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ANALYTICAL STUDIES ON DYNAMIC PROPERTIES OF INDIRECT OIL HEATERS

Purpose. To investigate the linearised mathematical model of a heater and identify the potential for lowering the order of the heater's transfer functions for further synthesis of control systems and improvement in the efficiency and safe operation of a track heater.

Methodology. After obtaining a linearised mathematical model of a heater with a transitory heat medium, the mathematical model equations are written in a standard format, followed by their Laplace transformation at zero initial conditions. With the help of the program product, an analytical study of the linearised model was performed, after which the Henkel singular values were used to simplify the order of the mathematical model transfer function from the seventh to the third and an assessment of the calculation accuracy loss after the function's approximation was conducted, which showed that the accuracy loss was not significant.

Findings. Having studied the linearised mathematical models of heaters with a transitory heat medium, it was determined that the transfer functions in the mathematical model are of the seventh order, which significantly complicates the creation of an automated heater control system. As a result, a simplified mathematical model with transfer functions of the third order is obtained, which significantly reduces the calculation complexity on modern microprocessors.

Originality. Linearised mathematical models of an indirect heater were developed, based on the assumption of insignificant deviations of the output values from their baseline values. The studies of the linearised model showed that the transfer functions of the oil track heater are of the seventh order. Using Henkel singular values, it was possible to achieve transfer functions of the third order, which would reduce the complexity of creating automated control systems for indirect oil and gas condensate heaters.

Practical value. Modern digital control systems for the oil heating process are created. Using the method for reducing the order of transfer functions by using Henkel singular values to three significantly facilitates the integration of these models into microprocessor-based controls. This helps to improve the efficiency and reliability of automatic control systems, ensuring a stable and safe oil heating process, which is critical for the smooth operation of technological processes.

Keywords: *linearised model, reduced model, Henkel singular values, indirect heaters*

Introduction. The oil is preheated in indirect (track) heaters before being supplied to the separation units or to the pipeline system (at low temperatures). Such heaters are constructed in the form of a bath filled with water, with heat exchanging pipes through which the working substance (oil) flows.

Natural gas combustion products pass through the combustion liners in the combustor. The efficiency of the indirect heater depends heavily on the automatic control system for the oil heating process. In most cases, to maintain the technological process at a given level, single-circuit control systems are used, but they are unable to provide the required control process indicators due to the presence of internal connections in the object (heater).

Indirect oil heaters are used in technological schemes to heat oil to reduce its viscosity and for heating gas before reducing the pressure to prevent the formation of hydrates [1, 2]. Indirect heaters are constructed in the form of a container (bath) filled with water that washes the heat exchanging pipes where the working substance (oil) enters. The water is heated by means of combustion liners through which the combustion products resulting from burning natural gas in the combustor pass.

The use of a water bath contributes to uniform heating of the working substance and, unlike direct heating, prevents the appearance of hot spots that can cause degradation of the process equipment and can lead to ruptures, explosions and leaks.

The oil leaving the heater must have a certain temperature, which is maintained by an automatic control system. An effective heating process control system can only be created based on a mathematical model that describes the dynamics of heat transfer from the combustion liners through the transitory heat medium to the service environment.

Literature review. Water bath track heaters are widely used. They are used to heat oil and gas. Due to the low power efficiency of heaters with a transitory heat medium, various measures have been proposed to improve their performance [1]. One of them is the optimisation of the heater geometry described in [1], for example, increasing the diameter and length of the combustion liner, insulation thickness and other measures described in the article, which increased the efficiency of heaters from 44 to 70 % and reduced fuel consumption. The results of mathematical modelling were validated practically at municipal gas distribution stations.

To prevent the formation of hydrates at the gas supply pressure reducing stations and to ensure normal gas flow through the pressure reducing stations [2, 3], the gas is preheated in an indirect intermediate heater. Energy losses caused by excessive heating of the liquefied gas or insufficiently optimised heating systems highlight the need for improved heating systems with a transitory heat medium. To increase efficiency, it is recommended to use a heat exchanger that receives heat from combustion gases [2] or to improve the heating control system by analysing thermodynamic equations and considering the deviation of natural gas from the ideal gas state [3].

The suggested system for regulating gas pressure within a gas pipeline [4] allows increasing power efficiency by 28.54 %, maintaining optimal gas pressure, and reducing fossil fuel consumption by optimising gas pressure regulation and the process of gas release from the system. The suggested system also allows achieving better environment performance by reducing pollutant emissions, namely 55.8 % less CO_2 and 55.56 % less CO.

In [5, 6], an evacuated tube solar water heater integrated with nanofluids was experimentally investigated. Since water-based nanofluids have been tested earlier, we will focus on oil-based nanofluids here.

Therefore, most papers analysing various aspects of heaters' operation consider the search for ways to save fuel (commercial) gas and reduce harmful emissions into the atmosphere.

Thus, in [7], research was carried out to determine the effect of ultrasound on heat transfer in a thermal battery. Experimental modelling has shown that ultrasonic vibration has a positive effect on increasing the heat transfer rate, which is more evident at lower liquid flow rates and lower heater power levels.

The presence of a large water bath in indirect oil and gas heaters results in low thermal conductance of the working substance heating system. To increase the heat transfer coefficient, the effect of ceramic nanotubes on the efficiency of indirect gas heaters was studied in [8]. The authors concluded that with a volume fraction of nanotubes of 0.025, the efficiency of the heat transfer process increased by 48 %.

For indirect water bath heaters in city-type gas distribution stations, the paper [9] proposes an optimal control system to obtain an acceptable temperature at the heater outlet based on the gas inlet conditions to the city station. The controller calculates the hydrate formation temperature in terms of the inlet gas pressure and transmits this information to the heater burner to adjust the fuel consumption. The use of the proposed system reduces fuel consumption and greenhouse gas emissions, while increasing system efficiency.

The twisted pipes in water bath heaters suggested in [10] make it possible to reduce the length of the gas pipe by 12.5 to 25 %, depending on the twist ratio, compared to the straight configuration. The article also found that twisted pipes increase the heat transfer coefficient and pressure in the system depending on the twist ratio.

Article [11] describes the study of the corrosion process in indirect heaters by assessing the mechanical properties of the heater, X-ray diffraction, scanning electron microscopy, etc. The causes that led to the corrosion of the pipe walls and the heater body are determined. As a result, the importance of maintaining the required level of working substance in the indirect heater bath always was identified.

The use of turbulators in water heaters described in [12] allowed increasing the fluid flow rate inside the spool, which increased the average Nusselt number and improved heat transfer by 20 %. This can result in a smaller heater size or reduced fuel consumption for heating the raw material. However, the authors of the article do not consider external disturbing factors that can affect the operation of turbulators by changing the temperature of the fluid.

The process of heating the transitory medium of an indirect heater using an electric heater is described in [13, 14]. Its effectiveness was also assessed. However, this method does not consider the fact that it is possible to use associated gas, which is released during oil extraction from a well, to heat the oil. These heaters can be further combined with indirect heaters to continue heating in times of power failure.

Publication reviews show that researchers pay much less attention to oil track heat installations, which is explained using associated gas to heat the service environment. Associated gas is not accounted for, and its excess is flared.

Thus, **the scientific problem remains relevant** to investigate ways to reduce the order of the transfer functions of the linearised mathematical model of a heater for both oil and gas condensate, to reduce the amount of modern computerised systems calculations and increase the efficiency of control systems for a heater with a transitory heat medium.

The purpose of the paper is to determine the method for optimising the linearised mathematical model of an indirect heater by reducing the order of transfer functions, to perform an analytical study of the simplified mathematical model and to estimate the approximation inaccuracy.

Procedure. In [15], a mathematical modelling of a heater with a transitory heat medium was performed using the heat balance equation based on a set of assumptions related to various physical parameters of the heater elements. After that, a linearised model of the heater was synthesised in [16].

The functional chart of the heater as an automatic control object, which is described in more detail in [17], is shown in Fig. 1, where the following notations are adopted: T_n^{in} – oil temperature at the inlet to the heater; G_n – mass oil consumption entering the heater; U – controlling influence on the execution unit installed on the natural gas supply line to the combustor. These quantities form a group of input values, including T_n^{in} and G_n – the perturbance.

The group of output values includes the following process parameters: T_n^{out} – oil temperature at the outlet of the heater (adjustable value); T_w – water temperature in the bath; T_{dg} – flue gas temperature in the duct.

The mathematical models of the heater obtained in [15] were linearised [16], which made it possible to obtain nine transfer functions (Fig. 1). Analytical expressions of such functions are quite cumbersome. Therefore, the study of the dynamic properties of an oil heater at small deviations of the output values from their baseline values is made on the example of a typical heat exchanger. The necessary parameters for the calculation are shown in Table 1.

The calculation of the heater's mathematical model parameters was performed using software created in the MatLab environment [18]. Therefore, in Table 1, in the "Machine variable" column, we provide the designations of the relevant values used in the calculations. The result of calculating the initial model parameters from [16] is shown in Table 2.

The parameters of the linearised models [16] were calculated using a program created in MatLab. The base values of the relevant variables are shown in Table 3.

Fig. 2 shows the static characteristic $G(U)$ of the execution unit (EU) [16], which describes the fuel gas consumption through the final control element (FCE) depending on the degree of its opening.

A linearised dependence $G(U)$ (Fig. 2) was chosen, which is described by the following formula

$$G(U) = a_0 + a_1 U.$$

The coefficients a_0 and a_1 are calculated on the assumption that the execution unit provides fuel gas consumption in the range of $G \in [G_{min}; G_{max}]$, where $G_{min} = 0.0129\,\frac{kg}{s}$; $G_{max} = 0.093\,\frac{kg}{s}$, and the degree of opening of the final control element is $U \in [0.2; 1.0]$. Based on the above-mentioned figures G and U, we obtained the following values of the coefficients: $a_0 = 0.0113$; $a_1 = 0.00806$.

Using the data presented in Tables 1–3, the parameters of the linearised models were calculated using software developed in the MatLab environment (Table 4). The values of the parameters of the linearised models [16], which are presented in Table 4, are the initial data for calculating the numerical values of the parameters of the transfer functions [16].

The analysis of the obtained transfer functions showed that their characteristic equations are of the seventh order.

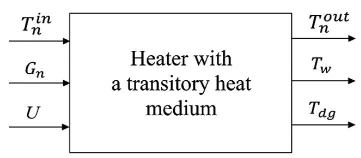


Fig. 1. Chart of an intermediate heater with perturbing, control and output parameters

Table 1

Initial data for the calculation

Parameter	Symbol	Machine variable	Unit of measurement	Value
Product weight in the pipe coil	M_n	Mn	Kg	$5.35 \cdot 10^3$
Average heat-absorption capacity of oil	C_n	Cn	$\frac{J}{kg \cdot K}$	2,038
Heat transfer coefficient from wall to environment	C_{sz_n}	Csz_n	$\frac{W}{m^2 \cdot K}$	562
Pipe coil exchange surface area	F_z	Fz	m^2	254
Pipe coil exchange weight	M_{zn}	Mzn	kg	$7.996 \cdot 10^3$
Heat-absorption capacity of pipe coil wall	c_z	cz	$\frac{J}{kg \cdot K}$	460
Heat transfer coefficient from water to wall	C_{w_sz}	cw_sz	$\frac{W}{m^2 \cdot K}$	175
Water weight	M_w	Mw	kg	$9.733 \cdot 10^4$
Water heat-absorption capacity	c_w	cw	$\frac{J}{kg \cdot K}$	3.841
Heat transfer coefficient from flue to water	C_{sdk_w}	Csdk	$\frac{W}{m^2 \cdot K}$	473
Flue surface area	F_{dk}	Fdk	m^2	28.5
Maximum calculated combustion temperature	$T_{\max}$	Tmax	K	$2.281 \cdot 10^3$
Preheater flue volume	V_{zdg}	Vzdg	m^3	3.013
Average flue gas density	ρ_{cpdg}	ro_cpdg	$\frac{kg}{m^3}$	0.275
Heat-absorption of combustion products at incinerator outlet	$c_{dg}^{\Uparrow}$	Cdg_in	$\frac{J}{kg \cdot K}$	1.321
Heat-absorption of combustion products at flue gas duct outlet	c_{dg}^{T}	Cdg	$\frac{J}{kg \cdot K}$	1.112
Fuel/air ratio	α	alpha	–	1.1
Mass flow of fuel gas	G	Bm	$\frac{kg}{s}$	0.0161
Amount of combustion products per 1 kg of fuel gas	G_{dg}	Mdg	–	19.05
Calorific heat value of fuel gas	q_g^H	q_dH	$\frac{J}{kg}$	$4.956 \cdot 10^7$
Blackness of exhaust duct wall surface	ε_c	Eps_c	–	0.85
Radiation coefficient of absolutely black body	C_0	C0	$\frac{W}{m^2 \cdot K^4}$	5.67
Blackness of flue gases at average gas temperature	ε_{dg}	Eps_dk	–	0.171
Absorption capacity of gas at surface temperature $T_{sdk.}$	A_{dg}	Adk	–	0.37
Flue weight	M_{dk}	Mdk	Kg	$2.239 \cdot 10^3$
Wall heat-absorption capacity	c_{dk}	Cdk	$\frac{J}{kg \cdot K}$	460
Coefficient that accounts for heat distribution in fire box from flare	ψ	Psi	–	1.2

As an example, Table 5 shows the parameters of the transfer functions $W_{11}(s)$, $W_{12}(s)$ and $W_{13}(s)$, which characterise the dynamics of the heater through the following channels of input transmission: "oil inlet temperature – oil outlet temperature of the heater", "mass consumption rate of oil inlet – oil outlet temperature of the heater" and "control action of the execution unit – oil outlet temperature of the heater" [16].

It should be noted that the numbering of the parameters of the transfer functions $W(s)$ (with the corresponding indices) in Table 5 corresponds to the presentation in $W(s)$ [19].

The high order of the transfer functions $W_{ij}(s)$ of the oil track heater creates difficulties in the synthesis of an automatic control system and the subsequent practical implementation of such a system on industrial controllers. Therefore, it is reasonable to approximate the transfer functions $W_{ij}(s)$ by lower-order transfer functions [19].

Reduction of heater transfer functions. The dynamic properties of objects described by linear or linearised mathematical models can be characterised in state space.

The Henkel singular values make it possible to construct a diagram of energy distribution by state of the controlled object. [20, 21].

The method, which is based on Henkel's singular numbers, allows you to build a diagram showing the distribution of energy by state. The number of dominant states in the diagram determines the order of the reduced model.

So, if a linear (linearised) model of a system or object is represented in the form of a transfer function, then it can be represented by the following differential equation

Table 2

Non-linearised model parameters

Parameter name	Formula	Machine variable	Unit of measurement	Numerical value
Non-linearised model – 1 (oil)				
Time constant	$\tau_n = \dfrac{M_n \cdot c_n}{C_{sz-n} F_z}$	tau_n	C	76.3815
Transfer ratio	$k_{n,1} = \dfrac{c_n}{C_{sz-n} F_z}$	Kn1	s/kg	0.0143
Linearised model – 2 (oil)				
Time constant	$\tau_{sz} = \dfrac{M_z \cdot c_z}{(C_{w\_sz} + C_{sz-n}) F_z}$	tau_sz	C	19.6485
Transfer ratio	$k_{wcp} = \dfrac{C_{w\_sz}}{C_{w\_sz} + C_{sz\_n}}$	Kw	Primary units	0.2374
Transfer ratio	$k_{ncp} = \dfrac{C_{sz\_n}}{C_{w\_sz} + C_{sz\_n}}$	Kn	Primary units	0.7626
Linearised model – 3 (water)				
Time constant	$\tau_w = \dfrac{M_w \cdot c_w}{C_{sdk\_w} \cdot F_{dk} + C_{w\_sz} F_z}$	tau_w	C	6,453.3282
Transfer ratio	$k_{sdk} = \dfrac{C_{sdk\_w} \cdot F_{dk}}{C_{sdk\_w} \cdot F_{dk} + C_{w\_sz} F_z}$	Ksdk	Primary units	0.2327
Transfer ratio	$k_{sz} = \dfrac{C_{w\_sz} \cdot F_z}{C_{sdk\_w} \cdot F_{dk} + C_{w\_sz} F_z}.$	Ksz	Primary units	0.7673
Non-linearised model – 4 (flue gases)				
Transfer ratio	$k_1 = C_{dg\Uparrow} \cdot B0_{\max}$	k1	$\dfrac{W}{K}$	341.2913
Transfer ratio	$k_2 = 0.5 \cdot 10^{-8}(\varepsilon_c + 1) C_0 F_{dk} \cdot \varepsilon_{dg}$	k2	$\dfrac{W}{K^4}$	2.5380*e*–07
Transfer ratio	$k_3 = 0.5 \cdot 10^{-8}(\varepsilon_c + 1) C_0 F_{dk} \cdot A_{dg}$	k3	$\dfrac{W}{K^4}$	5.4916*e*–07
Transfer ratio	$k_4 = 2.1 \cdot F_{dk}$	k4	m^2	59.8500
Transfer ratio	$k_5 = BT^{H}_{g\max}$	k5	W	4.7908*e*+05

Table 3

"Base" values of variables in linearised model

Title	Symbol	Machine variable	Unit of measurement	Base value
Oil inlet temperature	$T^{(in)}_{n,0}$	Ton_in	Kelvin	288
Oil outlet temperature	$T^{(out)}_{n,0}$	Ton_out	Kelvin	315.5
Mass oil consumption	$G_{n,0}$	G0n	kg/s	24.2
Outlet flue gas temperature	$T_{dg\Uparrow,0}$	T0dg	Kelvin	505.5
Flue wall temperature	$T_{sdk,0}$	T0sdk	Kelvin	420
Water temperature	$T_{w,0}$	T0w	Kelvin	355.5
Control signal	U_0	U0	–	0.6

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{j=0}^{n} b_j \frac{d^j u(t)}{dt^j}. \tag{1}$$

In Equation (1), the derivatives on the left and right sides have the same order *n*. In the case when $m < n$, where *m* is the order of the derivative of the variable $u(t)$, the coefficients $b_{m+1}, b_{m+2}, \ldots, b_n$ are set to zero.

Equation (1) is matched by a system of differential equations, each of which is a first-order differential equation [21], i. e.

$$\frac{dx_{i-1}(t)}{dt} = x_i(t) + \beta_{i-1} u(t), \quad i = \overline{2,n}; \tag{2}$$

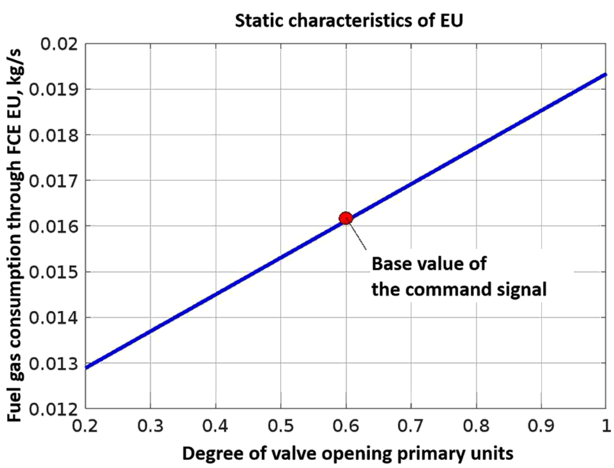


Fig. 2. Static characteristics of the execution unit

Table 4

Linearised model parameters

Title	Symbol	Machine variable	Unit of measurement	Value
Linearised model – 1				
Time constant	τ_n	tau_n	S	76.3815
Coefficient	$k_{\theta n}$	kt_n1 = – kt_n2	Primary units	0.6996
Coefficient	k_g	kg	Kg/(s deg. K) b	0.4283
Linearised model – 2				
Time constant	τ_{sz}	tau_sz	C	19.6485
Coefficient	k_{wcp}	Kw	Primary units	0.2374
Coefficient	k_{ncp}	Kn	Primary units	0.7626
Linearised model – 3				
Time constant	τ_w	tau_w	S	6,453.3282
Coefficient	k_{sdk}	Ksdk	1/s	0.2327
Coefficient	k_{sz}	Ksz	Primary units	0.7673
Linearised model – 4				
Time constant	$\tau_{\theta 1}$	tau_T	S	0.3177
Coefficient	$K_{\theta 1}$	K_T1	Primary units	0.3660
Coefficient	K_u	K_U	Kelvin	1.2078
Linearised model – 5				
Time constant	$\tau_{t,1}$	tau_1	S	73.0783
Coefficient	$K_{t,1}$	Kt1	Primary units	0.1317
Coefficient	$K_{t,2}$	Kt2	Primary units	0.9565

$$\frac{dx_n(t)}{dt} = -\frac{1}{a_n}\sum_{j=1}^{n} a_{j-1}x_j(t) + \beta_n u(t); \quad (3)$$

$$y(t) = x_1(t) + \beta_0 u(t). \quad (4)$$

The unknown quantities β_i, $i=\overline{0,n}$, are calculated as a solution to the matrix equation $A\overline{\beta}=\overline{b}$, i.e. $\overline{\beta}=A^{-1}\overline{b}$ [19], where is a $A_h = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\frac{a_o}{a_{n_h}} & -\frac{a_1}{a_{n_h}} & -\frac{a_2}{a_{n_h}} & \cdots & -\frac{a_{n_h-1}}{a_{n_h}} \end{bmatrix}$ matrix of dimension $n_h \times n_h$; $\overline{b} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdots \\ \beta_{n_h} \end{bmatrix}$ and $\overline{c} = \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$ are vectors of dimension $n_h \times 1$; $\overline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{n_h} \end{bmatrix}$ is a vector of states of dimension $n_h \times 1$ of the reduced model.

Henkel's method makes it possible to extract from the system of equations (2–4) those state variables that have the highest energy. Let there be such state variables n_h. Then, to describe the reduced model in (2–4), we need n to replace by n_h. The result is a reduced model.

To move from a reduced model that describes the dynamic properties of an object (system) in the state space to the transfer function $W_{ij}^{(r)}(s)$, equations (2–4) must be transformed by the Laplace transform at zero initial conditions. This conversion gave the following result

$$s\overline{X}(s) = A_h\overline{X}(s) + \overline{b}_h U(s);$$
$$Y(s) = \overline{c}^T\overline{X}(s) + \beta_0 U(s),$$

from which we determine the transfer function of the reduced model as shown in [19]

$$W^{(r)}(s) = \overline{c}^T(sI - A_h)^{-1}\overline{b}_h + \beta_0.$$

The accuracy of the approximation of the initial transfer function $W(s)$ by the reduced transfer function $W^{(r)}(s)$ was estimated by the maximum relative error of the deviation of the reduced transient response $h^{(r)}(t)$ from the transient response $h(t)$

$$\delta_r = \max_i\left\{\frac{\left|h(t_i) - h^{(r)}(t_i)\right|}{\max_i h(t_i)}\right\}\cdot 100\,\%, \quad i=\overline{1,N_r},$$

where N_r are the ordinates of the transient characteristics calculated in discrete time.

Results. The reduction of the heater's mathematical models made it possible to simplify the heater's transfer functions without a noticeable loss of accuracy (Table 6). In Table 6, the following indexation of transfer functions $W_{1j}(s)$, $j = 1, 2, 3$ is adopted, where the first digit "1" means the first output (oil temperature T_n^{out}), and the second index "j" means input values, the numbering of which in Fig. 1 is numbered from top to bottom.

The analysis of the results presented in Table 6 shows that there was a decrease in the orders of the transfer functions from seven (Table 5) to three without a noticeable loss of approximation accuracy.

As an example, Fig. 3, *a* shows the energy distribution by object states through the channel "oil inlet temperature – oil

Table 5

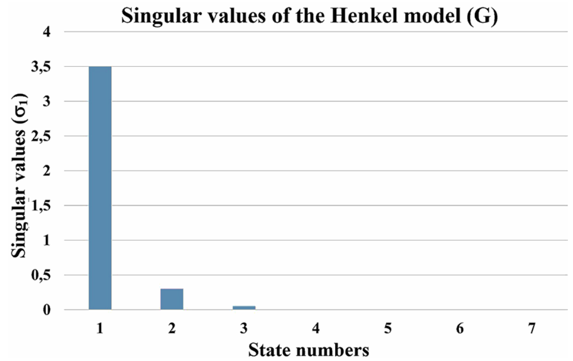
Transfer functions parameters

Transfer function $W_{11}(s)$		
i, j	b_j	a_i
0	–1.13e08	1.13e08
1	–3.709e08	3.73e08
2	–4.785e07	5.455e07
3	–1.972e06	2.811e06
4	–2.519e04	6.082e04
5	–13.34	581
6	1.0000	2.033
7	0.000237	3.2e–05
Transfer function $W_{12}(s)$		
0	–7.601e09	8.133e12
1	–2.492e10	2.684e13
2	–3.122e09	3.927e12
3	–1.325e08	2.024e11
4	–2.11e06	4.378e09
5	–1.128e04	4.182e07
6	–1.0000	1.463e05
7	–	2.303
Transfer function $W_{13}(s)$		
0	1.0000	2.353e09
1	0.06458	7.767e09
2	0.0006964	1.136e09
3	–	5.855e07
4	–	1.267e06
5	–	1.21e04
6	–	42.34
7	–	0.0006664

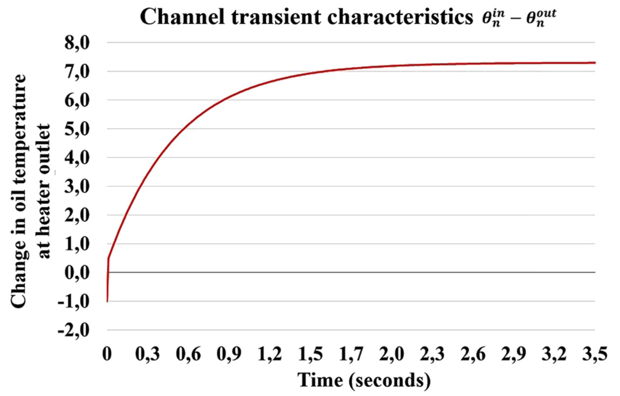
Table 6

Transfer function parameters of the heater after reduction

Transfer function $W_{11}(s)$			
i, j	b_j	a_i	Approximation error, %
0	–1.01e08	1.01e08	3.5987*e*–04
1	–5.569e06	7.397e06	
2	3e04	6.337e04	
3	7.406	1.000	
Transfer function $W_{12}(s)$			
i, j	b_j	a_i	Approximation error, %
0	–9.443e04	1.01e08	4.0980*e*–04
1	–4823	7.397e06	
2	–0.4342	6.337e04	
3	–	1.000	
Transfer function $W_{13}(s)$			
i, j	b_j	a_i	Approximation error, %
0	102.6	5.257e08	9.2313*e*–02
1	–13.4	1.145e07	
2	1.046	6.356e04	
3	–	1.000	



a



b

Fig. 3. Graphs of the analytical study of the transition functions of the reduced mathematical model:

a – diagram of the dominant states on the oil temperature input-output channel; b – ratio of frequency characteristics of the original and reduced models

outlet temperature". From Fig. 3, *a* there are three dominant states that determine the order of the reduced transfer function $W_{11}(s)$. Fig. 3, *b* gives an idea of the transient characteristics of the original and reduced models, which are constructed using the original and reduced transfer functions for a single jump-like input disturbance.

Reducing the order of the heater's corresponding transfer functions simplifies the synthesis of an automatic oil heating control system and greatly facilitates the programming of industrial controllers.

Conclusions.

1. Based on the assumption that the deviations of the output values from their "base" values are insignificant, linearised mathematical models of the indirect heater were obtained, and their parameters were calculated. Studies of linearised models of an oil track heater have shown that its transfer functions are of the seventh order, which complicates the synthesis of automatic control systems for the oil heating process.

2. Using Henkel singular values, the order of the transfer functions is reduced to three without significant loss of accuracy, which greatly simplifies the creation of modern digital control systems using microprocessor-based tools.

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Аналітичні дослідження динамічних властивостей непрямих нагрівачів нафти

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Мета. Дослідити лінеаризовану математичну модель нагрівача, виявити можливості пониження порядку передавальних функцій нагрівача для подальшого синтезу систем керування та для підвищення ефективності й безпечності роботи шляхового нагрівача.

Методика. Після отримання лінеаризованої математичної моделі нагрівача із проміжним теплоносієм рівняння математичної моделі записано в стандартній формі, після чого здійснене його перетворення за Лапласом при нульових початкових умовах. За допомогою програмного продукту здійснене аналітичне дослідження лінеаризованої моделі, після чого використані сингулярні числа Генкеля для спрощення порядку передавальної функції математичної моделі із сьомого до третього та здійснена оцінка втрати точності обчислень після апроксимації функцій, яка показала, що втрата точності не суттєва.

Результати. Дослідивши лінеаризовані математичні моделі нагрівачів із проміжним теплоносієм, визначено, що передавальні функції у вищезазначеній математичній моделі мають сьомий порядок, що суттєво ускладнює створення автоматизованої системи керування нагрівачем. У результаті отримана спрощена математична модель із передавальними функціями 3-го порядку, що суттєво зменшує складність обчислень на сучасних мікропроцесорах.

Наукова новизна. Розроблені лінеаризовані математичні моделі непрямого нагрівача, що були створені на основі припущення про незначні відхилення вихідних величин від їх базових значень. Проведені дослідження лінеаризованої моделі показали, що передавальні функції шляхового нагрівача нафти мають сьомий порядок. Використавши сингулярні числа Генкеля вдалося досягти передавальних функцій третього порядку, що дозволить зменшити складність створення автоматизованих систем керування непрямими нагрівачами нафтового й газового конденсатів.

Практична значимість. Створені сучасні цифрові системи керування процесом нагрівання нафти. Використання методу зниження порядку передавальних функцій за допомогою сингулярних чисел Генкеля до трьох значно полегшує інтеграцію цих моделей у мікропроцесорні засоби керування. Це сприяє підвищенню ефективності й надійності систем автоматичного керування, забезпечуючи стабільний і безпечний процес нагрівання нафти, що є критично важливим для безперебійного функціонування технологічних процесів.

Ключові слова: *лінеаризована модель, редукована модель, сингулярні числа Генкеля, непрямі нагрівачі*

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