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APPARENT POWER PLACE IN THE INSTANTANEOUS POWER OF A LINEAR QUADRIPOLE WITH A SINUSOIDAL CURRENT

Purpose. Justification of the fallacy of using the concept of "apparent power" for quadripole circuits in alternating sinusoidal current circuits on the basis of the instantaneous power balance.

Methodology. The apparent power in electric power is a generalizing value of energy processes that researchers use provided that other power components are determined. Based on the analysis of known studies, some were found, in which the authors question such a role in apparent parent. The well-known provisions of the electrical engineering theory are used with the application of mathematical methods, in particular trigonometry, the Euler transformation and the complex numbers theory to determine the instantaneous power components of sinusoidal current and voltage.

Findings. The instantaneous power components of sinusoidal current and voltage are analytically determined in trigonometric and complex form. The corresponding vectors are represented graphically on complex planes of zero and doubled frequency. Accordingly, it is indicated which components of the instantaneous power correspond to the apparent power; in addition, the phase shifts of the latter on the corresponding complex planes are determined. For an elementary electrical circuit, they are defined for all elements of the circuit provided that the balance of instantaneous power (Tellejen's theorem), active power, reactive power (Boucherot's theorem) is observed.

Originality. It has been proven that the order of determining the active power as the difference between the maximum value of the instantaneous power and apparent power determined by the effective values of voltage and current in sinusoidal current circuits cannot be accepted as general. Using the example of a sinusoidal current an elementary electrical circuit, it was found that for an element of electric energy transmission, the amplitude of power fluctuations, which in certain cases is called "apparent power" in general, can be less than the instantaneous power average value – active power.

Practical value. The obtained results can be used to improve the power component compensation algorithms for series and parallel power active filters.

Keywords: *electrical power, apparent power, instantaneous power, active power, reactive power*

Introduction. Apparent (total) power became widespread in electrical engineering after the publication of the work first edition of "Theory and Calculation of Alternating Current Phenomena" in 1897 under the authorship of the American engineer of Polish origin, Charles Proteus Steinmetz. The chapter of the book entitled "Power, and double frequency quantities in general" deals with the graphical interpretation of sinusoidal current and voltage time-oscillating dependences by corresponding vectors on the complex plane. The author introduces the concept of "total volt amperes of circuit". Along with the term "true power", which means active power, and the term "wattles power", the term "total apparent power" is also introduced. There is a statement that in the symbolic representation in the form of two-frequency vector products, powers can be combined and represented by a parallelogram of vectors in the same way as currents and e.m.f. in graphic or symbolic representation. In other words, the apparent power in symbolic terms (real and wattles) of a circuit or system is the sum of the powers of its individual components in symbolic terms. Finally, the author states that the first equation is obviously a direct consequence of the law of conservation of energy.

As a result, the conclusion is made: if the current in the generator, feeding the system, does not coincide in phase with the e.m.f., then in order to eliminate the wattles power, it is necessary to bring the current into phase with the e.m.f. generator, or make the load on the generator non-inductive by installing a device that produces wattles power anywhere in the circuit. That is, the compensation of wattles currents in the system occurs regardless of the location of the compensating device. Obviously, wattles currents will flow between the compensating device and the compensated wattles current source, and for this reason it may be advisable to move the compensator as close as possible to the compensated circuit. Thus, Steinmetz laid down the idea of compensation the power, that is not "watt".

In fact, the definition of instantaneous power components in IEEE Standard 1459-2010 "IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions" is based on the concept of "apparent power".

Literature review. Research into the correctness of using full power to display energy processes in electrical circuits, electrical networks, and electrical installations does not stop.

The phenomenon, responsible for the different apparent power measured in a subsystem of a star-connected three-phase system based on a voltage reference point was identified in [1] using specific instantaneous power components, as a result of the application of the law of conservation of energy. The authors proposed a component of the apparent power, which is called neutral displacement power, the square of which is the quadratic difference between the subsystem apparent powers, measured using two voltage reference points. The neutral displacement power is the component of the apparent power, which is determined using the values of the zero-sequence voltages and line currents in this subsystem. Expressions of the proposed power were obtained using Buchholz's apparent power equations.

In research [2], a new equation for apparent power is proposed, which takes into account the power losses of the neutral conductor, is compared with the normative one, and the conditions of their equivalence are found. The authors claim, that a new physical meaning of apparent power has been established – the geometric mean value of power losses and reverse short-circuit power of the power supply system.

The work [3] is devoted to the analysis of three-phase electric energy. The authors considered the identification of the apparent power and its components, as well as the assessment of power quality indicators in three-phase systems with an asymmetric load. The evaluation and simulation results show differences in the estimation of power factor, apparent power and unbalance power in the case of a three-phase system with and without a neutral.

On the other hand, in the research [4], it is stated that the Buchholz apparent power and its derivative unbalanced power is determined from the quadratic difference between the apparent power and the positive sequence apparent power, formulated by the authors in vector form for three-phase sinusoidal systems. The proposed unbalanced power vector contains three components that measure the unbalance effects, caused by active and reactive currents, and voltage unbalance effects, respectively. According to these new formulations, the total apparent and asymmetric power of several subsystems (sources or loads) can be obtained, respectively, as the norm of the vector sums of their total and asymmetric power vectors.

Just as Steinmetz developed his theory of alternating current in single-phase sinusoidal systems, the authors in [5] identified several formal relations between the Buchholz instantaneous and apparent power expressions in three-phase systems. Based on these relationships, a methodology for expressing the apparent power of Buchholz and its components in any three-phase system with a star configuration – sinusoidal or non-sinusoidal, balanced or unbalanced – was created using instantaneous power equations. The proposed method application to the system made it possible to determine a new value called the neutral displacement power, which measures the effect of phenomena caused by the operation of the neutral wire on the apparent power value of the source and load.

The absence of a generally accepted definition of the apparent power components in circles with non-sinusoidal conditions is noted in [6]. The authors note, that a number of decompositions of the resulting power have been proposed and analyzed over the years, usually in the compensation context. At the same time, the authors present a unique system that is able to simultaneously perform apparent power decomposition for non-sinusoidal conditions in real time. The system can calculate the power components associated with different decompositions, but the basic parameter is the apparent power.

The authors of the paper [7] investigated low-voltage distribution systems, which are usually asymmetrical. In this paper, the presentation of vector expressions for the analysis of asymmetric apparent powers and powers of three-phase linear systems is proposed. In addition, these vector expressions are extended to nonlinear systems to quantify the harmonics apparent power. These expressions were formulated on the Buchholtz power basis and are valid for systems with unbalanced voltages and currents.

Important comments are formulated in work [8] regarding the concept of apparent power and power factor as indicators of system power, that have existed for more than a hundred years, but still have not received a single, strict and acceptable definition. Instantaneous power is accurately determined, and the average power, measured over a selected period is widely distributed. Many ways of determining and measuring reactive and apparent power in single-phase and three-phase systems are based on different assumptions and give different results in real cases. The authors are based on the definitions of the IEEE 1459-2010 standard. In this paper, in the linear algebra of the vector space and the frequency domain, active conducting currents are formulated, as those that cause minimal losses in the network for the supplied power. Power factor measures the relative efficiency of power supply as determined by losses. Apparent power, according to early terminology, is the maximum power that can be obtained with the same initial losses in the line. It is identified without requiring the controversial concept of reactive and inactive components of power. Measurements, based on this approach, are independent of assumptions about sinusoidal signal waveforms, voltage and current balance, and frequency-dependent wire resistances and are applicable to power supply systems with any number of wires.

The concept of apparent power is used apparently under the conditions of solving optimization problems [9] in hybrid AC and DC networks. According to the authors' reasoning, converters with a parallel connection can increase the intensity of the exchange of apparent power between the AC and DC buses. At a certain stage of solving the optimization problem, an indicator called the apparent power deficit index is introduced. It should be noted that for direct current and pulsating current circuits, similar theses are not actually used in the sources, and in this case, they are probably erroneous.

The authors of the work [10] note that traditional power theories and one of their most important concepts – apparent power, are still a source of controversy, because they contain a number of shortcomings that incorrectly interpret the phenomena of power transfer and energy balance in the distorted network conditions. In recent years, advanced mathematical tools such as geometric algebra have been introduced to solve these problems. However, the use of such a tool in electrical circles requires greater consensus, improvements and refinements. The authors reviewed the theories of electric power for single-phase systems based on geometric algebra. An alternative expression for electric power in the geometric domain is given. But at the same time, the norm is compatible with the traditional apparent power, which is defined as the product of the rms voltage and rms current values. This does not significantly differ from the generally accepted approach.

The researchers [11] presented a simple but effective way of calculating electrical power, which avoids the need for direct measurement of phase shift and frequency. The authors note that the traditional approach to calculating active and reactive power in AC power systems requires measuring the phase shift between voltage and current to estimate the power factor. In principle, it is always necessary to identify specific signal points (for example, using the zero crossing detection method) and obtain their time shifts. In particular, the authors calculate the active power as the difference between the peak value of the instantaneous power and the apparent power. Reactive power and power factor are estimated using the same quantities. Thus, there is a certain division of opinion regarding the use of full power.

The purpose. Justification of the fallacy of using the concept of "full power" for quadripole circuits in alternating sinusoidal current circuits on the basis of the instantaneous power balance.

Main material and research results. The apparent power in circuits of single-phase alternating sinusoidal current can be determined in several ways. The first is based on the current I_{rms} and voltage U_{rms} RMS values [12], as follows

$$S = U_{rms} I_{rms}.$$

Second in active P and reactive Q powers [13]

$$S^2 = P^2 + Q^2,$$

which, in turn, are determined by the current and voltage RMS values and the phase shift between them φ

$$P = U_{rms} I_{rms} \cos(\varphi);$$

$$Q = U_{rms} I_{rms} \sin(\varphi).$$

In fact, both options correspond to the same geometric interpretation [14], which is sometimes called the "power triangle". It is known [14, 15] that in this case, for an arbitrary configuration of the circuit, which includes v – passive elements (consumers) and w – active elements of sources, the power balance, generalized by Bouchereau's theorem, is preserved

$$\sum_v P_v + \sum_w P_w = 0;$$

$$\sum_v Q_v + \sum_w Q_w = 0.$$

But at the same time, it is noted that the apparent power balance is not fulfilled

$$\sum_v S_v + \sum_w S_w \neq 0.$$

Such a balance can only be achieved by using a complex form for full power [16]

$$\dot{S}_v = P_v + jQ_v; \quad \dot{S}_w = P_w + jQ_w.$$

Then it becomes a fair expression

$$\sum_v \dot{S}_v + \sum_w \dot{S}_w = 0.$$

The situation becomes even more complicated if we are talking about non-sinusoidal current circuits, and it becomes even more complicated with an attempt to determine the apparent power of a multiphase (three-phase) circuit, but these issues are not the purpose of the current work.

The question arises – what is the fallacy of "full power"? The analysis of works such as [14, 16] shows the use of additional mathematical techniques that are not devoid of meaning for determining apparent power, but at the same time have a gap with the concept of instantaneous power.

Consider the power of sinusoidal voltage and current [17]

$$i = \sqrt{2}I\sin(\omega t + \psi_i); \quad u = \sqrt{2}U\sin(\omega t + \psi_u),$$

where U, I are voltage and current RMS values, respectively; ψ_u, ψ_i – voltage and current initial phase, respectively; ω – angular frequency; t – time.

Using the Euler transformation for current and voltage, we will write them in this form

$$i = \sqrt{2}I\frac{e^{j(\omega t+\psi_i)} - e^{-j(\omega t+\psi_i)}}{2j} = \frac{\dot{I}e^{j\omega t} - \dot{I}^* e^{-j\omega t}}{2j} \leftarrow$$

$$\leftarrow \dot{I} = \sqrt{2}Ie^{j\psi_i} = \sqrt{2}I\cos\psi_i + j\sqrt{2}I\sin\psi_i;$$

$$u = \sqrt{2}U\frac{e^{j(\omega t+\psi_u)} - e^{-j(\omega t+\psi_u)}}{2j} = \frac{\dot{U}e^{j\omega t} - \dot{U}^* e^{-j\omega t}}{2j} \leftarrow$$

$$\leftarrow \dot{U} = \sqrt{2}Ue^{j\psi_u} = \sqrt{2}U\cos\psi_u + j\sqrt{2}U\sin\psi_u.$$

In this way, there is a transition to the known vector image of the specified parameters on the complex plane, which rotates counterclockwise with the angular frequency ω (Fig. 1).

In this case, the instantaneous power will have the form

$$p = ui = \frac{UI}{2}\Big[-e^{j(\omega t+\psi_u)+j(\omega t+\psi_i)} + e^{j(\omega t+\psi_u)-j(\omega t+\psi_i)} + $$
$$+e^{-j(\omega t+\psi_u)+j(\omega t+\psi_i)} - e^{-j(\omega t+\psi_u)-j(\omega t+\psi_i)}\Big].$$

Opening the parentheses in the exponents, while not excluding from the review $\omega t - \omega t = 0$, we get

$$p = \frac{UI}{2}\Big[-e^{j(2\omega t+\psi_u+\psi_i)} + e^{j(0+\psi_u-\psi_i)} + e^{-j(0+\psi_u-\psi_i)} - e^{-j(2\omega t+\psi_u+\psi_i)}\Big].$$

Let us separate elements dependent on frequency and dependent on phase shifts

$$p = \frac{UI}{2}\Big[-e^{j(2\omega t)}e^{j(\psi_u+\psi_i)} + e^{j(0)}e^{j(\psi_u-\psi_i)} + $$
$$+e^{-j(0)}e^{-j(\psi_u-\psi_i)} - e^{-j(2\omega t)}e^{-j(\psi_u+\psi_i)}\Big].$$

Let us write down the component equations as follows

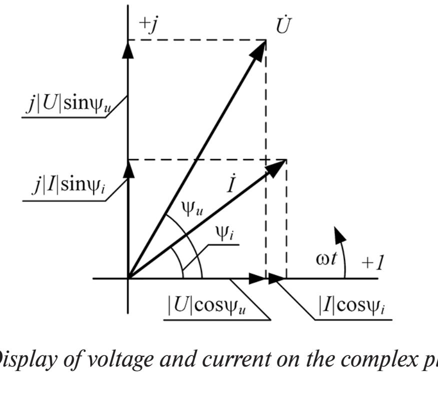

Fig. 1. Display of voltage and current on the complex plane

$$-e^{j(2\omega t)}e^{j(\psi_u+\psi_i)} = -(\cos(\psi_u+\psi_i) + j\sin(\psi_u+\psi_i))e^{j(2\omega t)};$$

$$-e^{-j(2\omega t)}e^{-j(\psi_u+\psi_i)} = -(\cos(\psi_u+\psi_i) - j\sin(\psi_u+\psi_i))e^{-j(2\omega t)};$$

$$e^{j(0)}e^{j(\psi_u-\psi_i)} = (\cos(\psi_u-\psi_i) + j\sin(\psi_u-\psi_i))e^{j(0)};$$

$$e^{-j(0)}e^{-j(\psi_u-\psi_i)} = (\cos(\psi_u-\psi_i) - j\sin(\psi_u-\psi_i))e^{-j(0)}.$$

We introduce the following power components

$$P_{a.1-1} = UI\cos(\psi_u - \psi_i); \quad P_{b.1-1} = UI\sin(\psi_u - \psi_i);$$

$$P_{a.1+1} = UI\cos(\psi_u + \psi_i); \quad P_{b.1+-1} = UI\sin(\psi_u + \psi_i).$$

Thus, the resulting instantaneous power equation will take the form

$$p = \frac{(P_{a.1-1} + jP_{b.1-1})e^{j(0)} + (P_{a.1-1} - jP_{b.1-1})e^{-j(0)}}{2} - $$
$$-\frac{(P_{a.1+1} + jP_{b.1+1})e^{j(2\omega t)} + (P_{a.1+1} - jP_{b.1+1})e^{-j(2\omega t)}}{2}.$$

Otherwise, this expression can be represented using complex conjugate numbers (marked *), as follows

$$p = \frac{\dot{P}_{1-1}e^{j(0)} + \dot{P}^*_{1-1}e^{-j(0)}}{2} - \frac{\dot{P}_{1+1}e^{j(2\omega t)} + \dot{P}^*_{1+1}e^{-j(2\omega t)}}{2},$$

where $\dot{P}_{1-1} = P_{a.1-1} + jP_{b.1-1}$, $\dot{P}_{1+1} = P_{a.1+1} + jP_{b.1+1}$.

In turn, these components can be rewritten in an indicative form

$$\dot{P}_{1-1} = |P_{1-1}|e^{j\psi_{1-1}}; \quad \dot{P}_{1+1} = |P_{1+1}|e^{j\psi_{1+1}}.$$

However, given the pairwise identical arguments of the sine and cosine functions, it is clear that the power module will be the same for both cases and correspond to the current definition of apparent power

$$|P_{1-1}| = \sqrt{P^2_{a.1-1} + P^2_{b.1-1}} = |P_{1+1}| = \sqrt{P^2_{a.1+1} + P^2_{b.1+1}} = UI.$$

However, the phase shift of these components is different

$$\psi_{1-1} = \mathrm{arctg}(P_{b.1-1}/P_{a.1-1}) =$$
$$= \mathrm{arctg}(\sin(\psi_u - \psi_i)/\cos(\psi_u - \psi_i)) = \psi_u - \psi_i,$$

similarly

$$\psi_{1+1} = \psi_u + \psi_i.$$

That is, there are two groups of power components [18] oscillating with angular frequencies determined by the sum and difference of current and voltage angular frequencies, 2ω and 0. Accordingly, these groups of components can be depicted on the complex plane as shown in Fig. 2. Note that in fact, two current vectors and voltages shown on the complex plane ωt (Fig. 2, *a*) form a group of two power vectors, which are represented in two planes 0 (Fig. 2, *b*) and $2\omega t$ (Fig. 2, *c*).

Thus, the apparent power only partially reflects the characteristics of the energy process – instantaneous power. To identify gaps in the defined and interpreted apparent power, let us ask one more question. Is it possible to talk about the apparent power S of a certain quadripole? We will search for an answer to the specified question by analytically calculating a fairly simple linear circle (Fig. 3). As a result, we will determine the distribution of the instantaneous power, according to Telegen's theorem, and the active, reactive, and apparent powers of all circuit elements. Let us conditionally highlight an ideal source, transmission elements (a quadripole is marked with a dotted line) and a load in the scheme.

We will calculate currents and voltages by the classical method using complex numbers. Let us assume that the parameter values of the circuit elements and the power source specified voltage are known

$$\dot{U}_s = \mathrm{Re}(\dot{U}_s) + j\,\mathrm{Im}(\dot{U}_s) = U_s e^{-j\varphi_s},$$

where U_s is source voltage RMS value; φ_s – source voltage initial phase.

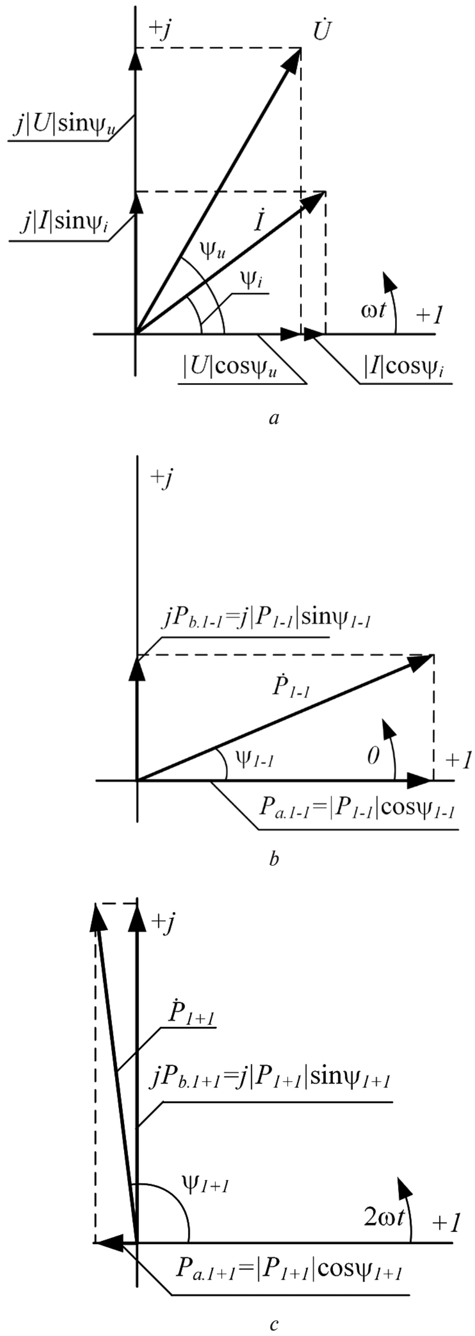

Fig. 2. Vector diagrams:

a – current and voltage; b – power vector on plane 0; c – power vector on the plane 2ωt

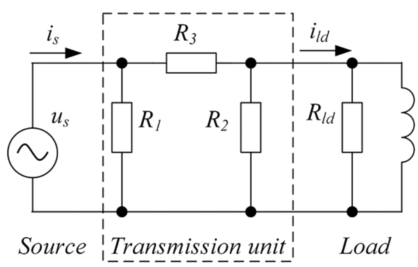

Fig. 3. Calculation scheme

The circuit total impedance for the power supply is

$$Z_{tot} = [R_1^{-1} + (R_3 + [R_2^{-1} + R_{ld}^{-1} - j(\omega L_{ld})^{-1}]^{-1})^{-1}]^{-1}.$$

Accordingly, the source current is

$$\dot I_s = \dot U_s / Z_{tot}.$$

The voltage drop between the source and the load will be

$$\Delta \dot U = \dot U_{R3} = \left(\dot I_s - \frac{\dot U_s}{R_1} \right) R_3.$$

Then the load voltage will be determined as follows

$$\dot U_{ld} = \dot U_{R2} = \dot U_s - \Delta \dot U.$$

Accordingly, taking into account the load impedance

$$Z_{ld} = [R_{ld}^{-1} - j(\omega L_{ld})^{-1}]^{-1}.$$

The load current will have the form

$$\dot I_{ld} = \dot U_{ld} / Z_{ld}.$$

Let us separate the parameters of voltage $\dot U_s$ and current $\dot I_s$ of the source, voltage $\dot U_{ld}$ and current $\dot I_{ld}$ of the load and each element of power transmission $\dot U_R$, $\dot I_R$. This is necessary to determine the instantaneous, active, and reactive powers of the source and load, respectively – powers at the input and output of the power transmission element [19]. To do this, we will use an approach based on the balance of instantaneous (Tellegen's theorem), active and reactive (Boucherot's theorem) capacities. According to Telegen's theorem, in the scheme the power balance is performed

$$p_s = (p_{R1} + p_{R2} + p_{R3}) + p_{Rld} + p_{Lld} = p_{tr} + p_{Rld} + p_{Lld}.$$

The instantaneous power at the input and output of the transmission element, as well as the power of each circuit element, will be considered in the following form

$$p_{el} = P_{a.1-1.el}\cos(0) + P_{b.1-1.el}\sin(0) + \\ + P_{a.1+1.el}\cos(2\omega t) + P_{b.1+1.el}\sin(2\omega t),$$

where $P_{a.1-1.el} = U_{el} I_{el} \cos(\psi_{u.el} - \psi_{i.el}) = P_{el}$ – the circuit element's active power; $P_{b.1-1.el} = -U_{el} I_{el} \sin(\psi_{u.el} - \psi_{i.el}) = Q_{el}$ – the circuit element's reactive power; $P_{a.1-1.el} = -U_{el} I_{el} \cos(\psi_{u.el} - \psi_{i.el})$ – a cosine oscillating power component; $P_{b.1+1.el} = U_{el} I_{el} \sin(\psi_{u.el} - \psi_{i.el})$ – a sine oscillating power component; U_{el}, I_{el} – voltage and current RMS value; $\psi_{u.el}$, $\psi_{i.el}$ – the voltage and current initial phase.

The specified voltage and current parameters (Fig. 1) are related to complex variables as follows

$$U_{el} = |\dot U_{el}|;\ \psi_{u.el} = \arg(\dot U_{el});\ I_{el} = |\dot I_{el}|;\ \psi_{i.el} = \arg(\dot I_{el}).$$

Quite often in known sources [19, 20] you can find another form of recording the instantaneous power of a sinusoidal current

$$p_{el} = P_{el}\cos(0) + Q_{el}\sin(0) + S_{el}\cos(2\omega t + \psi_{s.el}),$$

where $S_{el} = \sqrt{P_{a.0.el}^2 + P_{b.0.el}^2} = \sqrt{P_{el}^2 + Q_{el}^2} = \sqrt{P_{a.2.el}^2 + P_{b.2.el}^2}$ is the amplitude of the instantaneous power oscillating component; $\psi_{s.el} = \mathrm{arctg}(P_{b.2.el}/P_{a.2.el})$ – initial phase of the instantaneous power oscillating component.

Considering the complexity of the analytical calculation of the indicated circuit elements indicators, we will perform numerical calculations by setting the circuit parameters as follows $\dot U_s = 100e^{-j0^\circ}V$, $R_1 = R_2 = 200$ Ohm, $R_1 = 10$ Ohm, $\omega = 314.16\ s^{-1}$.

Consider two load cases: 1) $R_{ld} \to \infty$, $L_{ld} = 20/\omega H$ (reactive load); 2) $R_{ld} = 20$ Ohm, $L_{ld} \to \infty$ (active load).

The results of calculating currents, voltages and power components of the elements are summarized in Tables 1 and 2. The analysis of the calculation results for instantaneous power components shows the following. First, the power balance is performed according to all the components listed in the table, both active and reactive powers, and quadrature components of the oscillating power for both variations. Second, in all but one case, the identity holds $\sqrt{P_{el}^2 + Q_{el}^2} = \sqrt{P_{a.1+1.el}^2 + P_{b.1+1.el}^2}$.

Table 1

The currents and voltages calculation results

Element	*U*, *V*	*I*, A
case 1 $R_{ld} \to \infty$, $L_{ld} = 20/\omega H$		
Source	$100 + j0$	$2.737 - j3.69$
*R*1	$100 + j0$	$0.5 + j0$
*R*2	$77.63 + j36.97$	$0.39 + j0.19$
*R*3	$22.37 - j36.97$	$2.24 - j3.69$
Load	$77.63 + j36.97$	$1.848 - j3.88$
$Tr.El._{in}$	$100 + j0$	$2.737 - j3.69$
$Tr.El._{out}$	$77.63 + j36.97$	$1.848 - j3.88$
$Tr.El.\Delta$	–	–
case 2 $R_{ld} = 20$ Ohm, $L_{ld} \to \infty$		
Source	$100 + j0$	$4.05 + j0$
*R*1	$100 + j0$	$0.5 + j0$
*R*2	$64.52 + j0$	$0.32 + j0$
*R*3	$35.48 + j0$	$6.45 + j0$
Load	$64.52 + j0$	$3.23 + j0$
$Tr.El._{in}$	$100 + j0$	$4.05 + j0$
$Tr.El._{out}$	$64.52 + j0$	$3.23 + j0$
$Tr.El.\Delta$	–	–

Table 2

The power components calculation results

Element	*P*, W	*Q*, VAr	$P_{a.2}$, VA	$P_{b.2}$, VA
case 1 $R_{ld} \to \infty$, $L_{ld} = 20/\omega H$				
Source	273.66	369.69	–273.66	–369.69
*R*1	50	0	–50	0
*R*2	36.97	0	–23.30	28.70
*R*3	186.69	0	86.64	–165.37
Load	0	369.69	–287.0	–233.02
$Tr.El._{in}$	273.66	369.69	–273.66	–369.69
$Tr.El._{out}$	0	369.69	σ287.0	–233.02
$Tr.El.\Delta$	273.66	0	13.342	–136.67
case 2 $R_{ld} = 20$ Ohm, $L_{ld} \to \infty$				
Source	404.84	0	–404.84	0
*R*1	50	0	–50	0
*R*2	20.81	0	–20.81	0
*R*3	125.91	0	–125.91	0
Load	208.12	0	–208.12	0
$Tr.El._{in}$	404.84	0	–404.84	0
$Tr.El._{out}$	208.12	0	–208.12	0
$Tr.El.\Delta$	196.72	0	–196.72	0

Third, in the case of the power transmission element, the identity is not performed.

Accordingly, Fig. 4 shows time diagram of the circuit elements' instantaneous power for the two studied cases. As can be seen in the first case, the diagram of the instantaneous power of the transmission element p_{tr} has an oscillations amplitude, which coincides with the average value. In the second case (Fig. 2, *b*), the diagram has an amplitude of the variable component smaller than its average value. Thus, following the instantaneous power equation and the statements used in works [11, 21], the conclusion is that the apparent power of the power transmission element is less than the active power $S_{tr} < P_{tr}$.

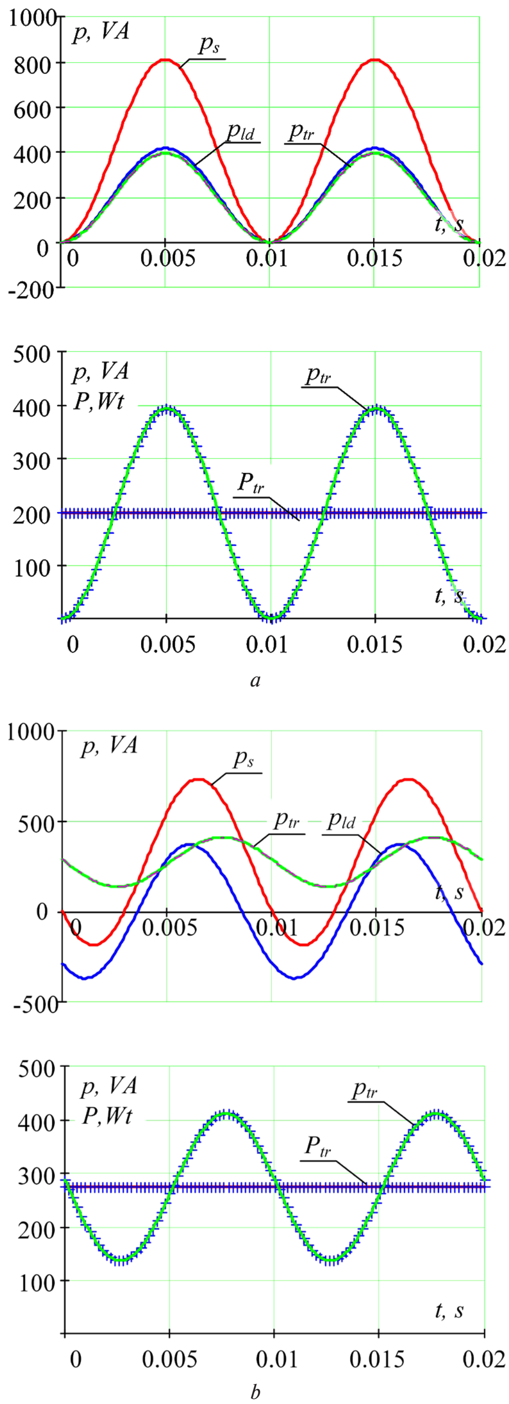

Fig. 4. Time diagrams of instantaneous power in circuit elements: a – for case 1; b – for case 2

To determine the reason, consider the time diagram of transmission element instantaneous power $p_{tr} = p_{R1} + p_{R2} + p_{R3}$ shown in Fig. 5 for option 1. Based on the orthogonal components given in Table 2, the instantaneous power of the resistors is determined by the following expressions

$$p_{R1} = 50 - 50\cos(2\omega t);$$

$$p_{R2} = 36.97 - 233\cos(2\omega t) + 28.7\sin(2\omega t) = \\ = 36.97 + 36.97\cos(2\omega t + 2.25);$$

$$p_{R3} = 186.69 + 86.64\cos(2\omega t) - 165.37\sin(2\omega t) = \\ = 186.69 + 186.69\cos(2\omega t - 1.09).$$

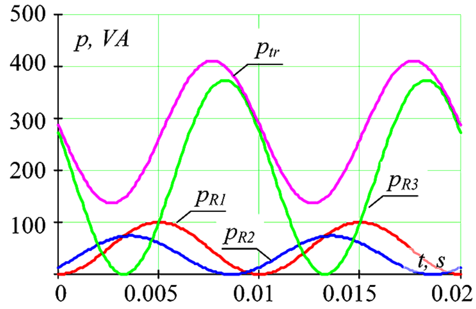

Fig. 5. The time diagram an instantaneous power on resistors transmission element

Thus, the phase shift between the variable components of the resistors' instantaneous powers leads to their interaction, in which their total instantaneous power has an amplitude of oscillations much smaller than the average value.

Conclusions and further research direction. As a result of the analysis of the known studies of the relationship between instantaneous power and apparent power in sinusoidal current circuits, it was established that some authors use the difference between the maximum value of instantaneous power and the apparent power determined by the RMS values of voltage and current to determine the active power. It is proved that this method cannot be used in the general case in sinusoidal current circuits.

Analytically, on the basis of current and voltage, the power components of the sinusoidal current are determined in a complex form, respectively, modules, arguments and orthogonal components, with a division into components of zero and doubled frequency, as a result, the place of "apparent power" is determined for such a case.

For an elementary electrical circuit of a sinusoidal current using Telegen's theorem for the instantaneous power balance and Boucherot's theorem for the active and reactive powers balance, their distribution for all elements of the circuit was determined, and it was found that for the electric energy transmission element the amplitude of power oscillation, which in certain cases is called "apparent power", can be less than the average value of instantaneous power – active power in the general case.

The obtained results can be used in the future to improve power component compensation algorithms for series and parallel power active filters.

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Місце повної потужності в миттєвій потужності лінійного чотириполюсника при синусоїдальному струмі

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Мета. Обґрунтування хибності використання поняття «повної потужності» для чотириполюсників у колах змінного синусоїдального струму на підставі балансу миттєвої потужності.

Методика. Повна потужність в електроенергетиці є узагальнюючою величиною енергетичних процесів, яку дослідники використовують за умови визначення інших компонент потужності. На підставі аналізу відомих досліджень виявлені такі, в яких автори ставлять під сумнів узагальнюючу роль повною потужності. Використані відомі положення теорії електротехніки із застосуванням математичних методів, зокрема тригонометрії, перетворення Ейлера й теорії комплексних чисел для визначення миттєвої потужності синусоїдальних струму та напруги.

Результати. Аналітично визначені складові миттєві потужності синусоїдального струму й напруги у тригонометричній і комплексній формі. Відповідні вектори репрезентовані графічно на комплексних площинах нульової та подвоєної частоти. Як наслідок зазначено, які складові миттєвої потужності відповідають повній потужності, окрім того визначені фазові зсуви останньої на відповідних комплексних площинах. Для елементарної електричної схеми, за умови дотримання балансу миттєвої потужності (теорема Телледжена), активної потужності, реактивної потужності (теорема Бушеро), їх визначено для всіх елементів схеми.

Наукова новизна. Доведено, що порядок визначення активної потужності як різниці між максимальним значенням миттєвої потужності й повною потужністю, визначеної через діючі значення напруги та струму, у колах синусоїдального струму, не може бути прийнятим як загальний. На прикладі елементарної електричної схеми синусоїдального струму виявлено, що для елементу передачі електричної енергії амплітуда коливань потужності, яку в певних випадках називають «повна потужність», у загальному випадку може бути менша за середнє значення миттєвої потужності – активну потужність.

Практична значимість. Отримані результати в подальшому можуть бути використані для удосконалення алгоритмів компенсації складових потужності для послідовних і паралельних силових активних фільтрів.

Ключові слова: *електрична потужність, повна потужність, миттєва потужність, активна потужність, реактивна потужність*

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