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STATIC CONTINUOUS BULK MATERIAL MODEL FOR INCLINED BUNKER SECTION

Purpose. Obtaining an analytical pattern of pressure distribution of bulk material based on the classical Jansen's model for an inclined outlet part of a hopper with an arbitrary cross-sectional shape.

Methodology. The work used a set of research methods, including scientific analysis and synthesis of available technical information regarding the current regulatory and professional approaches to determining the pressure from bulk material in container structures. Computer modeling methods based on the numerical method of structural mechanics – the finite element method – were also used. Analysis of the performance of structural options was carried out using the SCAD design and computing complex (Ukraine). A separate direction in the work was design developments, which included methods for engineering assessment of the accuracy and reliability of the results obtained.

Findings. An analytical expression for determining the vertical pressure of bulk material is obtained, which reflects in a closed form the regularities of its distribution for the case of a straight inclined rigid wall of the outlet part of the hopper container with an arbitrary cross-sectional shape. The pressure value of the bulk material according to this expression quantitatively exceeds the pressure value according to known analytical models. This gives grounds to believe that when loading the hopper structures, a change in the structure of the bulk material occurs, which is described in the literature as its loosening.

Originality. The conducted researches allowed for the first time to establish the regularities of the pressure distribution of bulk material during static operation of a hopper structure with straight inclined walls. The obtained expression is structurally the product of two power functions, in which the exponent is the expressions that reproduce the geometry of the outlet part of the hopper structure and the material of its side walls.

Practical value. The obtained expression allows to calculate the vertical and, if necessary, normal pressure of bulk material for straight inclined walls of hopper structures. It is proved that the pressure increases significantly with increasing its depth, which in the case of unloading the container should lead to the destruction of the static form of laying of bulk material. The developed model is the basis for a more detailed consideration of the characteristics of bulk material, such as the density of laying or the angle of laying.

Keywords: *bulk material, Jansens's model, bunker, hopper, container*

Introduction. Container structures are the main type of building structures for storing bulk materials. Such structures are distinguished by a fairly high diversity in their geometric dimensions and, accordingly, their total volume. Their useful capacity ranges from several tonnes to tens of thousands of tonnes, and their applications include traditional industries (metallurgy, mining, chemical, construction, etc.), agriculture, and transport.

Modern international standards (cited in [1]) are focused on different terminology related to container building structures (Table 1). Such terminological diversity introduces certain complications both in the process of designing container structures and in the assessment of their performance. It will also reduce the effectiveness of scientific research, as experts from different countries often do not understand each other.

Table 1

Intrinsic frequency spectrum of the load-bearing framework of the equipment compartment

Standard	Country	Terms
BS EN 1991-4:2006 [2]	Great Britain	silo, tank
DIN 1055-6:2005-03[3]	Germany	silo bin
ACI 313-97 [4]	USA	silo
ANSI/ASAE EP433 DEC1988 (R2011) [5]	USA	bin
AS 3774-1996 [6]	Australia	container
GB 50322-2011 [7]	Chine	silo
DBN V.2.2-8-98 [8]	Ukraine	silo, bunker

According to the European Standard EN 1993-4-1 [9], all container structures are grouped under one term – silo, which includes all other functional forms of containers – bin, bunker or grain tank. Therefore, from now on, we will also use this terminology.

Structurally, a silo is made in the form of a vertical part intended for direct accumulation of bulk material and an inclined outlet part intended for unloading the container (Fig. 1). For the outlet part, the European standard EN 1993-4-1 [9] uses the term hopper. However, nowadays hoppers are increasingly used as independent container structures in various industries and transport [10] (Fig. 2). Therefore, the issue of correct determination of loads from bulk material for hoppers is becoming increasingly important.

In Ukrainian practice, the main term, which is an indirect analogue of the English term hopper, is a bunker. It is quite clearly rooted in the current standards of Ukraine [7, 11]. Bunker structures are used in many modern industries, such as mining [12], metallurgical, chemical or construction industries. Bunkers are also an indispensable object of most transport hubs and transshipment stations on the railway or in port facilities.

The main use of bunkers is for short-term storage of bulk materials and their periodic supply to the technological process of the enterprise. This is also related to the need to ensure uninterrupted and trouble-free operation of bunkers, as this will directly affect the stability of the production and the enterprise as a whole.

The dominant type of bunker container at the moment is pyramidal-prismatic structures. Quite often they have a low prismatic part, but a developed pyramidal part. Such a con-

a

b

Fig. 1. Steel hoppers:

a *– with suspension bracket; b – with support bracket*

structive solution is very close to the hopper-type structures common in foreign practice. Thus, bunkers are in many ways one of the key sections of many technological processes related to the processing of bulk cargoes of various types.

Literature review. As shown in [13, 14], refinement of approaches to determining the load from bulk material on container elements is one of the priority tasks of the mechanics of granulated solids. This opens up the prospects for further improvement of the design solution itself, increasing its reliability and, at the same time, reducing material consumption [15]. At the same time, it is possible to significantly reduce the level of accidents of container structures and reduce the number of their failures during operation [16, 17]. This will also facilitate the rapid restoration of damaged container structures through the use of special heat-strengthened steels [18, 19] and provide opportunities to control the dynamic properties of such structures [20, 21].

In international standards [2–8], Janssen's continuum model is the basis of the model of bulk material load on the container elements. It describes the static equilibrium of a horizontal elementary layer of bulk material of infinitesimal thickness. As a result, we obtained an expression (1) for the dependence of the vertical axial pressure P_v on the depth of the bulk material z

$$P_v(z)=\frac{\gamma\cdot A}{K\cdot\mu\cdot U}\cdot\left(1-\exp^{-\frac{K\cdot\mu\cdot U\cdot z}{A}}\right), \tag{1}$$

where γ is specific gravity of the bulk material; A – area of the hopper cross-section; K – coefficient of lateral pressure of the bulk material; μ – coefficient of friction of the bulk material against the silo wall ($\mu = \text{tg}\,\varphi$); φ – angle of external friction of the bulk material against the hoper walls; U – perimeter of the hoper cross-section.

The simplicity and physical clarity of this model have made it very popular. Therefore, despite only qualitative confirmation by experimental data, this model became the basis for international standards [2–8].

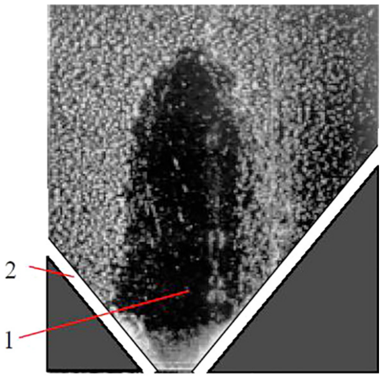

Fig. 2. Bulk material loosening during unloading:

1 – void in the bulk material; 2 – walls of the hopper

The main disadvantage of this model is the hypothesis of a stable relationship between the horizontal P_h and vertical P_v pressure of bulk material on the silo walls through the coefficient K (2)

$$Ph(z) = K\cdot P_v(z).$$

Therefore, to account for the quantitative difference with experimental data, different standards introduce different correction factors and even correction expressions. Their value can be as high as 3, meaning that it is assumed that the difference between the actual pressure of the bulk material and the pressure determined by Janssen's model can vary significantly.

There is also an opinion among experts that the hypothesis of a stable relationship between horizontal and vertical pressure is fulfilled the better, the higher the ratio of the silo height to its characteristic transverse dimension – the hydraulic radius $\rho=\frac{A}{U}$. Yet, there is currently no experimental confirmation of this theory. However, in the case of a hopper with an external configuration that tapers towards the bottom, the ratio of the structure height h to the hydraulic radius will go to infinity

$$\lim_{z\to h}\frac{h}{\rho}\to\infty.$$

This gives grounds to believe that the proposed approach in Janssen's model is close to the actual stress values and, in general, increases the reliability of this problem statement.

The tangential pressure on the silo walls is calculated using the coefficient μ

$$P_\tau(z) = \mu\cdot P_h(z).$$

However, in the case of non-flat walls (e.g. corrugated walls) this expression becomes a separate independent problem.

Among the alternative models, the most well-known is the hydrostatic pressure model, which describes the actual fluid pressure, according to expression

$$P_v(z) = \gamma\cdot z. \tag{2}$$

Reimbert's model is also used in some cases [22]. It is a further development of the hydrostatic model (5), but is not widely used:

$$P_v(z)=\gamma\cdot\frac{z}{\frac{4\cdot K\cdot\text{tg}\,\varphi\cdot z}{D}+1}, \tag{3}$$

where D is the width of the silo loading opening.

All of these models assume a horizontal surface of the bulk material during loading, which is partially stored.

A separate area of research is the development of dynamic models for determining the pressure of the bulk material. All such models aim to build a general differential equation of motion of bulk material during its unloading from a container. A generalisation of the main approaches in this direction is presented in [23]. However, the main difficulty in this case is the correctness of taking into account the phenomenon of bulk material loosening (Fig. 3), which is difficult to describe analytically.

Unsolved aspects of the problem. Janssen's model allows obtaining the static pressure of the bulk material for the case of vertical silo walls. However, the problem remains open for hoppers. International standards [2–8] use indirect approaches to determining the pressure. They consist in the fact that at the first stage, the vertical pressure is calculated according to Janssen's model, and at the second stage, this pressure is transformed using special expressions for inclined hopper walls. Such expressions have only a partial analytical justification and are largely based on empirical data.

Purpose. The purpose of the study is to substantiate the model of static pressure of bulk material in a hopper type container.

To achieve the purpose, the following tasks were formulated and solved:

- to develop a theoretical Janssen's model of the static work of bulk material in a hopper-type container structure;
- to develop a mathematical Janssen's model of the static work of bulk material in a hopper-type container structure;
- based on the mathematical model, to obtain an expression for determining the vertical pressure from bulk material in a hopper-type container structure;
- to compare the obtained solution in quantitative and qualitative terms with solutions based on existing models, in particular the hydrostatic model, Janssen's model and Reimbert's model;
- to use the obtained solution to assess the bearing capacity of a real hopper-type container structure.

The object of study is a hopper with rectilinear walls inclined to the horizon at an angle **α**. The walls are assumed to be absolutely rigid, which makes it impossible for them to deform under the influence of bulk material.

Description of the research methodology. The design scheme is a hopper filled with bulk material (Fig. 4). An elementary horizontal layer of infinitesimal thickness dz is separated from the bulk material. The bulk material is considered to be at static rest. The following forces act on this layer: F – load from the dead weight of the overlying mass of bulk material; Q – reaction from the underlying mass of bulk material; dG – dead weight of the elementary layer of bulk material; dN – reaction from the hopper wall, which can be decomposed into a normal component dN_n and a tangential component dN_τ.

Since the elementary horizontal layer of the bulk material under consideration is in equilibrium, the equilibrium equation is the sum of the projections of all applied forces on the vertical axis Z

$$F - Q + dG - dN \cdot \sin(90 - \alpha + \varphi) = 0. \qquad (4)$$

Let us consider all the components of this equation.

The load from the dead weight of the overlying mass of bulk material is represented as $F = P_v \cdot A$ (where A is the area of the elementary horizontal layer).

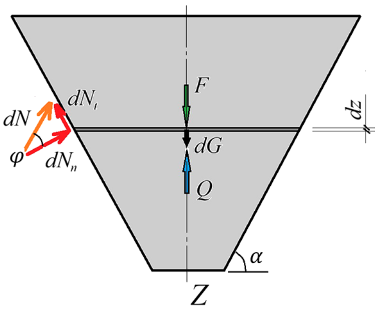

Fig. 3. Continuum model of bulk material in hopper

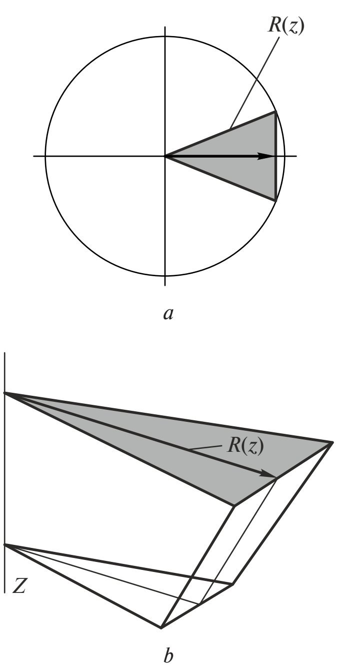

Fig. 4. Segment of hopper:

a – cross-section; b – spatial fragment

The reaction from the underlying mass of bulk material is represented as $Q = F + dF$.

The dead weight of an elementary layer of bulk material is represented as $dG = \gamma \cdot A \cdot dz$.

The reaction from the hopper wall is represented through its components, which are related to the load from the dead weight of the overlying mass of bulk material

$$dN = \sqrt{dN_n^2 + dN_\tau^2} = \sqrt{(K \cdot P_v \cdot U \cdot dz)^2 + (K \cdot \mu \cdot P_v \cdot U \cdot dz)^2} =$$

$$= K \cdot P_v \cdot U \cdot dz \cdot \sqrt{1 + \mathrm{tg}^2 \varphi} = \frac{K \cdot P_v \cdot U}{\cos\varphi} \cdot dz.$$

Substituting all the components in equation (4) after mathematical transformations, we obtain expression

$$\frac{d(P_v \cdot A)}{dz} + K \cdot P_v \cdot U \cdot \frac{\cos(\alpha - \varphi)}{\cos\varphi} - \gamma \cdot A = 0,$$

which can be reduced to expression

$$\frac{dP_v}{dz} \cdot A + P_v \cdot \frac{dA}{dz} + K \cdot P_v \cdot U \cdot \frac{\cos(\alpha - \varphi)}{\cos\varphi} - \gamma \cdot A = 0,$$

or to the form of expression

$$\frac{dP_v}{dz} + P_v \cdot \left(K \cdot \frac{U}{A} \cdot \frac{\cos(\alpha - \varphi)}{\cos\varphi} + \frac{dA}{dz} \cdot \frac{1}{A} \right) - \gamma = 0. \qquad (5)$$

The cross-sectional area of the hopper is represented for the case of a regular polygon with the number of sides n and the radius of the inscribed circle R (Fig. 4) as expression (6), and its first derivative as expression (7)

$$A(z) = n \cdot \mathrm{tg}\frac{\pi}{n} \cdot R(z)^2; \qquad (6)$$

$$\frac{dA}{dz} = 2 \cdot n \cdot \mathrm{tg}\frac{\pi}{n} \cdot R(z) \cdot \frac{dR}{dz}, \qquad (7)$$

where n is the number of sides in the polygonal cross-section of hopper.

The perimeter of the hopper cross-section is represented by expression

$$U(z) = 2 \cdot n \cdot \mathrm{tg}\frac{\pi}{n} \cdot R(z). \qquad (8)$$

For a hopper with rectilinear sidewalls and a loading opening width D, we write

$$R(z)=\frac{D}{2}-\frac{z}{\mathrm{tg}\alpha} \quad \text{and} \quad \frac{dR}{dz}=-\frac{1}{\mathrm{tg}\alpha}.$$

Substituting expressions (6–8) into expression (5), after mathematical transformations, we obtain a linear inhomogeneous differential equation of the first order in the form of expression

$$\frac{dP_v}{dz}+P_v\cdot\frac{4\cdot\left(K\cdot\mathrm{tg}\alpha\cdot\dfrac{\cos(\alpha-\varphi)}{\cos\varphi}-1\right)}{D\cdot\mathrm{tg}\alpha-2\cdot z}-\gamma=0.$$

We also note that this expression does not depend on the geometric shape of the hopper cross section.

The general solution to this equation is known in the form of expression

$$P_v=\exp^{-\int\left(\frac{4\cdot\left(K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}-1\right)}{D\cdot\mathrm{tg}\alpha-2\cdot z}\right)dz}\times$$
$$\times\left(\int\gamma\cdot\exp^{\int\left(\frac{4\cdot\left(K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}-1\right)}{D\cdot\mathrm{tg}\alpha-2\cdot z}\right)\cdot dz}\cdot dz+C\right).$$

Such a solution is closed and requires additional definition of an arbitrary constant after integration.

After integration, we have expression

$$P_v=(D\cdot\mathrm{tg}\alpha-2\cdot z)^{2\cdot\left(K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}-1\right)}\times$$
$$\times\left(\gamma\cdot\frac{(D\cdot\mathrm{tg}\alpha-2\cdot z)^{3-2\cdot K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}}}{2\cdot\left(2\cdot K\cdot\mathrm{tg}\alpha\cdot\dfrac{\cos(\alpha-\varphi)}{\cos\varphi}-3\right)}+\tilde{N}\right).$$

Using the boundary conditions $P_v=0$ at $z=0$, which physically mean that there is no pressure of the bulk material at its upper boundary, we obtain the expression for determining an arbitrary constant

$$C=-\gamma\cdot\frac{(D\cdot\mathrm{tg}\alpha)^{3-2\cdot K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}}}{2\cdot\left(2\cdot K\cdot\mathrm{tg}\alpha\cdot\dfrac{\cos(\alpha-\varphi)}{\cos\varphi}-3\right)}.$$

After performing further mathematical transformations, we finally have an expression for determining the vertical pressure in the hopper in the form of (9)

$$P_v=\gamma\cdot(D\cdot\mathrm{tg}\alpha-2\cdot z)^{2\cdot\left(K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}-1\right)}\frac{(D\cdot\mathrm{tg}\alpha-2\cdot z)^{3-2\cdot K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}}-(D\cdot\mathrm{tg}\alpha)^{3-2\cdot K\cdot\mathrm{tg}\alpha\cdot\frac{\cos(\alpha-\varphi)}{\cos\varphi}}}{2\cdot\left(2\cdot K\cdot\mathrm{tg}\alpha\cdot\dfrac{\cos(\alpha-\varphi)}{\cos\varphi}-3\right)}. \quad (9)$$

The resulting expression is structurally the product of two power functions, where the exponents are expressions that represent the geometry of the hopper structure and the material of its side walls.

Presentation of the main material and scientific results. Let us analyse the resulting expression (9). To allow for a quantitative comparison of the obtained expression with Janssen's model, let us substitute expressions (6) and (8) into expression (1), taking $R(z)=D/2$. In this case, we obtain expression (10) in the form

$$P_v(z)=\frac{\gamma\cdot D}{4\cdot K\cdot\mathrm{tg}\varphi}\cdot\left(1-\exp^{-\frac{4\cdot K\cdot\mathrm{tg}\varphi\cdot z}{D}}\right). \quad (10)$$

The graph (Fig. 5) shows the curves of the vertical pressure distribution of bulk material for the known models (2, 3, 10) and the obtained model (9). For the sake of concreteness, the following parameters were taken: $\gamma=800$ kg/m^3; $D=10$ m; $K=0.333$; $\varphi=30°$; $\alpha=50°$.

As can be seen for the obtained distribution the pressure gradually increases, exceeding the hydrostatic pressure in its value. Such an increase in pressure, in our opinion, is due to a structural change in the properties of the bulk material during loading. The maximum pressure, according to the theoretical model, should be observed in the outlet zone. However, in this zone, the horizontality of the layer is disturbed so much that in practice, hollow voids are formed (Fig. 3). In this case, the bulk material seems to be "squeezed out" of the horizontal layer.

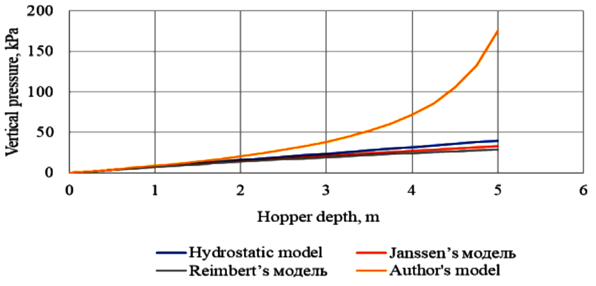

Fig. 5. Vertical pressure of bulk material in hopper

The size of the loosening zone of the bulk material in the hopper depends on the physical and mechanical characteristics of the bulk material and requires additional study.

As a confirmation (albeit indirect) of the validity of the proposed model of bulk material, we present information on the damaged hopper at one of the industrial enterprises of

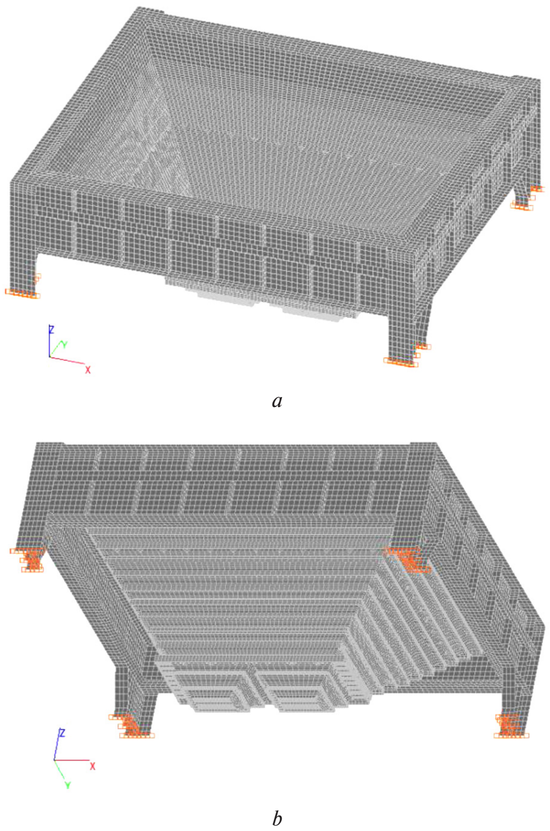

Fig. 6. Calculation model of an industrial pyramidal hopper: a – inner view; b – external view

Ukraine. The hopper is designed to store sinter. The structure has overall dimensions of 18 × 15 m in plan with a total height of about 9 m. The hopper is designed as a prismatic structure with a 43° angle of inclination of the funnel faces to the horizon. The calculation model of the hopper is shown in Fig. 6.

It is a combined finite element scheme. The bunker wall, horizontal stiffening ribs of the bunker, and elements of the bunker beams are modeled by plate finite elements of universal isoparametric type 44. Vertical stiffening ribs, as well as elements of the rigid transverse frame are modeled by rod finite elements of general type 5. The connection of different types of finite elements is provided for by nodes without additional contour nodes.

The boundary conditions are rigid fastening of the vertical load-bearing columns of the bunker overpass at the point of their support on the reinforced concrete supporting platform. In Fig. 6, the fastening is highlighted in orange. The scheme of the applied pressure from the bulk material is shown in Fig. 7.

The hopper was designed for loading according to the hydrostatic pressure model. According to current views, this pressure is considered to be the maximum possible in a hopper and should ensure its design bearing capacity. However, in practice, the hopper structure suffered serious damage that required immediate repair and reinforcement work (Fig. 8). This suggests that the actual loads from the bulk material were higher than those calculated by the hydrostatic model.

This situation was modeled by the finite element method [24, 25] based on the SCAD design and computing complex (Ukraine). Table 2 presents the main results obtained – the distribution of maximum equivalent stresses according to Mises for the case of the hydrostatic model of pressure of bulk material and the pressure model proposed in this work. The numbering of the stiffening ribs is given from top to bottom. It is seen that in the second case the stress level exceeds the permissible strength level of the steel (230 MPa), from which the bunker is made. This confirms the emergency nature of the situation with this bunker, which occurs in operational practice.

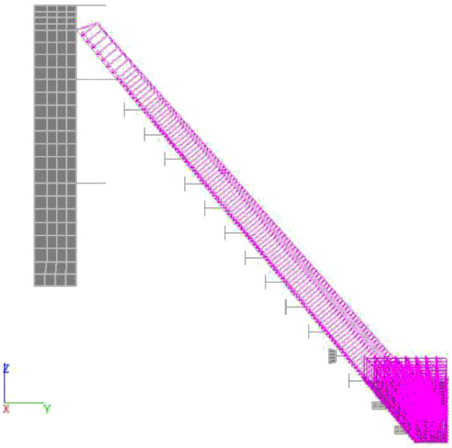

Fig. 7. Scheme of loading from bulk material (fragment of calculation scheme)

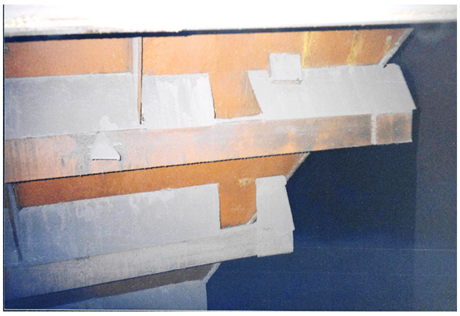

Fig. 8. Reinforcement of an industrial pyramidal bunker

Also, the facts of excess pressure from bulk material for inclined parts of container structures of the values laid down in the current standards [2–8] were established in the course of a number of modern experimental studies [26]. Such research studies were carried out both on models of container structures and on real objects. In work [27], similar experimental data were obtained for the bottom of container structures, which can be considered as a straight wall of a hopper funnel with a minimum angle of inclination to the horizon. According to these works, the experimental values of pressure from bulk material obtained exceed the pressure values according to known models in the standards [2–8] by up to 2 times.

Table 2

Stressed state of the sinter hopper

Structure element of bunker	Stresses (MPa) for model of pressure of bulk material	
	hydrostatic	authors'
Long side 18 m		
Bunker beam	182	209
Rib No. 1	103	154
Rib No. 2	119	165
Rib No. 3	127	197
Rib No. 4	139	221
Rib No. 5	152	243
Rib No. 6	143	250
Rib No. 7	137	268
Rib No. 8	124	274
Rib No. 9	120	284
Rib No. 10	112	297
Rib No. 11	98	319
Rib No. 12	84	322
Rib No. 13	76	343
Rib No. 14	65	320
Rib No. 15	61	306
Rib No. 16	54	254
Short side 15 m		
Bunker beam	162	221
Rib No. 1	76	111
Rib No. 2	98	124
Rib No. 3	101	132
Rib No. 4	118	154
Rib No. 5	138	168
Rib No. 6	120	183
Rib No. 7	117	201
Rib No. 8	106	211
Rib No. 9	95	224
Rib No. 10	88	254
Rib No. 11	79	287
Rib No. 12	71	302
Rib No. 13	63	337
Rib No. 14	57	306
Rib No. 15	50	280
Rib No. 16	43	259

This also indirectly confirms the reliability of the bulk material pressure model proposed by the authors for structures with straight inclined hopper-type walls.

Conclusions. The developed Janssen's continuum model of static equilibrium for the case of a hopper-type container structure with horizontal side faces allows obtaining an analytical expression for determining the vertical pressure in bulk material. The resulting solution is the product of two power functions. The exponents of these functions reflect the geometry of the hopper structure and the material of its side walls.

A quantitative comparison of the obtained solution with the existing hydrostatic pressure model, Janssen's model and Reimbert's model revealed that the vertical pressure increases asymptotically along the depth of the bulk material. At the same time, its value exceeds the hydrostatic pressure. Most likely, this situation is the main factor in changing the structure of the bulk material during its loading into the hopper. However, this issue requires further study.

The experience of operating large-sized hoppers indicates their underestimated bearing capacity, which indirectly confirms the reliability of the model of bulk material and the nature of its pressure distribution on structural elements proposed by the authors.

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Статична континуальна модель сипучого матеріалу для похилої частини бункера

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Мета. Отримання аналітичної закономірності розподілу тиску сипучого матеріалу на основі класичної моделі

Янсена для похилої випускної частини бункерної ємності з довільною формою поперечного перерізу.

Методика. У роботі використовувався комплекс методів дослідження, що включає науковий аналіз і синтез наявної технічної інформації стосовно чинних у світі нормативних і фахових підходів до визначення тиску від сипучого матеріалу в ємнісних конструкціях. Також застосовувалися методи комп'ютерного моделювання на базі чисельного методу будівельної механіки – методу скінчених елементів. Аналіз роботи конструктивних варіантів проводився із використанням проєктно-обчислювального комплексу SCAD (Україна). Окремим напрямом у роботі були конструкторські розробки, що включали методи інженерної оцінки точності й достовірності отриманих результатів.

Результати. Отримано аналітичний вираз для визначення вертикального тиску сипучого матеріалу, що відображає в замкненому вигляді закономірності його розподілу для випадку прямолінійної похилої жорсткої стінки випускної частини бункерної ємності з довільною формою поперечного перерізу. Величина тиску сипучого матеріалу, відповідно до цього виразу, перевищує в кількісному відношенні величину тиску за відомими аналітичними моделями. Це дає підстави вважати, що при завантаженні бункерної конструкції відбувається зміна структури сипучого матеріалу, яка описується в літературі як його розпушування.

Наукова новизна. Проведені дослідження дозволили вперше встановити закономірності розподілу тиску сипучого матеріалу при статичній роботі бункерної конструкції із прямолінійними похилими стінками. Отриманий вираз структурно являє собою добуток двох ступеневих функцій, в яких показником є вирази, що відтворюють геометрію випускної частини бункерної конструкції та матеріал виготовлення його бічних стінок.

Практична значимість. Отриманий вираз дозволяє розрахувати вертикальний і, за необхідності, нормальний тиск сипучого матеріалу для прямолінійних похилих стінок бункерних конструкцій. Доведене суттєве підвищення тиску у міру збільшення глибини бункера, що у випадку розвантаження ємності має призводити до руйнування статичної форми укладки сипучого матеріалу. Розроблена модель Янсена є основою для більш детального врахування характеристик сипучого матеріалу, таких як щільність укладки або кут укладки.

Ключові слова: *сипучий матеріал, модель Янсена, бункер, хопер, ємність*

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