

S. V. Biehichev¹,
orcid.org/0000-0001-9861-8754,
H. S. Ishutina*¹,
orcid.org/0000-0002-0665-3040,
L. A. Chumak¹,
orcid.org/0000-0002-3858-8028,
A. P. Hoichuk²,
orcid.org/0000-0003-2477-9488

1 – Ukrainian State University of Science and Technologies,
Dnipro, Ukraine

2 – Dnipro University of Technology, Dnipro, Ukraine

* Corresponding author e-mail: ishutina.hanna@pdaba.edu.ua

ASSESSING THE RELIABILITY OF A SURVEYING AND GEODETIC NETWORK BASED ON A MARKOV MODEL

Purpose. To build a graph of states and transitions of the surveying-geodetic network (SGN), which includes 16 points. To study the functioning of constructed discrete-continuous stochastic Markov models of the surveying-geodetic network with full and current recovery. To perform a numerical calculation of reliability, safety and efficiency indicators: readiness ratio, limit probability states, mean time to failure, mean time between failures.

Methodology. A model of SGN functioning is built in the form of a graph of states and transitions with current and full recovery. Based on the model in the Mathcad software, the availability factor, mean time to failure, mean time between failures are calculated. The following graphs are built: readiness functions, probabilities of operation until the first failure, and frequency of getting into an emergency situation.

Findings. Constructed discrete-continuous stochastic Markov reliability models of the boundary probability states, mean time between failures, mean time to the first failure have been analyzed. The probability of fault-free operation is presented graphically in the form of a transition graph, which describes the logic of the operation of the SGN. Based on the graph of states and transitions (graphical model) according to the Kolmogorov-Chapman algorithm, an analytical model of the reliability behavior of the surveying-geodetic network was built. A system of linear Kolmogorov-Chapman differential equations was compiled and solved. The distribution of probabilities of being in each state of the surveying-geodetic network has been obtained.

Originality. For the first time in surveying practice, a reasonable choice of a discrete-continuous stochastic model of the functioning of a surveying-geodetic network based on the application of the state space method has been made. This model most fully describes the process of functioning (behavior) of the dynamic system. Dependencies between reliability indicators and safety indicators are established. It is recommended to use a model with ongoing network recovery, which allows you to maintain a given level of reliability through timely maintenance (recovery).

Practical value. The most expedient time for restoration of geodetic points with certain failure intensity parameters has been determined. It has been done in order to maintain the SGN in an operational state with a given level of reliability. Current network recovery makes it possible to maintain reliability at the desired level. In the case of complete restoration of the SGN, the readiness factor will be lower, but such a system will be significantly cheaper.

Keywords: *surveyor-geodetic network (SGN), availability factor, probability of failure-free operation, Kolmogorov-Chapman equation, Markov model*

Introduction. Surveying and geodetic works are performed at all stages of mineral deposit development, they are an integral part of all stages of mining production. Without the correct organization of the surveying service, the correct, rational and safe conduct of mining operations are impossible. Underground surveying networks are the basis for surveying mine workings, solving mining and geological tasks. They are created by the method of polygonometry traverse. Surveying networks are created on the basis of reference network points and serve directly to perform surveying tasks. During mining operations, there is a need to restore and reconstruct the underground support network as a result of the increase in the length of workings, the loss of the stable position of points or their destruction, and the instability of rocks. Maintenance of a reliable state of the reference net-

work, restoration of points based on the results of monitoring and assessment of network reliability ensures the quality and reliability of surveying works.

Unsolved aspects of the problem. The quality and reliability of the data of surveying and geodetic works depends on many factors: the qualifications of the executors, the availability of geodetic instruments of the appropriate accuracy, the optimal method of observation, the reliability of the surveying and geodetic network. Unfortunately, the last component is not paid enough attention in practice. In accordance with the Rules [1], repeated measurements are carried out at least once every 15 years with the determination of the coordinates of all points of the planned support networks. According to the Instruction [2], permanent points are laid in places that ensure their long-term stability and safety. Points can be laid in the roof, soil and in the sides of the product. The surveying points are mainly fixed on the fastening of the products.

As a result of the negative impact of natural and anthropogenic factors (mining operations), points are displaced (and sometimes destroyed), which negatively affects the reliability of the network.

The work of both Ukrainian and foreign scientists is devoted to the issue of studying the internal reliability of geodetic networks: Gladilin V. M. [3, 4], Litnarovich R. M., Rodionova Y. V., Savchyn I. R., Tretyak K. R. [5, 6], Uspenskyi M. S., M. Amin Alizadeh-Khameneh [7], Amiri-Simkooei [8, 9], S. Baselga [10], Waldemar Odziemczyk [11], W. A. Baarda, Guanqing Li [12], Martin Štroner [13, 14], Erik W. Grafarend, Waldemar Krupinski, Koch K. R. [15], Bonimani M. L. [16], Roffatto V. F. [17, 18] Gilad Even-Tzur [19], and others.

The works by S. V. Biehichev, H. S. Ishutina. [20, 21] were first to be devoted to the issue of modeling the external reliability of the surveying-geodetic network, which depends on the constancy of the spatial coordinates of the geodetic points (invariance of their position). The application of reliability assessment methods will allow determining the operational condition of the technical system (surveying and geodetic network) and restore its condition. The use of reliable starting points of SGN will significantly increase the accuracy and reliability of surveying survey data: surveying or taking to the ground mining workings, transfer of elevation marks, construction of surveying-geodetic reference and surveying networks.

The consequences of mining are the development of deformation processes, the appearance of rock pressure, the appearance of cracks in the side rocks, deformation and destruction of the walls of the workings and fasteners. Heaving and mountain pressure have a negative impact on the stable position of the points of the underground surveying network fixed in the roof of the workings. If a group of permanent points is damaged by mountain pressure, according to the Instruction [2], the coordinates of these points may be used as starting points under the following conditions:

- at the points that have moved, control measurements of the lengths of both sides are made and their gyroscopic orientation is performed;
- by comparing the old and new values of length and direction angles, the possible displacement of the points is evaluated and corrections to their coordinates are found. With this, linear correction to the point of the planned position, required for the day off, should be no more than 0.15 m.

To avoid significant errors during surveying, the mining engineer-surveyor works in a conventional coordinate system and performs gyroscopic orientation of the mine workings. According to the Instruction [2], with the development of mining operations, the underground support network is reconstructed if necessary. The network is subject to reconstruction when, due to the loss and disruption of points, it becomes impossible to ensure the further development of the network and the execution of surveys of preparatory and cleaning works with the necessary accuracy and reliability.

The analysis of the performed studies showed that the problem of assessing the reliability of SGN is an urgent scientific task in our time. It is important to be able to choose stable geodetic points based on the reliability assessment to create a reliable SGN in the place where the development of mining operations is planned.

Literature review. The construction of surveying plan-altitude support and recording networks on the earth's surface is regulated in the Rules [1]. According to the Rules, the surveying-geodetic network is a set of evenly spaced points on the territory of the mining enterprise, which are fixed with the help of special centers and signs.

Works on the construction of a surveying reference network on the earth's surface and surveying of the earth's surface are carried out in accordance with the Law of Ukraine [22], Order [23] and other normative legal acts in the field of topographical, geodetic and cartographic activities. Chapter III of the Rules [1]

states that the surveying-geodetic network is created according to a certain project, and if necessary, it is reconstructed.

The studies performed in [7] showed that the reliability of the geodetic networks of the designed tunnels in Sweden is quite low due to the weak geometry of the network (limited space in the lateral part of the tunnels). The authors concluded that adding more station settings and involving observations from bracket points on the tunnel wall could reduce uncertainty and improve network reliability. The inclusion of orientation measurements (gyroscopic tracking) has a significant impact on preventing the rapid decline of network accuracy in long tunnels.

The publication [12] investigated the stability of geodetic points of the underground network of a 20-km-long buried tunnel, with the aim of determining its safety and the accuracy of connecting elements of tunnel structures. A submerged tunnel has unique engineering characteristics, construction conditions, and higher alignment and watertight requirements.

In the article [14], the authors single out three methods for optimizing geodetic measurements: choosing a coordinate system, optimizing the network configuration and the number of repetitions; improving the existing network by adding points and/or observations. In engineering searches, the problem of second-order optimization needs to be solved – the location of the points of the geodetic network, the execution of the minimum necessary number of repetitions while observing the required accuracy of measurements. Previously applied general mathematical optimization methods did not consider the nature of geodetic measurements and rational requirements for optimization results. The article presents a new method for optimizing the leveling network based on the gradual selection of the most advantageous measurement to meet the accuracy requirements. The goal of the proposed optimization was to fulfill the selected accuracy criteria with the minimum number of measurement repetitions. At the same time, the authors did not consider such an important parameter as the reliability of the geodetic network. The main attention was paid to ensuring the accuracy of measurements.

The publication [24] proposed a new method for applying the reliability theory in geodetic measurements. It is based on a probabilistic determination of reliability, a system of tolerances and methods of localizing gross errors. Equations are derived for determining the average number of states of points of polygonometry and their standards.

In the work [25], the authors, based on their own experience, draw attention to the fact that before the deadline for geodetic works, there are cases where a sufficient number of points are missing due to their destruction. This, in turn, leads to gross miscalculations, lack of control and low reliability of geodetic work results. The authors believe that in order to calculate the optimal number of points in the polygonal network, it is necessary to apply mathematical methods of decision-making, that is, first to compile a model of the functioning of the geodesic network in the form of a graph of states based on the theory of Markov random processes. The publication also draws attention to the fact that geodetic networks in most cases are not restored during their use, but are created anew over time.

Unsolved aspects of the problem. There are several logic-probabilistic static methods that are widely used to assess the reliability of various complex technical systems:

- the method of reliability prediction, which is intended for reliability assessment at the stage of technical system design;
- the method of reliability block diagram (RBD) – for systems consisting of separate modules at the level of systems or complexes;
- the method of Fault Tree Analysis (FTA).

These methods allow you to calculate the probability of failure-free operation, the probability of failure, the average time of operation before failure, but the obtained indicators are static and can be calculated for a certain moment of operation of the technical system (surveying-geodetic network). For the completeness of information on the functioning of the technical

system, it is necessary to have an idea of the change in reliability indicators over time, therefore, the considered static methods do not meet these requirements. This means that in order to solve the problem, it is necessary to apply another type of model that best meets the set requirements – the Markov model.

Markov models are discrete-continuous stochastic random processes. The probability of a technical system being in different states varies discretely in continuous time. An analytical model is built on the basis of the state graph using the Kolmogorov-Chapman algorithm. By solving the system of linear differential equations, we obtain the probability distribution of the system being in each state.

Markov models are suitable both for studying the functional behavior of a technical system and reliability. That is, it is possible to obtain information about changes in the state and structure of the system during its operation, the interaction between its elements when they fail or are restored. Markov processes are stochastic, they describe the behavior of random events in time. The set of states (the set of all states in which the SGN is) is called the state space or phase space.

The process of functioning of any SGN can be represented as a sequence of changes in certain states known to us. When an event occurs, the technical system transitions from one state to another, while the duration of the state is a random value. Markov discrete-continuous random process has certain properties:

- ordinariness (sequence of events, impossibility of simultaneity of two events);
- stationarity (invariance over time of the probabilistic characteristics of a random process);
- no after-action ($\lambda = \text{const}$), that is, each subsequent action depends only on the current state and does not depend on previous transitions. The transition probabilities are uniquely determined by the state.

Purpose. The purpose of this work is to analyze the functioning of the SGN using a Markov model in the form of a graph of states and transitions. To study the technical system (SGN) by the state space method. To compare two constructed Markov reliability models for determining the probability of fault-free operation of the surveying-geodetic network during a given time, taking into account its restoration (current and complete). To perform a numerical calculation of reliability, safety and efficiency indicators: readiness ratio, limit probability states, average time between failures, average duration of SGN's failure-free operation.

Methods. Let us apply the state space method for the technical system (SGN), which consists of 16 points. Let us generate the states in which the SGN can be present and build a graph of SGN's states and transitions (Fig. 1), which is a diagram that displays the transition from state to SGN state depending on the state of the points. The graph is a graphic model of the SGN system and its functioning, which is created on the basis of logic and makes it possible to quantitatively calculate reliability indicators. The system can be in each state for a certain time. At the beginning of its operation, the SGN was in state 1, when all points were operational with the state vector (16.0).

If one of the SGN points fails with failure intensity λ , the system will go into state 2. At the same time, the state vector will be (15.1). In state 3, one more item (14.2) will fail, so the number of serviceable items will be 14, and 2 non-serviceable items. The last state 9 (8.8) is the state when SGN loses reliability, half of the network points are inoperable. It makes no sense to consider further possible inoperable states. Fig. 1, a shows a complete system recovery. After the loss of reliability of all points, the stage of their recovery with the intensity of full recovery comes – μp .

Let us have a scheme with current restoration (Fig. 1, b), then when transitioning to state 2 at the maintenance stage, the geodetic point is restored in the event of its destruction or re-determination of its position is carried out with the restoration intensity μp and the system returns to state 1 again. Provided

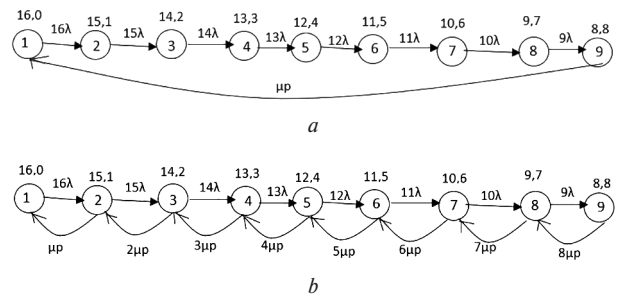


Fig. 1. Graph of states and transitions of the MGM, including 16 geodetic points:

a – for a system with full recovery; b – for a system with current recovery

that during the restoration of the geodetic point in state 2, one more point fails – the system will go to state 3, etc. The diagram (Fig. 1) shows that states 1–8 are operational, and state 9 is inoperable. It makes no sense to consider the states of further failure of the geodetic points of the network, because the number of operational elements of the network will be less than half. Given that periodic monitoring of MGM is carried out once every 15 years, there is a possibility of failure of all elements during this time. The current restoration of the network (Fig. 1, b) can be carried out under the condition of systematic geomonitoring and timely reconstruction (restoration) of the network. The periodicity of monitoring and inspection (reconstruction) of the network depends on the intensity of failure of its elements – geodetic points that are negatively affected by both natural and man-made factors. Let us consider and analyze the system with full recovery (Fig. 1, a). Let us take the initial data: $\lambda = 3 \cdot 10^{-3}$ – intensity of failures, $T_r = 10$ days – recovery time; $\mu p = 0.013$ is the intensity of full recovery.

$$\mu_p = \frac{1}{8T_r}$$

Let us construct a system of linear Kolmogorov-Chapman differential equations for the graph shown in Fig. 1, a.

$$\begin{aligned} \frac{dP1(t)}{dt} &= -16\lambda \cdot P1(t) + \mu_p \cdot P9(t); \\ \frac{dP2(t)}{dt} &= -15\lambda \cdot P2(t); & \frac{dP3(t)}{dt} &= -14\lambda \cdot P3(t); \\ \frac{dP4(t)}{dt} &= -13\lambda \cdot P4(t); & \frac{dP5(t)}{dt} &= -12\lambda \cdot P5(t); \\ \frac{dP6(t)}{dt} &= -11\lambda \cdot P6(t); & \frac{dP7(t)}{dt} &= -10\lambda \cdot P7(t); \\ \frac{dP8(t)}{dt} &= -9\lambda \cdot P8(t); & \frac{dP9(t)}{dt} &= -\mu_p \cdot P9(t) + 9\lambda \cdot P8(t). \end{aligned}$$

In the MathCad software, we will write a graph of states and transitions in the form of a matrix

$$MG = \begin{pmatrix} 1 & 2 & 16\lambda \\ 2 & 3 & 15\lambda \\ 3 & 4 & 14\lambda \\ 4 & 5 & 13\lambda \\ 5 & 6 & 12\lambda \\ 6 & 7 & 11\lambda \\ 7 & 8 & 10\lambda \\ 8 & 9 & 9\lambda \\ 9 & 1 & \mu_p \end{pmatrix}$$

We form a complete matrix of the intensity of transitions from state to state

$$\dot{A} = \begin{pmatrix} -0.048 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.013 \\ 0.048 & -0.045 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.045 & -0.042 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.042 & -0.039 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.039 & -0.036 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.036 & -0.033 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.033 & -0.03 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.03 & -0.027 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.027 & -0.013 \end{pmatrix}.$$

We form and solve a system of equations relative to the average duration of stay in normal functioning states before the first failure and determine the mean time to failure (MTTF)

$$MTTF = \sum_k T_k = 221.$$

$$T = \begin{pmatrix} 20.833 \\ 22.222 \\ 23.81 \\ 25.641 \\ 27.778 \\ 30.303 \\ 33.333 \\ 37.037 \end{pmatrix};$$

Let us determine the MTBF (Mean time between failures)

$$MTBF = MTTF + T_p = 231.$$

Fig. 2 shows the solution of a system of linear differential equations in Mathcad using the Runge-Kutt-Merson method (MR)

Fig. 3 shows a graph of the probability of operation until the first failure. According to the schedule, the working time before the first failure will be 524 days.

We form the AM matrix.

$$\dot{AM} = \begin{pmatrix} -16\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_p \\ 16\lambda & -15\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15\lambda & -14\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 14\lambda & -13\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13\lambda & -12\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12\lambda & -11\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11\lambda & -10\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10\lambda & -9\lambda & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix};$$

$$PTS = (AM)^{-1} \cdot D = \begin{pmatrix} 0.069 \\ 0.074 \\ 0.079 \\ 0.085 \\ 0.092 \\ 0.101 \\ 0.111 \\ 0.123 \\ 0.266 \end{pmatrix}.$$

We calculate the readiness factor (KG) and plot the graph of the readiness function KG(t) (Fig. 4) and the graph of the frequency of failures w(t) (Fig. 5).

$$KG = \sum_k PTS_k = 0.734; \quad w(t) = 9\lambda \cdot MR_{t,s}.$$

In the graph (Fig. 4), during the first 200 days of operation of the MGM, a decrease in the readiness factor is observed from 1 to 0.677, and then due to full recovery, the readiness factor increases to $KG = 0.734$. The probability of getting into an emergency situation increases during the first 160 days of operation (Fig. 5). The emergency peak is on the mark $w_{\max} = 5.368 \cdot 10^{-3}$. As a result of network repair, the emergency system switches to stationary mode ($w_{av} = 3.331 \cdot 10^{-3}$).

To increase the availability factor, the recovery time can be halved, i.e. $T_r = 5$ days with unchanged failure intensity values, then we will get the following parameters: $MTTF = 221$ days; $MTBF = 226$ days; $KG = 0.847$ (Fig. 6). On the two hundredth day

of system operation $KG_{\min} = 0.807$ (Fig. 6) and as a result of recovery, it takes a constant value $KG_{av} = 0.847$ during the end of the functioning of the technical system. Therefore, due to the reduction of maintenance (recovery) time, it is possible to increase the readiness factor of SGN, while the safety of the technical system is almost unchanged (the frequency of failures remains constant).

If we double the intensity of failures ($\lambda = 6 \cdot 10^{-3}$) while keeping other parameters unchanged ($T_r = 10$ days), we get: a doubling of $MTTF = 110$ days and $MTBF = 120$ days, a decrease in the availability factor ($KG = 0.58$) and an increase the frequency of failures is more than 1.6–1.9 times ($w_{\max} = 10.695 \cdot 10^{-3}$; $w_{av} = 5.242 \cdot 10^{-3}$) (Fig. 7).

	0	1	2	3	4
0	0	1	0	0	0
1	27.62	0.266	0.367	0.238	0.096
2	55.239	0.071	0.203	0.275	0.231
3	82.859	0.019	0.085	0.179	0.236
4	110.479	$4.977 \cdot 10^{-3}$	0.031	0.092	0.169
5	138.098	$1.322 \cdot 10^{-3}$	0.011	0.042	0.1
6	165.718	$3.511 \cdot 10^{-3}$	$3.618 \cdot 10^{-3}$	0.017	0.053
7	193.338	$9.325 \cdot 10^{-3}$	$1.173 \cdot 10^{-3}$	$6.914 \cdot 10^{-3}$	0.025
8	220.957	$2.477 \cdot 10^{-3}$	$3.727 \cdot 10^{-4}$	$2.628 \cdot 10^{-3}$	0.012
9	248.577	$6.579 \cdot 10^{-3}$	$1.166 \cdot 10^{-4}$	$9.691 \cdot 10^{-4}$	$5.011 \cdot 10^{-3}$
10	276.197	$1.747 \cdot 10^{-6}$	$3.607 \cdot 10^{-5}$	$3.49 \cdot 10^{-4}$	$2.101 \cdot 10^{-3}$
11	303.816	$4.641 \cdot 10^{-7}$	$1.105 \cdot 10^{-5}$	$1.233 \cdot 10^{-4}$	$8.561 \cdot 10^{-4}$
12	331.436	$1.233 \cdot 10^{-7}$	$3.358 \cdot 10^{-6}$	$4.289 \cdot 10^{-5}$	$3.408 \cdot 10^{-4}$
13	359.056	$3.274 \cdot 10^{-8}$	$1.014 \cdot 10^{-6}$	$1.473 \cdot 10^{-5}$	$1.331 \cdot 10^{-4}$
14	386.675	$8.696 \cdot 10^{-9}$	$3.047 \cdot 10^{-7}$	$5.005 \cdot 10^{-6}$	$5.115 \cdot 10^{-5}$
15	414.295	$2.31 \cdot 10^{-9}$	$9.112 \cdot 10^{-8}$	$1.685 \cdot 10^{-6}$...

Fig. 2. Solution in Mathcad of a system of differential equations for a system with full recovery

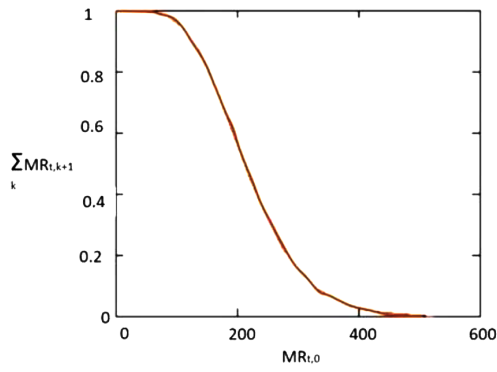


Fig. 3. Probability of operation until the first failure of the system with full recovery

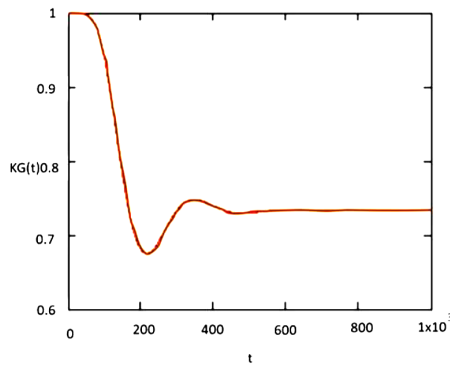


Fig. 4. Readiness function graph for a system with full recovery

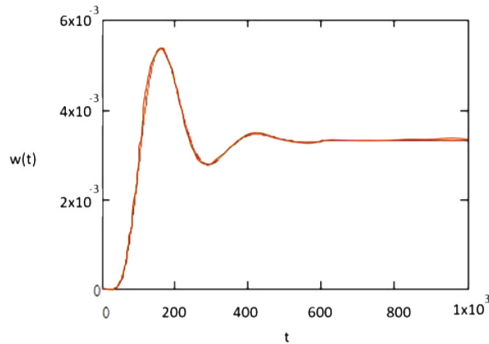


Fig. 5. Graph of failure rate \$w(t)\$ for a system with full recovery

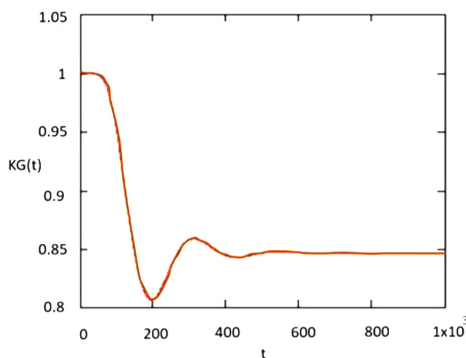


Fig. 6. Graph of system readiness function with full recovery after halving the repair time

Let us consider the scheme of the system with current recovery (Fig. 1, b) and analyze it. Let the original data remain unchanged:

$\lambda = 3 \cdot 10^{-3}$ – intensity of failures;

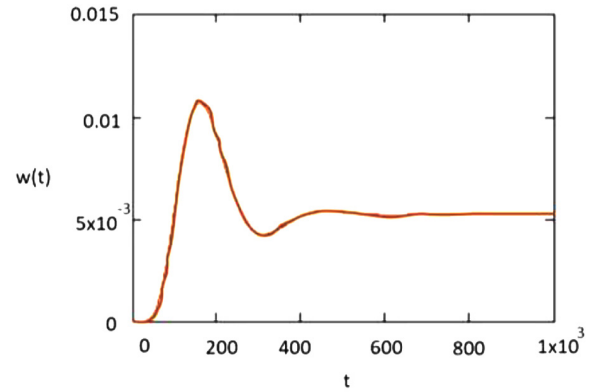


Fig. 7. Graph of system failure with full recovery after increasing failure intensity

$T_r = 10$ h – recovery time;

$\mu_p = 0.013$ – the intensity of the current recovery.

The graph of states and transitions in Mathcad will take the form

$$MG = \begin{pmatrix} 1 & 2 & 8\lambda \\ 2 & 1 & \mu_\partial \\ 2 & 3 & 7\lambda \\ 3 & 2 & 2\mu_\partial \\ 3 & 4 & 6\lambda \\ 4 & 3 & 3\mu_\partial \\ 4 & 5 & 5\lambda \\ 5 & 4 & 4\mu_\partial \\ 5 & 6 & 4\lambda \\ 6 & 5 & 5\mu_\partial \\ 6 & 7 & 3\lambda \\ 7 & 6 & 6\mu_\partial \\ 7 & 8 & 2\lambda \\ 8 & 7 & 7\mu_\partial \\ 8 & 9 & \lambda \\ 9 & 1 & 8\mu_p \end{pmatrix}$$

Let us construct a system of linear Kolmogorov-Chapman differential equations for the graph shown in Fig. 1, b

$$\frac{dP1(t)}{dt} = -16\lambda \cdot P1(t) + \mu_p \cdot P2(t);$$

$$\frac{dP2(t)}{dt} = -15\lambda \cdot P2(t) + 2\mu_p \cdot P3(t);$$

$$\frac{dP3(t)}{dt} = -14\lambda \cdot P3(t) + 3\mu_p \cdot P4(t);$$

$$\frac{dP4(t)}{dt} = -13\lambda \cdot P4(t) + 4\mu_p \cdot P5(t);$$

$$\frac{dP5(t)}{dt} = -12\lambda \cdot P5(t) + 5\mu_p \cdot P6(t);$$

$$\frac{dP6(t)}{dt} = -11\lambda \cdot P6(t) + 6\mu_p \cdot P7(t);$$

$$\frac{dP7(t)}{dt} = -10\lambda \cdot P7(t) + 7\mu_p \cdot P8(t);$$

$$\frac{dP8(t)}{dt} = -9\lambda \cdot P8(t) + 8\mu_p \cdot P9(t);$$

$$\frac{dP9(t)}{dt} = -8\mu_p \cdot P9(t) + 9\lambda \cdot P8(t).$$

We form a complete matrix of the intensity of transitions from state to state for a system with current recovery

$$\dot{A} = \begin{pmatrix} -0.48 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.48 & -0.55 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.45 & -0.62 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.42 & -0.69 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.39 & -0.76 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.36 & -0.83 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.33 & -0.9 & 0.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & -0.97 & 0.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.27 & -0.8 \end{pmatrix}.$$

We will get the following indicators: $MTTF = 171.6$ days, $MTBF = 181.6$ days. The readiness factor will be $KG = 0.987$, which fully corresponds to the specified level of reliability of MGM thanks to the current restoration of the network and is maintained throughout its operation. The graph of the readiness function of the surveying-geodetic network with current restoration is shown in Fig. 8.

If we compare the graphs of the readiness function of MGM with full (Fig. 4) and current recovery (Fig. 8) under the condition of the same values of failure intensity and recovery time, we can conclude that due to current recovery, it is possible to maintain a given level of system reliability. On the other hand, a system with current recovery is more expensive compared to a similar system with full recovery.

The function of the frequency of SGN with current recovery in an emergency situation is shown in Fig. 9. The graph shows that the accident rate of SGN with current recovery ($w = 0.01$) is an order of magnitude higher than the accident rate of a similar SGN with full recovery ($w = 0.003$). Therefore, it is not always possible to increase the reliability of the network to increase its safety.

It should be remembered that low-reliability MGM points lead to loss of network performance and high system failure. At the same time, due to technical maintenance (current recovery), the reliability of the network can be significantly increased, but the probability of getting into an emergency situation will also be high. With current recovery, the availability factor will be higher than with full recovery, but this negatively affects the cost of maintenance of the network to maintain its reliable operation.

Conclusions. The application of Markov stochastic discrete-continuous models allows research of technical systems at the

stage of their design. There is no system yet at all, but we know how to correctly choose the recovery time, how to choose the limit values of reliability, consider the behavior and functioning in the dynamics of the Markov model depending on the intensity of failures. You can analyze and see the relationship between reliability indicators and safety indicators and choose the optimal initial characteristics of the technical system to ensure the efficiency and accident-free operational state of the SGN.

Recovery provides an opportunity to maintain reliability at a given level. With a complete restoration, the readiness factor will be lower, but such a system will be significantly cheaper. Carrying out the current restoration (reconstruction) of the system requires additional expenditure of time and resources for the reconstruction of the SGN based on the results of monitoring, but it ensures the maintenance of reliability at the given level. It is advisable to choose the optimal parameters of SGN, taking into account cost, reliability and accuracy. Each technical system is designed for a certain time of operation. When using very reliable elements (geodesic points), the service life of the technical system increases. That is, at the expense of unreasonable additional costs, we have a prolonged time of using the technical system, which is not always a rational solution in cases of creating temporary local networks (the system does not fully develop its resource).

The use of a reliable surveying-geodetic network will allow increasing the accuracy and quality of conducting surveying works with obtaining reliable and reliable results of surveying mining workings, determining the areas of mining leads, the accuracy of determining mineral reserves, etc.

The research results have a scientific novelty, because in surveying practice, for the first time, the question of the reliable state of the reference network for the restoration of geodetic and surveying points has been investigated based on the results of monitoring and assessment of the reliability of the network, which ensures the quality and reliability of the performance of surveying works. This article shows the results of the first stage of research – the development of a mathematical apparatus for determining the reliability of surveying and geodetic networks (SGN).

During practical calculations, regardless of the number of points of the surveying-geodetic network, the length and configuration of the courses, the parameters will adequately describe the current situation regarding the reliability of this particular network.

The application of the given model for practical determination of the reliability of real SGNs is planned after the completion of theoretical studies on improving the mathematical apparatus of the proposed model.

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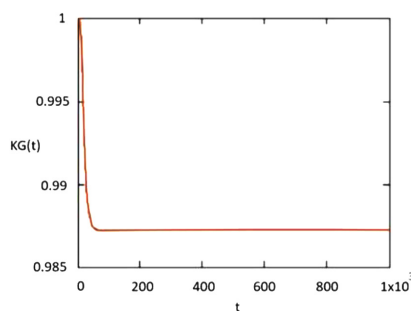


Fig. 8. Graph of SGN readiness function with current recovery

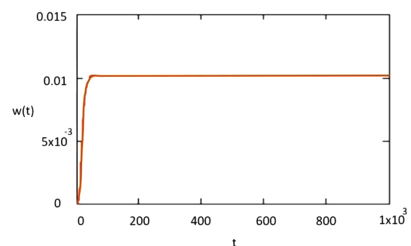


Fig. 9. SGN accident schedule with ongoing recovery

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Оцінка надійності маркшейдерсько-геодезичної мережі на основі марківської моделі

С. В. Бегічев¹, Г. С. Ішутіна^{*1}, Л. О. Чумак¹,
А. П. Гойчук²

1 – Український державний університет науки та технологій, м. Дніпро, Україна

2 – Національний технічний університет «Дніпровська політехніка», м. Дніпро, Україна

* Автор-кореспондент e-mail: ishutina.hanna@pdaba.edu.ua

Мета. Побудова графу станів і переходів маркшейдерсько-геодезичної мережі (МГМ), що включає 16 пунктів. Дослідження функціонування побудованих дискретно-неперервних стохастичних марківських моделей мережі з повним і поточним відновленням. Виконати чисельний розрахунок показників надійності, безпечності та ефективності: коефіцієнта готовності, граничних імовірнісних станів, середнього часу між відмовами, середньої тривалості безвідмовної роботи МГМ.

Методика. Побудована модель функціонування МГМ у вигляді графу станів і переходів із поточним і повним відновленням. На основі моделі у програмному засобі Mathcad розраховані коефіцієнт готовності, середній час роботи до першої відмови. Побудовані графіки: функції готовності, імовірності роботи до першої відмови та частоти потрапляння в аварійну ситуацію.

Результати. Проаналізовані побудовані дискретно-неперервні стохастичні марківські надійнісні моделі граничних імовірнісних станів, середнього часу між відмовами, середнього часу до першої відмови. Представлена графічно імовірність безвідмовної роботи у вигляді графу переходів, що описує логіку функціонування МГМ. На основі графа станів і переходів (графічної моделі) за алгоритмом Колмогорова-Чепмена побудована аналітична модель надійнісної поведінки маркшейдерсько-геодезичної мережі. Складена й розв'язана система лінійних диференціальних рівнянь Колмогорова-Чепмена. Отриманий розподіл імовірностей перебування в кожному стані МГМ.

Наукова новизна. Уперше в маркшейдерській практиці виконано обґрунтований вибір дискретно-неперервної стохастичної моделі функціонування мережі на основі застосування метода простору станів, що найбільш повно описує процес функціонування (поведінки) динамічної системи. Встановлені залежності між показниками надійності й показниками безпечності. Рекомендовано застосовувати модель із поточним відновленням мережі, що дозволяє підтримувати заданий рівень надійності шляхом своєчасного технічного обслуговування (відновлення).

Практична значимість. Визначений найбільш доцільний час проведення відновлення геодезичних (маркшейдерських) пунктів при певних параметрах інтенсивності відмов з метою підтримання МГМ у працездатному стані із заданим рівнем надійності. Поточне відновлення мережі дає можливість утримувати надійність на бажаному рівні. При повному відновленні МГМ коефіцієнт готовності буде нижче, проте така система буде суттєво дешевше.

Ключові слова: маркшейдерсько-геодезична мережа, коефіцієнт готовності, безвідмовна робота, рівняння Колмогорова-Чепмена, марківська модель

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