the need to adhere to driver scheduling restrictions is fatigue, which causes road accidents. However, driver scheduling is only a safety measure [13]. In a broader sense, fatigue monitoring is needed. This should be a feedback loop in scheduling. In these studies, we did not consider such conditions.

There are known methods for planning the use of a truck fleet and separate methods for organizing the work of transport crews [14, 15]. We note that these methods are not interconnected. This does not contribute to high synchronization of processes. The article [16] presents the only connection between these subjects of the transport process, which consists in setting time windows that drivers must adhere to and operating trucks within these windows. The change of drivers in this work is provided only within the framework of tour trips. The criterion for solving the problem here is maximum profit. In this regard, such studies do not indicate ways to comply with the rules of the AETR.

For freight road transportation, it is permissible that short-term rest or driver replacement can be carried out at any permitted points of the transport network, where stopping and/or parking are allowed. But for passenger buses, such actions are not possible. Therefore, integer mathematical programming methods that are suitable for passenger transportation [13, 16, 17] cannot be applied to freight transportation. Using integer programming, only a quasi-optimal schedule for a truck driver with a minimum duration can be found. For example, the research and development of methods for optimizing schedules in [17] is quite accessible and appropriate for different conditions of the problem, but it is unsuitable for our purpose.

Algorithms for building schedules are usually tested on models [18, 19]. Based on this, a conclusion is made about the effectiveness of the algorithm. However, there is no universal method that would prove itself well for all types of tasks. As a rule, when testing a method, permissible errors of evaluation of results within 3.41–14.26 % are used. However, when considering our task, we cannot be sure that we will achieve the specified accuracy. Therefore, it was decided to compare the results of the method with practical data that are massively achieved by carriers.

For routing and scheduling conditions that change rapidly, dynamic planning and routing problems have recently been used [20, 21]. In this case, time windows and resource constraints are used. Operational control strategies are defined and compared through the allowed tolerances. The results show that some tolerance for deviations from the current planned solution usually leads to better solutions. The problem we are solving has, on the one hand, more significant tolerances for the route configuration, and on the other hand, it has strict restrictions on the permissible driver operating modes.

Efficient driver scheduling on intercity automobile routes demands optimization strategies that balance operational performance, energy use, and human factors. Insights from recent railway studies offer valuable parallels. Research has shown that driver behavior – particularly during acceleration and braking – significantly influences energy consumption in electric rail vehicles, emphasizing the role of assistance systems and adaptive scheduling [22]. Route-specific analyses of electric multiple units further highlight how terrain, environmental factors, and operational habits contribute to energy inefficiencies, which can be mitigated through data-driven optimization models [23]. Additionally, quantified studies on locomotive energy use demonstrate the value of predictive modeling in managing energy demand across different driving phases [24]. These findings underscore the potential of integrated, data-based approaches to enhance the efficiency and sustainability of rotational driver scheduling in intercity road transport.

Unsolved aspects of the problem. The main positive feature, which we use to solve this problem is the presence of a larger field of possible solutions, which improves the quality of the resulting solution. According to the latest current research on routing problems and methods for solving them, it is possible to propose a set of routes that differ slightly in terms of their overall efficiency (indicators of total mileage, travel time, and wasted mileage) within the tolerance range. Such equivalent routes can be used with equal success to fulfill a set of transportation orders. However, the different structure of such routes suggests that they are good material for subsequent scheduling problems. In this case, the converse statement cannot be used in general [25, 26]. This fact shapes the direction of our research. First, routing problems with multi-variant interchanges will be solved. After that a heuristic method for optimizing driver schedules will be applied.

Purpose. The purpose of our study is to solve the complex problem of simultaneous routing of several trucks on intercity routes, as well as drawing up driver work schedules, provided that the work and rest of the drivers are synchronized. Achieving this goal will allow for the application of a variable work method, which will lead to increased productivity of trucks on routes and a reduction in the need for hired drivers. To achieve this goal, the following tasks were formulated and solved.

1. Display data on initial transport tasks – orders for cargo transportation by a fleet of trucks. Take into account the presence of cyclical orders (regular customers, the use of several trips with cargo on one order).

2. Apply the method for constructing optimal duration schedules for drivers while observing the AETR rules and the shift work method. Develop the appropriate algorithm.

3. Test the improved algorithm on practical data.

Formulation of a typical operational task. Let us assume that the carrier has trucks M_k at their disposal, where k is the truck type index. The truck type means, first of all, its carrying capacity. The carrier fulfills orders for the transportation of piece universal cargo in packages. Simultaneous reception of J orders for cargo transportation is possible only if all orders are fulfilled no later than a certain allowable period T. And for this to happen, the carrying capacity of the truck fleet must be no less than the specified volume of transport work. The fulfillment of each order can consist of $n \geq 1$ trips, since the amount of cargo in the order can exceed the carrying capacity of any of the available truck types. Let us also assume that the average duration of all trips with/without cargo on a given transport network as well as downtime duration are values that can be predicted with sufficient accuracy. If the customer's transportation volume does not change over time and a truck type is selected for its transportation, then the number of trips between the specified points of departure and destination of cargo is constant. Road transport downtime cannot be avoided due to the cyclical nature of road transport. However, the total length of idle runs and unproductive downtime can be minimized by developing rational routes and the structure of the transport process [27]. By applying a certain sequence of trips with cargo, without cargo, involving the optimal number of vehicles of the required carrying capacity, it is possible to achieve the minimum duration of the transport process so that the condition T_J < T is met; the minimum required number of vehicles, drivers, and conditions 251/2006 will be involved to fulfill all J given orders. This means that the content of the operational task is reduced to building the optimal schedule in terms of duration [28].

Let us represent the transport process of fulfilling orders J as a set of operations $Q = \{1, 2, ..., q\}$. In scheduling theory, an operation is usually called a process or part of the process of servicing a separate order by one technical mean. In this study, the operations of the general transport process are truck trips to the loading point (plus loading), truck trips with cargo to the unloading point (plus unloading), transit trips with cargo between points of the transport network, and idle trips. Each operation $i \in Q$ can be estimated by a positive real number t_i – the average duration of its execution. The moment of the start of the i-th operation is denoted by t_i^b, the moment of the end of the i-th operation is denoted by t_i^e. So, $t_i^e = t_i^b + t_i$. The schedule of execution of the entire process is considered to be uniquely determined if the beginning or end times of all operations are known. Let us introduce the value $a_{ij} = t_j^b - t_i^b$, where i is the operation that is performed by the previous one, and j is the next one by the same vehicle. If the sequence of operations $i \to j$ is strictly defined in this process, then the inequality $a_{ij} \geq t_i$ must hold. Since $a_{ij} \geq t_j^b - t_i^b$, then $-a_{ji} \geq -(t_j^b - t_i^b)$, i.e. $a_{ij} + a_{ji} = 0$. Similarly, we can write the following for the end times of operations, namely $a_{ij} \geq the - t_i^e$, $-a_{ji} \geq -(the - t_i^e)$. And this, in turn, means

$$-a_{ji} \leq t_j^e - t_i^e \leq a_{ij}. \quad (1)$$

Expression (1) is used to check the uniqueness of the schedule of operations of the transport process, in which operation i is performed before operation j. The value a_{ij} has the physical meaning of the time tolerance for observing the schedule of operations, which, in the general case, is subject to inequalities: the beginning of the next operation j cannot occur before the completion of the previous operation i. There may be delays in the execution of operations, in connection with which the inequality in the expression is applied (1).

The negative value $-a_{ji}$ is used in scheduling theory to model the maximum duration of consecutive operations. For example, two operations Nos. 1 and 2 are given, which have durations, respectively $t_1 = 1.5$ hours, $t_2 =$ $= 2.0$ hours. Two fictitious operations S – formal start and F – formal end of the process – are also given, the durations of which are equal to zero. The time tolerance of the start of operation No. 1 is $a_{S.1} = 1$ hour. The time tolerance of the completion of operation No. 2 is $a_{1.2} =$ $= 2.5$ hours. The formal time tolerance of the completion of the process is $a_{F.S} = 3$ hours. The moment of completion of operation No. 1 is calculated as $t_1^e = 0 + a_{S.1} = 1$ hour. The moment of completion of operation No. 2 is calculated as $t_2^e = t_1^e + a_{1.2} = 1 + 2.5 = 3.5$ hours, and $t_F^e = 3.5$ hours. If we use condition (1) to check such a schedule, it turns out that the inequality holds.

$$-a_{F.S} \leq t_F^e - t_S^e;$$
$$-(-3.5) = 3.5 - 0.$$

If the absolute value of $a_{F,S}$ is reduced by 0.5 h, then condition (1) will not be fulfilled and the resulting schedule is invalid. In this way, it is possible to reduce the total duration of the process if there is a possibility to apply a different sequence of operations. The duration of the process T_J of executing J orders is determined by the moment of completion of the last operation from the entire set Q of given

$$T_J = \max_{i \in Q} \{t_i^e\}. \quad (2)$$

If a fictitious process completion operation F is used, then $T_J = t_F^e$. Obviously, the tightest schedule meets the criterion of minimum order fulfillment duration. In such a schedule, the carrier's available resources can be used most efficiently.

It should be noted that the duration of the entire project involves not only runs with cargo and idle runs, but also forced downtime of freight transport, which are associated with loading and unloading operations, as well as with each shift and between-shift rest of drivers, which are stipulated by rules of AETR. In order to reduce non-productive downtime of trucks during the rest of the drivers, carriers use the method of shift work. Under this method a driver who is scheduled to start a shift/inter-shift rest instead of parking the truck hands it over to another driver who is ready to start a new shift. If the driver shift is incomplete, then this is not taken into account, since the drivers can be involved in the transportation as soon as they have rested the appropriate time. The driver can rest and transfer the vehicle to his partner only in the depot.

Thus, the main task of dispatching control of a fleet of M_k trucks and a team of drivers is to make a schedule of its execution according to criterion (2) for each vehicle from the set of M_k, which is involved in the fulfillment of orders. This means that it is necessary to justify the required number of trucks of k type which work simultaneously on the routes and to specify the earliest finishing moments of each trip provided by pre-compiled routes. The duration of continuous driving and the duration of each driver's shift should not exceed the rules.

Method. To display the transportation process of fulfillment of a set of orders on the transport network, the mixed graph ordering methodology [27] was applied. The transport network graph $H(G, V)$ is considered, where G is the set of vertices $G = \{D, g_1, ..., g_s\}$, (of s transport points plus depot D), V is the set of edges that is, connecting paths (it is assumed that all paths of the network are two-way ones (Fig. 1).

Vertex D symbolizes the carrier's depot, where all routes start and end. Weight coefficients $a_{i,j}$, are assigned to each edge of the graph (in Fig. 1 marked above the edges), which are the time tolerance for observing the schedule of operations duration of the trips along the specified communication path in the forward and reverse directions, where i, j are numbers of vertices and $D, g_i, g_j \in Q$.

In this study we also use additional weight coefficients $q_{i,j}$, which are the cargo flows between the vertices g_i and g_j. The volume of cargo flow $q_{i,j}$ is equal to the amount of cargo Q that must be shipped from a point g_i(marked with a "+" sign in Fig. 1), or the same amount of cargo that must be delivered to a point g_j (marked with a "–" sign in Fig. 1). For example, according to the presented scheme, there can be two cargo flows from point

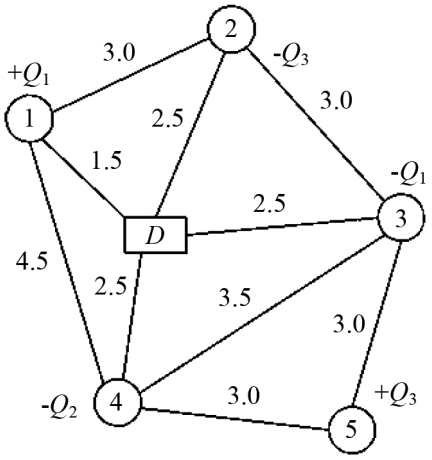

Fig. 1. Scheme of the transport network graph H(G, V): 1, 2,..., 5 – transport point numbers; D – depot; $+Q_1$,...., $+Q_3$ – cargo to be shipped, $-Q_1$,..., $-Q_3$ – demand for cargo to be delivered according to orders; numerical values above the edges are average duration of travel between the corresponding points D, 1,..., 5, in the forward and reverse directions, hours

g_5, transiting through point g_3 and to point g_2, $q_{5.3} = q_{3.2} = Q_3$. The choice of edges to which the value $q_{i,j}$ is assigned depends on the cargo delivery route. If there is no direct connection between any two vertices g_i and g_j, then $a_{i,j} = +\infty$, and at the same time, the planned cargo flow $q_{i,j}$ also does not exist. Graph H also shows potential cargo flows ($+q_{i,j}$; $-q_{i,j}$). If, with $q_{i,j} > 0$ and $a_{i,j} = +\infty$, for any two vertices i, j but $a_{j,\xi} \neq +\infty$ (g_ξ – some transit vertices) and $a_{\xi,i} \neq +\infty$, then the flow $q_{i,j}$ decomposes into two elementary ones: $q_{i,j} = q_{i,\xi} + q_{\xi,j}$. Based on the transport network diagram and given cargo stocks and requirements, it is possible to construct a cargo flow matrix ($q_{i,j}$). Therefore, each non-zero element of the matrix ($q_{i,j}$) corresponds to a pair of adjacent vertices of graph H. Each cargo flow $q_{i,j}$ is a total value consisting of elementary correspondence

$$q_{i.j} = \sum_{m=1}^{Z} q_{i.j.m}, \quad (3)$$

where $q_{i.j.m}$ is a multiple component of the weight of the arc V of graph H, $Z = [Q_{i,j}/q_n]$, q_n is a value limiting the elementary freight flow. One of the features of this model, in contrast to known models of transport flows, is that any vertices of the set of sources G_φ can simultaneously be vertices of the set of sinks G_ψ: $G_\varphi \cap G_\psi \neq \{\varnothing\}$. Such a vertex is vertex g_2 in Fig. 1, where $q_{i.j.m}$ is a multiple component weight of arc V of graph H, $Z = [Q_{i,j}/q_n]$, q_n is the value limiting elementary cargo flow, square brackets mean division with rounding up to a larger whole number.

Applying the shortest route construction method to graph H [26], we obtained graph $A(G, U)$, which reflects all the necessary runs (with and without cargo) that need to be made in order to complete the transportation project as a whole (Fig. 2) and their durations $a_{i,j}$.

Graph A is obtained from graph H by replacing the set of edges V with the set of arcs U. The set of vertices G remains unchanged with the only difference that the depot D point is duplicated as the start and end of all routes. However, it is necessary to specify the number of trucks involved and their type. This may be a suboptimal assumption, but it leads to a multi-variable solution. For

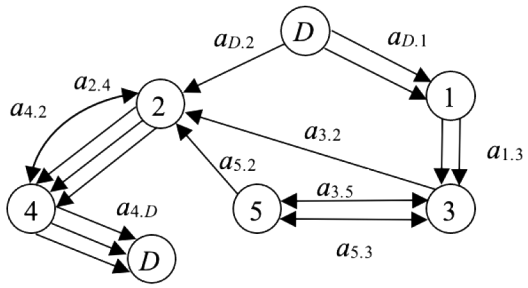

Fig. 2. The graph of the optimal duration of runs on a given transport network of three trucks

example, it is predetermined in Fig. 2 that the routes are carried out by three trucks of the same type. In graph A, there are the same vertices G as in graph H, U is a set of arcs that reflect the mandatory trips for the project.

The total time expenditure of all runs according to graph A is constant and does not depend on the number of trucks involved. Thus, by optimizing the route scheme relative to the initial graph H, it was found that the duration of all runs is 70 hours, regardless of the number of vehicles involved. However, as can be seen from Fig. 2, graph A contains cycles. If there are cycles in the initial graph A, this means that it is impossible to develop a unique driver schedule. And this means that the duration of the project, taking into account the duration of truck downtime, has a sufficiently wide numerical range. So, from the obtained graph A, one can get three variants of optimal routes if there are three trucks.

Option 1: $D-1-3-2-4-D$; $D-1-3-5-2-4-D$; $D-2-4-2-4-D$.

Option 2: $D-1-3-5-3-5-2-4-$ D; $D-1-3-2-4-D$; $D-2-4-2-4-D$.

Option 3: $D-1-3-5-3-2-4-2-4-D$; $D-1-3-5-2-4-D$; $D-2-4-D$.

In order to get rid of this uncertainty, as well as to find the optimal active schedule of transport crews, such transformations of graph A into graph $B(G_1, U, V_1)$ were applied, where G_1 is the new set of vertices with a doubled ones of mutual connections. First, each vertex of graph A, to which a multiple arc from the vertex g_i, is matched, was replaced by multiple vertices, which were introduced according to a number of arcs. Thus, from vertex g_2 of graph A (Fig. 2) to vertex g_4, 4 multiple arcs fit, each of weight $a_{2.4}$. Vertex g_4 was replaced in graph B by vertices $g_{2.1}$, $g_{2.2}$, $g_{2.3}$, $g_{2.4}$. Multiple vertices are connected to each other in pairs by edges. The aim of it is to: a) ensure the possibility of using several trucks for runs between transport points of the same name; b) determine the sequence of runs between transport points of the same name. Also, graph B contains two fictitious vertices – S and F, symbolizing the formal *beginning* and *finishing* of the project. The fictitious vertices are connected by arcs to D. An example of new resulting graph B is shown in Fig. 3.

Arcs and edges of graph B have a weights $a_{i,j} > 0$, except for those arcs that connect multiple vertices. Multiple vertices are connected by edges of weight $a_{i,j} = 0$.

For each arc of B, such a requirement (1) must be met. There t_i^e, t_j^e are the moments of sending the truck, respectively, from the i-th and from the j^{th} transport point. An arc with a negative weight, $-a_{i,j}$ reflects the time limit for order fulfillment (Fig. 4).

Then, if $t_{D1}^e = 0$, $a_{D1.1} = 2.5$ hours, $a_{1.2} = 3.0$ hours, $a_{2.D2} = 2.5$ hours, $a_{2.D1} = 4.5$ hours (the maximum allowed by AETR duration for continuous vehicle driving), and $a_{D2.D1} = 9$ hours (the maximum allowed duration of the driver's shift), then constraint (1) for the path D_1-1-2 is not fulfilled by the duration of continuous driving, and it is fulfilled by the maximum duration of the shift for the path $D_1-1-2-D_2$. If condition (1) is not fulfilled for any edge or arc, then such edge and arc are called conflicting and measures are taken to resolve the conflict. So, in this example, it is necessary to use some intermediate point between points g_1 and g_2 for a short-term rest of the driver, or apply a driver replacement.

Interoperation links marked $-\infty$ are, in fact, non-existent, or are not accepted for consideration.

In the model of the disjunctive (mixed) graph B, a set of edges V is also specified, each $[i,j]$ of which is matched with a couple of values $a_{i,j}$, $a_{j,i}$. If there is connection between g_i, g_j, this means their independence. Cargo delivery to these points can be carried out simultaneously, provided there is a sufficient number of trucks.

If m vehicles can be used for transportation, then they must be used on all routes synchronously. Transportation is carried out sequentially and in parallel. This means that in graph B, you need to select as many chains of operations as there are trucks involved. Each chain starts at vertex S (the start of the entire process) and ends at vertex F (the finish of the entire process). In this version of the problem, we are looking for minimal idle trips with the smallest process time delays. Therefore, the searched chains must pass along the shortest routes that were designed earlier. If the chain reaches some vertex g_y, and then there is no path in graph B with a non-negative or non-zero weight, then the chain goes to vertex F. The transport cycle for this truck will be considered completed, despite the fact that the time reserve for the execution of other, still unfulfilled orders. Also, graph B is supplemented with arcs with negative weights, which have the following meaning:

a) from each point of all routes to the depot – $a_{j.D}$, which means checking the limit of the maximum time of

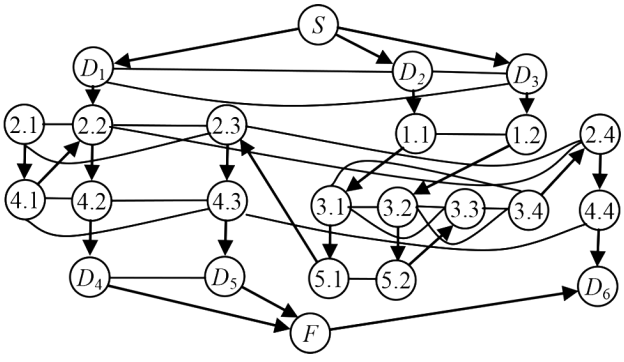

Fig. 3. Initial unordered mixed graph

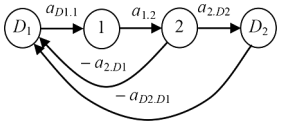

Fig. 4. Graphical modeling of restrictions on the duration of operations

continuous driving, as well as the maximum duration of the change of drivers;

b) from the fictitious vertex F to the fictitious vertex S, which means limiting the duration of the entire project?

Algorithm for an active optimal truck scheduling development using mixed graphs. The algorithm we proposed corresponds to the "branches and bounds" method [29, 30]. To develop the algorithm we have adapted the algorithm widely used to construct an optimal schedule to the conditions of the task we have set. Let us define some terms used. A path in graph $B(G_1, U, V_1)$ is called a sequence of arcs

$$U_k = \left\{\left(g_1, g_i\right), \left(g_i, g_j\right), \ldots \left(g_p, g_s\right)\right\}, \quad (4)$$

there all vertices, which $g \in G_1$ are different, and the initial and final vertices D_k denote the depot for the truck of M_k set. A contour is a closed path in graph B. The sum of the weights of the arcs included makes up the weight of the path. The weight of the path is shown in numerals within the interval $(-\infty, +\infty)$, that is, it is a real number. In this regard, the term contour is used, or a path of positive (or negative) weight. The path of the highest positive weight in graph B, which connects vertices g_i and g_j, is denoted by J_{ij}. If we do not have path from the vertex g_i to the vertex g_j in graph $B(G_1, U, V_1)$ resulting from the elimination of some edges, then $\vartheta_{ij} = -\infty$. In order for the sought schedule $\{t^e_S, \ldots, t^e_i, \ldots, t^e_F\}$ to be unique (there was no time mismatch), condition (1) must be met. The earliest moment of the truck leaving any i^{th} point, which is included in the route for the execution of some order $J_x \in J$, is searched from the ratio

$$t^e_i = \max_{J_x}\left\{0, \vartheta_{S.i}\right\}, \quad J_x \in J, \quad (5)$$

where $\vartheta_{S.i}$ is the path from the fictitious vertex S to the vertex g_i in graph B.

Since the execution of all orders occurs simultaneously, the moment of completion of any J_x order can be found from the expression

$$t^e_x = t^b_x + \vartheta_{S.F}, \quad (6)$$

where t^b_x is the moment of the earliest beginning of J_x order; $\vartheta_{S.D}$ is the path in graph B from the fictitious vertex S to the fictitious vertex F, when the order J_x is executed.

The frame for which equation (5) is true for every vehicles and for all orders should be called active scheduling. Each early starting moment of order J_x is the value t^e_x, it should be defined by equation (6). So, we get a one-to-one correlation between the set of all active schedules originating from graph B and the set $P(B)$ of all graphs that do not include cycles. So, the main idea of algorithm is to find the distribution made by graph $B(G_1, U, V_1)$, which does not contain cycles of positive weight, and therefore edges of V_1. Sequential analysis of options was applied with the selection the set graphs $P(B)$ and looking for the optimal sequence according to value (2). We used the procedure of successive partitioning of $P(B)$ into subsets aimed at organizing the search with respect to avoiding unproductive selection of non-optimal options. The set $P(B)$ is first divided into subsets $P(B_1), P(B_2), \ldots, P(B_h)$, where B_h are graphs obtained from B as a result of replacement or elimination of one or more edges. Edges were replaced by arcs in two versions. Thus, the edge $[i, j]$ can be replaced by the arc (i, j) or (j, i). Next, we divide the subsets of $P(B_h)$ into further subsets. As a result of a finite number of such actions, we get all possible sorting-breaking options. This procedure can be shown in the form of a tree (Z_y, W_y), where Z_y is the set of vertices which denote the initial graphs $B(G_1, U_\varphi, V_\varphi)$, U_φ U, V_φ V, and W_y are a set of arcs, that is, traversal paths. The root of this tree is graph $B(G_1, U, V_1)$. At each step the lower optimality score is calculated

$$f^\circ = \inf\left(\min\left\{ F\begin{pmatrix} t^b_1, t^b_2(G,U'), \\ \ldots, t^b_p(G,U') \end{pmatrix} \right\} \middle| (G,U') \in P(B) \right), \quad (7)$$

which is described in relation (2). We call vertex ZF final if the inequality holds

$$f^\circ(B_1) \le r_m, \quad (8)$$

r_m is current record low score

$$r_m = \min\{f^\circ(B) \mid B_1 \in Z_y\}. \quad (9)$$

If the inequality (8) holds, then we do not consider vertex Z_y in the future, and we consider the path that includes vertex Z_y in the graph (Z_y, W_y) to be false. The procedure ends when all vertices, except the one sought, are discarded.

The content of operations with graph B is as follows. If two vertices g_i, g_j of graph B belong to different routes and are intended for simultaneous execution by different trucks, then there might not be temporal connection between them, and the edge of the unordered model $[i, j] \in V_1$ is eliminated. If the same vertices are performed consecutively, then the edge $[i, j] \in V_1$ is replaced by an arc (i, j) of weight $a_{i,j}$, or an arc (j, i) of weight $a_{j,i}$.

In addition to operating with the main graph $B(G_1, U, V_1)$, we create and operate with the auxiliary undirected graph $C_m(G_m, V_m)$, whose vertices belong to the subset G_m. Therefore, the number of such graphs will be m. The graph is built like this. If the operation α_1 with the main graph is the destruction of the edge $[i, j]$, then graph $C_m(\alpha_1)$ is obtained from graph $C_m(\alpha_0) = (G_m, \varnothing)$ as a result of the addition of the edge $[i, j]$. In case the operation α_1 is presented by the replacement of the edge $[i, j]$ by an arc (i, j) or (j, i) in graph $B(G_1, U, V_1)$, then graph $C_m(\alpha_1)$ can be obtained from graph $C_m(\alpha_0)$, identifying vertices g_i, g_j with one vertex. It is obvious that the number of vertices in graph C_m is not greater than number of available trucks of carrier type M_m, $m = 1..k$. Having calculated the chromatic value of graph $\chi(C_m(\alpha_n))$, we can determine the restriction imposed on the final optimal version of the schedule by the number of trucks fleet

$$\chi(C_m(\alpha_n)) \le M_m. \quad (10)$$

This condition means limited opportunities (to fulfill several orders at the same time) due to the lack of the necessary number of rolling stock, organizationally established sequence of execution, etc.

To carry out an on-purpose search in the graph, the understanding of a conflicting edge, i.e. one for which the condition of inequalities (1) is not fulfilled, is used. Among the conflicting edges of graph $B(G_1, U, V_1)$, one can find the most conflicting ones, i.e. those whose transformation leads to a greater searching effect. To perform this, it is necessary to find the value for each conflicting edge

$$h_{ij} = t_j^e(G_1,U) + \vartheta_j(G_1,U) + a_{i.j} - \vartheta(G_1,U), \quad (11)$$

there $\vartheta_j(G_1, U)$ means the maximum value of the chain in graph B, which starts at the vertex g_j; $\vartheta(G, U)$ is the maximum B graph chain. When one selects the most conflicting edge after parameter of $\min(h_{i,j}, h_{j,i})$. The edge for which the found value is the largest will be called the most conflicting edge.

There are 11 stages of proposed algorithm.

1. Checking if graph $B(\alpha_1) = B(G_1, U)$ includes a cycle of positive weight. Let us consider α_1 as l-th stage of the algorithm. If a contour is present, then go to step 10.

2. Searching for the earliest start of arrival of trucks at each point of the network according to formula (5). Find the latest completion of orders according to formula (6). In case the values we found do not correspond to the directives, then go to stage 10.

3. Searching for the set of conflicting graph edges $B(\alpha_1)$ and the most conflicting one of them. If there are none, go to stage 11.

4. Replacing the conflicting edge $[i, j]$ in column $B(\alpha_{1.1})$ with arc $(i.j)$, in column $B(\alpha_{1.2})$ with arc $(j.i)$; destroying the edge $[i,j]$ in column $B(\alpha_{1.3})$.

5. Creating the corresponding addition graphs $C_m(\alpha_{1.1})$, $C_m(\alpha_{1.2})$, $C_m(\alpha_{1.3})$. Calculating the chromatic number of each of the addition graphs $\chi(C_m(\alpha_1))$.

6. In the case of inequality (10) holds for the graph C_m, then go to stage 10.

7. If loops are present in the graph $C_m(\alpha_{1.z})$, $C_m(_{i,i}) = 1$, then go to stage 10.

8. Do stages 2, 3, 9 in sequence for $B(\alpha_{z+1})$, $B(\alpha_{z+2})$, $B(\alpha_{z+3})$.

9. Calculating the current record r_m among the entered graphs according to formula (7). If $r_m(A(\alpha_{1+z})) \le \le (A(\alpha_{1+z-1}))$, then go to stage 11, go to stage 4 if opposite.

10. State that it is impossible to construct a schedule based on it.

11. The wanted schedule is $\{t_1^e, t_2^e, \ldots, t_F^e\}$. We look for the objective function using expression (2).

Results of applying the algorithm. The developed algorithm was applied, first to the test model (Figs. 1–3). The initial data is taken from the operational information of the transport company of Ukraine. A small enterprise carries out intercity freight transportation in interregional connections and has 5 freight road trains with a nominal carrying capacity of 20 tons at its disposal. Fig. 1 shows the transport network on which the carrier fulfills orders for the transportation of goods. The graph shows an estimate of the average duration of runs along the route sections and a fragment of freight flows. This is an example of the initial pattern of a disordered transportation volume.

In accordance with the direction of freight flows, their size and the load of trucks, such trips with goods should be carried out: $q_{1.3} - 2$ trips; $q_{2.4} - 4$ trips; $q_{3.5} - 3$ trips; $q_{5.2} - 1$ trip. The enterprise did not use a shift management method yet. Drivers are assigned to each of the five trucks personally. Between-shift rest is carried out by drivers only at the depot. Driver schedules were modeled and optimized when applied to specified orders of 2, 3, and 4 trucks. The option with one truck is not applicable, because the total duration of such a project exceeds 56 hours, that is, the allowed working time of one driver. The use of 5 trucks did not change the solution obtained with $M = 4$. According to the algorithm, the unordered mixed graph was transformed into a cycle-free graph. An example of such a graph with $M = 3$ trucks is shown in Fig. 5. The graph shows 3 routes. If we add the necessary time for cargo work and the time for changing drivers to the conditions for the execution of these routes, we will get the drivers' work schedule, the main parameters of which are presented in Table 1.

The simulation was carried out both with and without the shift method. The results of the algorithm show that the shift method of drivers has advantages over the single-trip method, which are manifested in a significant reduction in truck downtime. As can be seen from the table, the duration of forced downtime when using the shift method does not exceed 30.6 % of the duration of the truck's movement (36 hours) and 23.4 % of the duration of the drivers' work. In the absence of the shift method, the duration of idle transport exceeds the duration of movement by 1.4 times. However, the shift method may have such a drawback that the unnecessary number of drivers increases. In this case, it becomes necessary to apply coordination of shifts of a team of drivers driving one vehicle. Thus, the number of drivers required to fulfill the orders given in the example is twice as large as in the single-trip method. The application of the driver schedule optimization algorithm, in general, allowed us to reduce the number of trucks required to fulfill a set of orders by reducing the duration of the project as a whole and the duration of forced vehicle downtime.

The advantages of using the driver scheduling optimization algorithm can be considered by such example more specifically. The same carrier, as in the previous example, carries out interregional cargo transportation in Ukraine. Freight road trains transport cargo on fixed shuttle routes are 1) Lviv – Ivano-Frankivsk; 2) Ivano-Frankivsk – Drohobych; 3) Drohobych – Kamianets-Podilskyi; 4) Kamianets-Podilskyi – Uzhhorod; 5) Ivano-Frankivsk – Uzhhorod; 6) Lviv – Lutsk; 7) Lutsk – Kamianets-Podilskyi; 8) Kamianets-Podilskyi – Drohobych; 9) Drohobych – Lviv. It means that the company serves routes, which are connected to the central point, Lviv City, where the company's truck fleet Depo is located. Return trips of routes were considered with the same parameters as direct ones. The indicators of the selected 9 routes, which are operated without the use of the variable driver work method, but in compliance with the AETR norms, are shown in Table 2.

The total mileage of trucks on the routes is 4,834 km. Significant downtime on simple routes is due to the fact

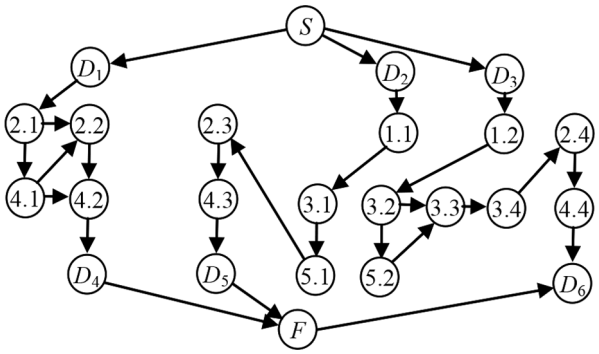

Fig. 5. An example of an ordered graph, initial for building a work schedule for three trucks

Table 1

Results of construction of active work schedule of drivers

Number of trucks	Number of drivers (shift method)	Total duration of the project, shift method/unchanged, hours	Routes	Duration of movement, hours	Duration of work of drivers, shift method/ single method, hours	Duration of idle time of the truck, shift method single method, hours
2	4	47/56	D-1-3-5-2-4-2-4-D	34	42/54	8/20
			D-1-3-5-3-5-2-4-2-4-D	36	47/56	11/20
3	6	32/52	D-2-4-2-4-D	20	24/35	4/15
			D-1-3-5-2-4-D	22	28/52	6/30
			D-1-3-5-3-5-2-4-D	28	32/–	4/–
4	8	34/–	D-1-3-5-2-4-D	28	34/–	6/–
			D-2-4-D	10	12/21	2/11
			D-1-3-5-2-4-D	22	28/–	6/–
			D-2-4-D	10	12/21	2/11

that drivers must apply the rule of minimum rest between shifts. Since the company mainly uses a single method of work of drivers assigned to trucks, the downtime of trucks can last more than double the downtime associated with driver rest. Nine trucks are required to service all routes simultaneously. If there are 9 free trucks, the entire project will be completed in 34.8 hours (determined by the duration of the longest shuttle route Lviv – Kamianets-Podilskyi – Uzhhorod –Kamianets-Podilskyi, taking into account the zero mileage (from the depot to the place of first loading and from the place of last unloading to the depot). However, since the carrier has only 5 road trains that perform such transport tasks, the duration of the entire project increases significantly. This is due to the fact that drivers who have completed the previous task must have a rest between shifts, and the trucks assigned to them, accordingly, are idle. According to the data in Table 2, if there are 5 trucks, the duration of the project will be 119.3 hours. This project will be carried out by 5 trucks and 5 drivers. Drivers have at least 2 inter-shift rest periods of 11 hours each. The duration of idle time and idle and zero trips of trucks will be up to 123 % of the nominal time fund.

If we apply our improved algorithm to scheduled transportation, we can obtain complex routes. Their number depends on the number of trucks involved (Table 3). The example uses 2 road trains. The configuration of routes is formed according to the criteria of minimum mileage and minimum truck downtime, which is associated with driver rest (Fig. 6).

Table 2

Indicators of selected pendulum routes

Route No.	Half route length, km	Duration of one trip with cargo, hours	Idle duration, hours	Duration of zero run, hours	Idle mileage of trucks, km
1	133	2.2	1.5	1.2	30
2	139	2.5	1.1	4.8	17
3	327	5.7	12.6	1.2	34
4	458	9.2	13.5	7.8	22
5	298	4.8	12.5	4.8	45
6	152	2.4	0.75	0.8	6
7	296	5.5	11.2	3.5	11
8	335	5.3	1.6	9.2	28
9	79	1.2	0.5	0.8	1.5

The parameters of the synthesized complex routes Nos. 1 and 2 are shown in Table 2.

The table shows that the total mileage of trucks on the routes is 3,033 km, and the required number of drivers is 6. However, this number of drivers is determined on the grounds that complex routes Nos. 1, 2 are independent. If the work of drivers on these two routes is synchronized, the required number of drivers for two trucks has to be reduced up to 4. In general, the level of transportation organization depends on the number of trucks involved (Fig. 7). Fig. 7 shows indicators of the duration of the transportation project and the number of trucks involved, provided that the algorithm we have improved is used, are inversely proportional to a certain extent. Thus, with the number of vehicles exceeding 5 units, the duration of the entire project does not decrease, since the idle time of trucks increases significantly. This result can be explained by the fact that a fleet of trucks that interact to perform a set of transport tasks is characterized by a synergistic effect. A larger number of trucks does not enhance this effect, but becomes redundant.

The required number of drivers according to the data in Table 2 is 1/3 higher than the same number according to Table 3. This means that the variable driving method not only has a positive effect on increasing the efficiency of truck operation, but also on solving the problem of driver shortage.

Conclusions. The method of driver shift work, which allows increasing the productivity of freight transport in intercity road transportation, can be successfully applied provided that the drivers' work schedules and the routes and schedules of truck trips are synchronized. The algorithm we developed, unlike previous developments, allows solving the complex problem of routing and driver scheduling with a guaranteed optimum based on a two-stage approach and the use of heuristics for the second stage of the algorithm.

The improved algorithm allows one to build complex routes on which idle transport exceeds the duration of

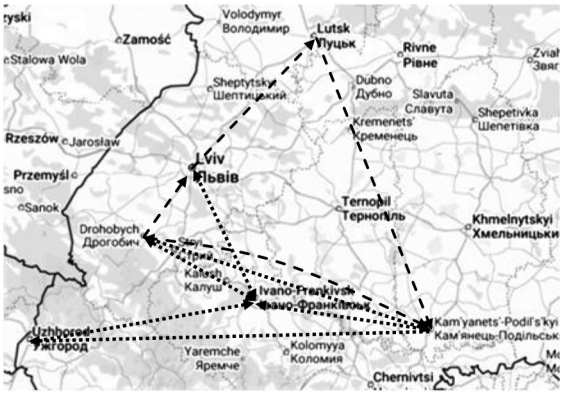

Fig. 6. The scheme of two complex ring routes, which are developed using the algorithm and the variable trip method: route No.1 Lviv – Ivano-Frankivsk – Drohobych – Kamianets-Podilskyi – Uzhhorod –Ivano-Frankivsk –Uzhhorod – Ivano-Frankivsk – Lviv; route No.2 Lviv – Lutsk – Kamianets-Podilskyi – Drohobych – Ivano-Frankivsk – Uzhhorod – Drohobych – Lviv

Table 3

Route indicators developed based on the algorithm using the driver shift method

Route No.	Route length, km	Driving duration, hours	Idle time, hours	Number of drivers	Number of driver changes	Idle and zero mileage, km	Driver change points
1	1,508	25	2.1	3	4	35	Drohobych, Uzhhorod, Lviv
2	1,472	23.3	3.2	3	4	18	Kamianets-Podilskyi, Uzhhorod, Lviv

movement by 1.4 times. The idle mileage of trucks does not exceed 30.4 % of the total.

The efficiency of using a larger number of vehicles is noticeable to a certain extent. After all, a larger number of vehicles complicates the structure of the transport process and, accordingly, leads to organizational downtime.

As for the use of drivers' labor, the number of trucks involved does not have a significant impact on their productivity, since drivers are primarily subject to the EUTR rules. After all, the duration of drivers' work and their rest is essential if the fleet of vehicles involved in transportation is sufficiently productive.

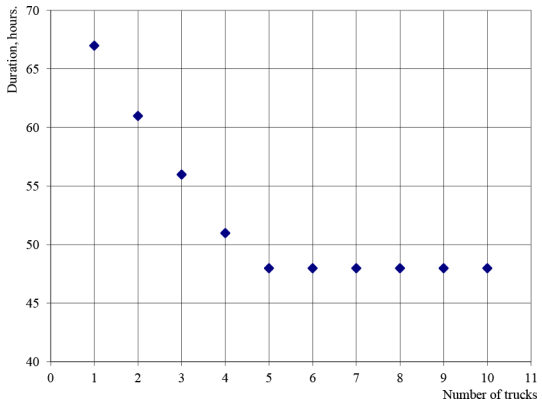

Fig. 7. Dependence of the duration of runs and cycle time on the number of vehicles

Prospective research will concern solving the current problem of driver shortage in Europe and controlling and organizing their safe work.

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Алгоритм планування роботи водіїв на міжміських маршрутах: застосування вахтового методу роботи

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Мета. Розробити методику побудови оптимального розкладу роботи групи вантажних автомобілів і водіїв, що забезпечує гарантоване розв'язання проблеми недостатньої продуктивності вантажного автопарку, з одного боку, та зниження дефіциту водіїв у разі застосування змінного методу їх роботи. При цьому дотримуються обмеження тривалості роботи й відпочинку водіїв. Проблема низької продуктивності проявляється також у надмірній тривалості доставки вантажів і занадто довготривалому простої вантажних автомобілів.

Методика. Оптимальний розклад вантажних автомобілів і водіїв отримано внаслідок розв'язання задач синхронної маршрутизації декількох автомобілів за критерієм мінімального пробігу та встановлення моментів початку/завершення транспортно-технологічних операцій. Ці дві задачі розв'язані за допомогою вдосконаленого двоетапного алгоритму впорядкування змішаних орієнтованих графів, що містять цикли. Загальний алгоритм упорядкування містить у собі методи лінійного програмування на першому етапі, а також методи «розділяй і володарюй» і допоміжного розфарбовування графів на другому етапі.

Результати. Розроблена методика побудови оптимального розкладу роботи водіїв і вантажних автомобілів, що дає змогу скласти змінний розклад роботи бригад водіїв. Запропоновано відповідний практичний алгоритм. При цьому дотримані правила Європейського Союзу щодо режиму праці й відпочинку водіїв. Також скорочується загальна тривалість перевезення вантажів і мінімізується час простою автомобілів, необхідний для відпочинку водіїв, закріплених за ними. Ефективність алгоритму була перевірена на практичних даних з діяльності вантажоперевізників. Було зафіксовано, що тривалість руху автомобілів із вантажем на день може бути збільшена на 30,6 %. Загальна продуктивність вантажних автомобілів може бути збільшена на 23,4 % без порушення Європейської угоди про роботу екіпажів транспортних засобів.

Наукова новизна. Уперше задачі маршрутизації автомобільних вантажних перевезень та оптимізації розкладів автомобілів розв'язані синхронно. Отримані найбільш щільні графіки роботи водіїв декількох транспортних засобів відповідно до норм Європейської угоди про роботу екіпажів транспортних засобів на комплексних маршрутах.

Практична значимість. Розроблена методика дає змогу координувати експлуатацію вантажних транспортних засобів на мережі міжміських перевезень, зменшуючи їхні простої, холості пробіги, а також скорочуючи витрати часу бригад водіїв при застосуванні змінного методу.

Ключові слова: *вантажні перевезення, маршрутизація вантажівок, оптимальний розклад, змішаний граф*

The manuscript was submitted 04.03.25.