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RESEARCH ON STOCHASTIC PROPERTIES OF TIME SERIES DATA ON CHEMICAL ANALYSIS OF CAST IRON

Purpose. To provide a procedure for identifying chaotic processes in a dynamic system and to examine time series, describing the chemical composition of cast iron at the blast furnace output with the purpose of identifying the nonlinearity of the investigated system and detecting the presence of chaotic processes in it.

Methodology. The determination of the unique characteristics of the attractor of a dynamic chaotic system based on the time series of cast iron's chemical composition values was carried out using methods of nonlinear dynamics and dynamic chaos theory, such as the autocorrelation function method, correlation and fractal dimensions.

Findings. The methods of nonlinear dynamics and dynamic chaos theory were used to study the behavior of time series data on the chemical composition of cast iron at the blast furnace output. The presence was identified of chaotic processes with a fractal structure in the studied dynamic system, leading to the inefficiency of traditional analysis methods based on the Gaussian properties of stochastic processes.

Originality. For the first time, the possibility and feasibility of applying chaos theory methods for the analysis and prediction of time series data on the chemical composition of cast iron at the blast furnace output were substantiated.

For the first time, the nonlinearity of the studied dynamic system was identified, and chaotic processes were discovered within it by determining the unique characteristics of the strange attractor of the system using the analyzed time series, such as embedding dimension, time delay, and the largest Lyapunov exponent.

Practical value. The obtained results open up the possibility for more effective and qualitative analysis of the behavior of the studied dynamic system by developing new tools for assessment and prediction that are adequate to the nature of the ongoing processes.

Keywords: *nonlinear dynamics, dynamic chaos, strange attractor, fractal properties of time series, spectrogram*

Introduction. In modern metallurgy of cast iron production, the rapid development of technology presents the industry with significant challenges, particularly concerning the improvement of product quality and the optimization of production processes. One of the key components of this process is the control and assurance of the required chemical composition of cast iron at the stage of release from the blast furnace. In this context, automated control and prediction of the chemical composition of pig iron become essential tools for enhancing production efficiency and quality.

The control, prediction, and management of the blast furnace process are complex and multifaceted. This process is characterized by non-stationary parameters, a large number of interdependent variables, a high level of noise, significant inertia, and time delays. Furthermore, the specific features of the technological process and the structural characteristics of the blast furnace complicate direct measurements. This creates serious obstacles to obtaining accurate information about the state of the object, significantly reducing the capabilities of control systems.

Improving the control and prediction of the chemical composition of cast iron requires the implementation of modern automated systems that ensure high accuracy and speed in obtaining data. Through the use of advanced information processing methods and mathematical modeling, such systems can predict changes in chemical composition in real time, allowing timely adjustments to technological parameters and preventing possible deviations from set norms.

Modern automated control systems not only increase measurement accuracy but also reduce the impact of the human factor on the production process. The use of advanced technologies, such as machine learning and artificial intelligence, allows for the development of sophisticated algorithms that efficiently process large volumes of data and provide a high level of adaptability to changing production conditions.

Thus, the integration of automated systems for controlling and predicting the chemical composition of cast iron is an integral part of modern metallurgy. This contributes to the improvement of the quality of the final product, the optimization of production processes, and the reduction of costs. Ultimately, the application of advanced technologies in metallurgy opens new opportunities for the industry, ensuring its sustainable development and competitiveness in the global market.

Literature review. Until recently, it was believed that the time sequences of data representing the results of the chemical analysis of cast iron at the blast furnace output followed a Gaussian distribution [1]. However, studies [2, 3] on time series data of chemical composition have proposed and confirmed the hypothesis of the fractal properties of the investigated time series. This, in turn, has provided the impetus for applying the principles of chaos theory to study the dynamic properties of the investigated time series.

Presentation of the main research material. At the initial stage of the study, an analysis of the time series generated by the dynamic system was conducted. This analysis identified the unique characteristics of the system's attractor, specifically the embedding parameters: the dimension of the embedding space and the signal time delay. Determining the embedding parameters is essential for further analysis of the time series, as the obtained indicators are input parameters for calculating the largest Lyapunov exponent [4].

The results of the first stage of processing chaotic processes allow for the identification of the chaotic process through the determination of the largest Lyapunov exponent and the phase reconstruction of the chaotic process [5]. At this stage, the main challenge is the need to process a large volume of output data, as with a small volume of initial information, it is practically impossible to qualitatively reconstruct the attractor and calculate the largest Lyapunov exponent. Therefore, it is necessary to use software tools with the required functionality for the analysis. Currently, one of the most effective and multifunctional software products for analyzing time series of various natures is Matlab. This tool allows for the analysis using both linear and nonlinear methods. Matlab's flexibility in programming is one of its key advantages. On the one hand, it offers a wide range of built-in functions that significantly facilitate the analysis process, and on the other hand, it provides the ability to develop custom algorithms, allowing it to be adapted to specific user needs.

Due to its versatility, Matlab has become an indispensable tool in many fields of science and engineering. It is used for signal processing, data analysis, system modeling, and for developing and testing new algorithms. Matlab's built-in functions allow for quick and efficient execution of complex mathematical calculations, greatly simplifying the work of researchers and engineers.

Additionally, Matlab has an extensive library of tools for data visualization. This allows for easy creation of graphs, charts, and other visual representations of data, promoting a deeper understanding of the studied processes and phenomena. This functionality enables users to quickly and clearly assess the results of their analyses.

Another significant advantage of Matlab is its ability to integrate with other software products and programming languages. This allows Matlab to be used as part of complex systems consisting of various components, providing flexibility in the development of multifunctional solutions.

The stages of identifying chaotic processes in a dynamic system are determined by the presence of a strange attractor in such systems, which has a fractal dimension and the property of scale invariance.

The first step in identifying a chaotic process is the analysis of the time series generated by the dynamic system. This analysis is crucial as it allows for the determination of the unique characteristics of the system's attractor, including the dimension of the embedding space and the signal time delay.

The process of analyzing the time series begins with the construction of the phase space, which allows for the visualization of the system's dynamic behavior. This is done using the embedding method, which involves transforming the one-dimensional time series into a multidimensional space. Choosing the correct embedding parameters, such as the dimension of the embedding space and the signal time delay, is critically important for accurately reflecting the system's dynamics.

The dimension of the embedding space determines the number of dimensions needed to adequately represent the system's phase space. It is chosen in such a way as to minimize the loss of information about the system's dynamics. Too small a dimension of the embedding space can lead to overlapping trajectories, complicating the identification of the attractor, while too large a dimension of the embedding space can introduce unnecessary dimensions, increasing computational costs.

The signal time delay is determined using methods such as the autocorrelation function or mutual information. It should be chosen to maximally preserve the system's dynamic structure, revealing hidden patterns in the time series [6].

The obtained indicators are input parameters for calculating the value of the largest Lyapunov exponent for the time series, a positive value of which identifies the presence of chaotic processes in the studied system.

This study was conducted using the Matlab and Fractan software environments, based on real data on the percentage content of silicon in pig iron obtained at different times at Blast Furnace No. 3 (BF-3) of the Ilyich Iron and Steel Works in Mariupol [7].

For the quantitative characterization and identification of patterns associated with the system's dynamics, a detailed analysis of the geometric image of the dynamic regime – the attractor, which is the so-called attracting set of the system's trajectories in D-dimensional phase (or pseudo-phase) space, is necessary. The coordinates of the phase space are the dynamic variables of the process. Each type of dynamic behavior corresponds to its own attractor and, accordingly, its geometric image – the phase portrait [8]. For example, the dynamics of a conventional chemical reaction corresponds to an attractor of the stable point type. Regular oscillations correspond to a stable limit cycle. These classical attractors correspond to classical geometric regions: a point, a closed curve (circle, ellipse, etc.), or a toroidal surface. In contrast, disordered phase portrait trajectories indicate the presence of a chaotic attractor. This class of attractors also includes the so-called strange attractor, whose geometric image in phase space is a fractal object.

The appearance of the strange attractor for the studied time series in normalized coordinate axes is shown in Fig. 1. Here, the attraction region, which is a dense "core", is clearly visible. This attraction region indicates the presence of structured dynamics in the system, where trajectories tend to be attracted to a certain region of phase space, forming a complex but organized geometric structure. The strange attractor demonstrates characteristic properties of chaotic systems, such as sensitivity to initial conditions and the presence of self-similar structures at different scales.

At the same time, for a random sequence, as mentioned above, the points of the reconstructed pseudo-attractor form an unstructured cloud in lag space. This means that random processes lack any organized structure, and the points are chaotically arranged without an apparent regular order. Such sequences do not exhibit the properties of attraction to certain regions of phase space, which is characteristic of deterministic dynamic systems.

The studies conducted by Takens [9] prove that by using only one coordinate of the dynamic system, it is possible to reconstruct the original attractor in the space of points with delays $[x(t), x(t+\tau), ..., x(t+(m-1)\cdot\tau) \in R^m]$, in such a way that it retains the most important topological properties and dynamics of the original attractor. The dimension of the embedding space m is determined by the formula

$$m \geq 2\{d\} + 1, \quad (1)$$

where d is the fractal dimension of the attractor. The curly braces indicate that only the integer value of d is considered.

Therefore, first and foremost, to conduct a thorough analysis of chaotic processes, it is necessary to determine the embedding parameters of the dynamic system required for maximizing the predictability of the chaotic process, specifically the appropriate signal time delay τ and the embedding dimension m [10].

Different methods are used to choose the time delay of the signal, including the autocorrelation function method and the mutual information method. These methods are based on the assumption that the most suitable time delay is the minimal delay at which the coordinates of the reconstructed attractor become maximally independent.

1. Autocorrelation Function Method. For each value of time delay τ autocorrelation Function $R(\tau)$ is determined by formula, where the correlation coefficient between the original time series and its modification is calculated using a time delay of τ steps.

$$\lambda = \frac{1}{\Delta t} \ln \frac{\varepsilon(t)}{\varepsilon(t_0)}.$$

The selection of the most appropriate time delay is performed according to the first zero-crossing or near-zero value of the autocorrelation function [11].

$$R(\tau) = \frac{1}{n-\tau} \sum_{t=1}^{N-\tau} y(t) \cdot y(t+\tau),$$

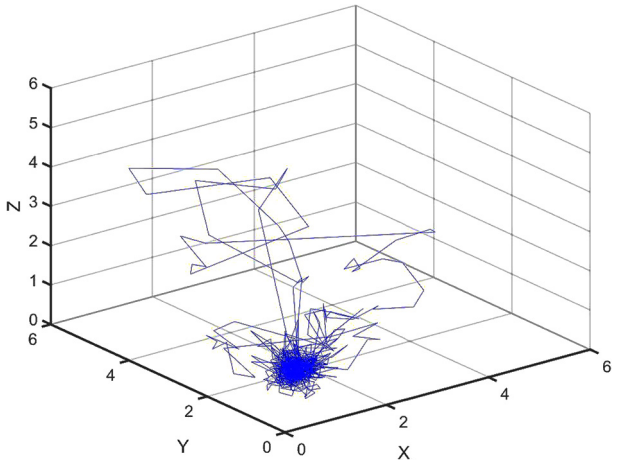


Fig. 1. Phase Portrait of the Strange Attractor

where $y(t) = x(t) - E\{x\}$ is a centered version of the time series.

For the investigated time series, the first approximation of the autocorrelation function approaches zero at lag 1 (Fig. 2), hence, according to this method, the most suitable time delay is 1 [12, 13].

The simplicity of calculation of this method, which does not require extensive computations, makes it one of the most accessible and widely used by many researchers. However, it relies on the assumption of non-correlation, which can lead to obtaining inappropriate values for time delay in chaotic analysis of dynamic systems

2. Mutual Information Method. The mutual information function, defined by the following formula, involves partitioning $(a, b) \in R^1$ – minimum interval containing all values of the investigated time series in L equal parts and denoted A_i when the event occurs, when $x(t)$ belongs to the interval i, and B_j occurrence of an event $x(t + \tau)$ belongs to the interval j.

$$I(\tau) = \sum_{i=1}^{L}\sum_{j=1}^{L} P(A_i B_j) \cdot \log_2 \frac{P(A_i B_j)}{P(A_i B_j)},$$

where P is the probability of a certain event occurring.

The most suitable time delay by this method is chosen according to the first minimum of the function $I(\tau)$.

The mutual information method provides more accurate results than the autocorrelation function method, as it relies on the independence property. However, it is significantly more complex to compute. Automation of this method, however, eliminates this drawback.

Next, it is necessary to choose the embedding dimension m, which can be estimated using various methods.

1. The correlation dimension method. To determine the value of the correlation dimension, the Grassberger-Procaccia algorithm is used, with which we calculate D_k, value F from a sample of points $\{x_i\}, i = \overline{1, M}$.

The correlation dimension is calculated by the formula

$$D_k = \lim_{\varepsilon \to 0} \frac{\ln \sum_{i=1}^{N(\varepsilon)} p_i^2}{\ln \varepsilon} = \lim_{\varepsilon \to 0} \frac{\ln C(\varepsilon)}{\ln \varepsilon},$$

where $C(\varepsilon)$ is the correlation integral which is calculated by the formula

$$C(\varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^2 = \lim_{M \to \infty} \frac{1}{M(M-1)} \sum_{i=1}^{M}\sum_{j=1}^{M} \theta\left(\varepsilon - \left\| x_i - x_j \right\|\right),$$

where θ is the Heaviside function, which is defined as

$$\theta(x) = \begin{cases} 0, & \text{when } x < 0 \\ 1, & \text{when } x > 0 \end{cases}.$$

The Grassberger-Procaccia algorithm consists of the following steps:

1. The correlation integral $C(\varepsilon)$ is computed for different ε.

2. The obtained dependence $C(\varepsilon)$ on ε is represented in a coordinate system with double logarithmic scale.

3. A search for a linear segment is conducted.

4. Angular coefficient d for the found segment; it will be an estimate of the correlation dimension for F.

This algorithm allows determining the embedding dimensionality m for the investigated time series.

For example, let there be a time series of data, and a time delay τ found by any method, fixing this whole number m and applying the idea of pseudo-phase reconstruction, we can obtain a set of points

$$x_i = x(i), x(i - \tau), ..., x(i - (m - 1)\tau) \in R^m,$$

where $i = \overline{\left[(m-1)\tau + 1\right], N}$.

For the value m and the obtained sample $x(t)$ the correlation dimension $D_k(m)$ is calculated using the method described above. This procedure is repeated several times, considering sequentially $m = 1, 2, 3, ...$ As the value m increases, saturation of the corresponding value $D_k(m)$ is observed, thus, the calculated value of the correlation dimension does not exceed the maximum value as the embedding dimension increases. If saturation $D_k(m)$ does not occur, then the investigated signal is likely generated not by a dynamic system but rather by noise. When calculating the correlation dimension using the method described above with the FRACTAN software, conflicting results were obtained. The automatic calculation of the correlation dimension in this program showed that the estimate for this dimension for the time series data on silicon content in cast iron is 8, while the embedding dimension is 12. The graph depicting the correlation integral for the time series on silicon content in cast iron, as a function of the experimental embedding dimension, is presented in Fig. 3.

2. False nearest neighbours algorithm. False nearest neighbours algorithm is based on Takens' theorem on embedding: with appropriate choices of τ and $m$the original and reconstructed attractors should be topologically equivalent (homeomorphic). Since trajectories of the original attractor do not intersect, trajectories of the reconstructed attractor should also not intersect. Self-intersections of trajectories in the reconstructed attractor imply that the embedding dimension is less than the fractal dimension of the attractor.

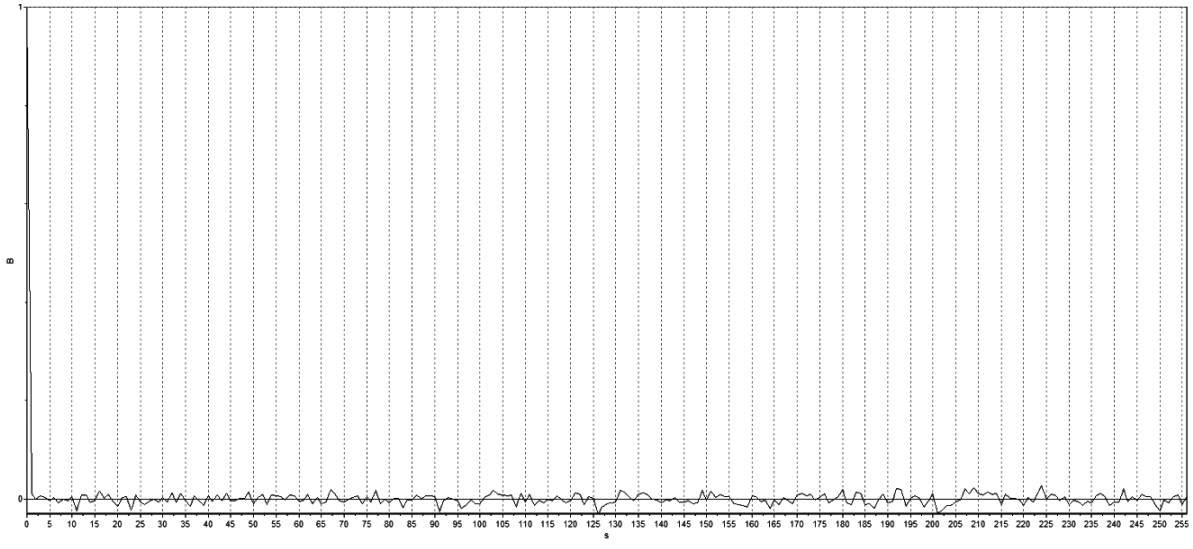


Fig. 2. Autocorrelation function

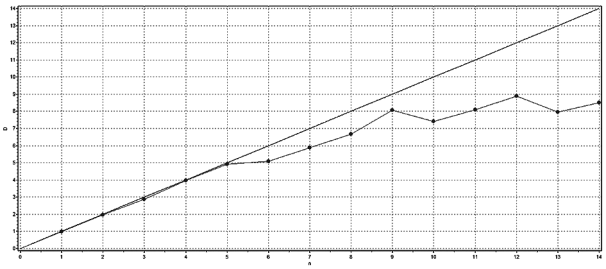


Fig. 3. Graph of dependence of the correlation integral for the time series on the content of silicon in cast iron, on the experimental dimension of the investment

Self-intersections will be absent provided that all neighboring points of the reconstructed attractor R^m will also be neighboring R^{m+1}.

Using this method, it is possible to determine the minimum embedding dimension such that when transition $m + 1$ the number of points of the attractor that are close to each other in R^m and those that are distant in R^{m+1}, will be relatively small.

The calculated value of m by this method determines the minimum embedding dimension where reconstruction of the attractor without self-intersections is possible.

False nearest neighbors algorithm is performed in several stages:

1. At the minimum embedding dimension $m = 1$, the search for the nearest neighbor is performed $\bar{x}(j)$ for each point $\bar{x}(i)$ of the investigated time series.

2. Calculation of the distance $\|\bar{x}(i) - \bar{x}(j)\|$.

3. Calculated distance between these points on the next step $\|\bar{x}(i+1) - \bar{x}(j+1)\|$.

4.Calculated R_i using the following formula

$$R_i = \frac{\|\bar{x}(i+1) - \bar{x}(j+1)\|}{\|\bar{x}(i) - \bar{x}(j)\|}.$$

If $R_i > R_t$ where R_t is threshold (recommended value is 2), then the point $\bar{x}(j)$ is considered a false nearest neighbor relative to the point $\bar{x}(i)$.

In our case, the calculated embedding dimension value using the false nearest neighbours algorithm is 5.

3. The Hurst exponent method. The Hurst exponent method is frequently used in the analysis of time series due to the persistent Hurst exponent, which distinguishes random time series from non-random ones. This method is also known as *R/S* analysis.

R/S analysis method allows identifying the following properties of time series:

1. The value of the Hurst exponent and the corresponding noise color.

2. Presence of long-term memory and estimation of its depth.

3. Presence of trend persistence.

4. Presence of cycles.

For a series of observations $E = \{e_i\}$, $i = 1, 2, 3, ..., N$, where N is a general number of observations, the Hurst exponent is calculated using the following algorithm:

1. A fixed time interval is determined $T = n$, and for each k from $N - n$ the first ones are determined n observations e_1, $e_2, ..., e_n$ and their average value is calculated M_{1n}.

2. A time series of cumulative deviations is generated using the formula

$$x_{1m} = \sum_{i=1}^{m} (e_i - M_{1n}),\ m < n.$$

3. For $k = 2$ a series of observations is selected $e_1, e_2, ..., e_{n+1}$ and stages 1 and 2 are repeated, thereby determining the average M_{2n} and X_{2n}.

4. The calculation process is repeated for all k from 1 to N thereby generating a series of cumulative deviation values X_{km}.

5. The range of the sequence is calculated, the series of cumulative deviation values by formula

$$R_n = \max(X_{km}) - \min(X_{km}).$$

For comparing different types of series, Hurst suggested dividing this range by the standard deviation of the original observations.

Let S be the sample standard deviation of the series $\{e_i\}$, 1, 2, ..., N. Then, normalizing the range R_n by dividing it S, the magnitude $\frac{R_n}{S}$ is determined – dimensionless variation of the series, which increases with the interval $T = n$.

The following regularity was discovered by Hurst

$$\frac{R_n}{S} = (an)^H, \quad (2)$$

where a is constant; n – the number of observations; H – Hurst exponent.

Taking the logarithm of (2), we obtain

$$\ln\frac{R_n}{S} = H \ln n + \ln a. \quad (3)$$

Using the method of least squares, linear regression is performed for points with abscissa $\ln n$ and ordinate $\ln\frac{R_n}{S}$ and the slope of the regression line determines the value of the Hurst exponent, which is an important fractal characteristic of the time series (Fig. 4).

The value of the Hurst exponent can be interpreted as follows:

- $0 \le H < 0.5$ – the time series exhibits anti-persistence (pink noise) and a tendency towards alternating trends;
- $H = 0.5$ – the degree of persistence of the time series is characterized as Brownian motion (white noise) with no trend;
- $0.5 < H < 1$ – means a persistent time series (brown noise) characterized by trend persistence.

Taking into account the Hurst exponent, the fractal dimension can be determined by formula

$$D = 2 - H.$$

For the investigated time series data on silicon content in liquid cast iron at the output of the blast furnace, the Hurst exponent was found to be 0.6189 ± 0.1261. According to formula (3), this determines a fractal dimension of 1.3811.

The obtained estimates of the Hurst exponent and fractal dimension for the analyzed time series indicate that the series exhibits long-term memory and is characterized as persistent. Thus, the calculated values of the fractal dimension obtained using the Hurst exponent method correspond to the conditions of formula (1).

The largest Lyapunov exponent, which characterizes the degree of exponential divergence of close trajectories, having a positive value means that any two close trajectories diverge rap-

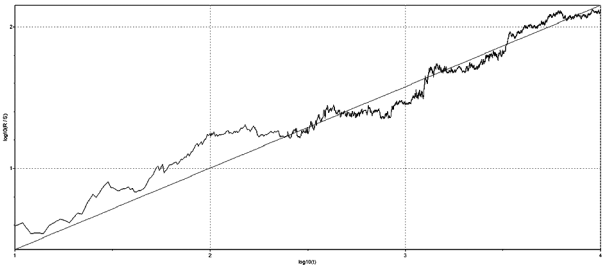


Fig. 4. The R/S analysis plot for the time series on silicon content in cast iron

idly over time. Therefore, the system is sensitive to initial conditions, identifying the dynamic system in terms of the presence of chaotic behavior [16]. The calculation of the largest Lyapunov exponent for the investigated time series was performed using the Matlab software environment and the "lyap_exp" function.

The largest Lyapunov exponent for the time series of silicon content in iron at the blast furnace discharge was calculated to be 0.3521 and 0.3579 for time series 1 and 2, respectively. These values confirm the hypothesis proposed in this study regarding the presence of chaotic properties in the analyzed time series. The magnitude of the largest Lyapunov exponent itself is not highly informative; its sign is of primary importance. A positive value indicates that the analyzed dynamic system exhibits chaotic properties.

In nonlinear dynamic systems, visual assessment plays a crucial role due to the often complex and unpredictable dynamics. Data visualization aids in better understanding the system's behavior. One of the key tools for analyzing nonlinear time series is the spectrogram of the largest Lyapunov exponent (Fig. 5). The spectrogram helps identify characteristic features of chaotic behavior, stability, or instability of the system under investigation. Interpretation of this spectrogram requires careful attention because correct analysis can provide insights into the nature of dynamic processes occurring within the system.

Visual analysis of the spectrogram of the largest Lyapunov exponent helps identify critical transition points, detect phases of stability and instability, and forecast possible changes in system behavior.

In the context of time series analysis, a spectrogram can help visualize changes in the frequency domain of the data series, which can be valuable for analyzing its dynamics and understanding the nature of oscillations [17, 18].

As seen in Fig. 5, the spectrogram predominantly shows yellow color (positive region), with occasional patches of green (closer to negative values), indicating relatively stable periods when the system is in a more predictable state [19, 20]. However, areas of orange and green colors closer to negative values may suggest transitions or shifts towards more chaotic behavior of the system. This could be associated with increased uncertainty or changes in the dynamics of the system, leading to more complex or less predictable behavior.

To demonstrate the capabilities of nonlinear dynamics and chaos theory, a study was conducted on the bifurcation process of time series data from chemical analysis of iron at the blast furnace output over different operational periods.

Phase bifurcation analysis of nonlinear dynamic systems allows identifying bifurcation zones (where the system abruptly changes its behavior). In the conditions of blast furnace production, this may indicate various critical situations such as ore breakthroughs or raw material changes. Characteristics of bifurcation zones include:

1. Unpredictability. Bifurcation zones typically involve several branches of the attractor (stable operating regimes), one of which the system will follow. However, it is impossible to predict in advance which new attractor the system will occupy.

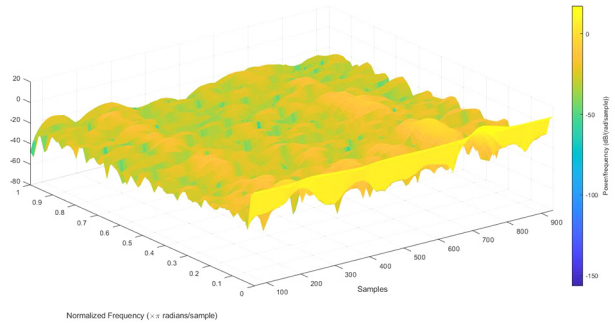


Fig. 5. The 3D spectrogram of the largest Lyapunov exponent for the investigated time series

2. Short-term nature. Bifurcation zones are transient and divide longer stable system regimes.

3. Cascade effects. In certain situations, bifurcation zones can trigger cascade effects, where changes in parameters in one zone provoke further changes in other zones.

On the bifurcation diagram, each point represents a value of the chemical analysis parameter of cast iron at a specific moment in time. The horizontal axis represents time (the number of points), and the vertical axis represents the value of the chemical analysis parameter. When bifurcation points become pivotal moments that alter the structure or dynamics of the system, further data analysis becomes necessary.

For investigating bifurcation zones, a Matlab program was developed to construct a bifurcation diagram. To enhance informativeness, two experiments were conducted analyzing data on chemical composition to identify bifurcation zones. The first experiment utilized 1,000 data points of chemical composition, while the second used 1,400 data points. The program's output resulted in graphs depicted in Fig. 6.

Based on the obtained bifurcation diagrams, the following conclusions can be drawn:

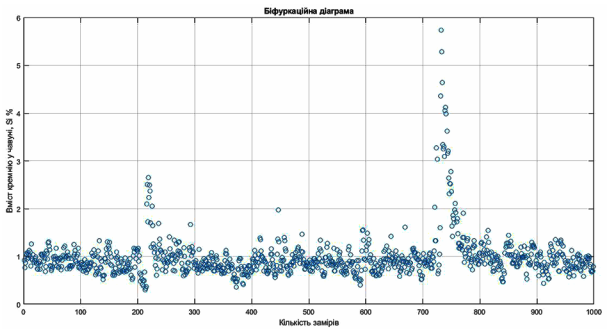
1. The diagrams clearly show points where the chemical analysis values significantly change or abrupt transitions occur. These points may indicate the presence of bifurcations or transitions of the system from one state to another.

2. Stable values of the chemical analysis are visible on the diagrams, indicating the existence of stable states within the system.

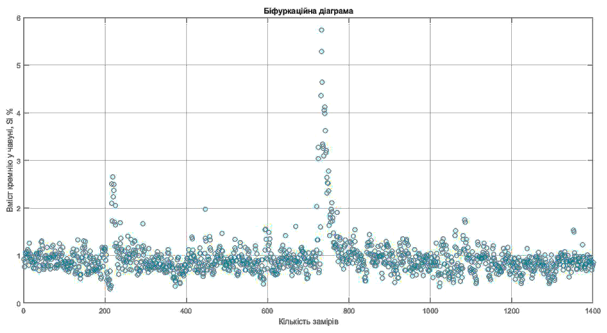
3. Transitions between different values of the chemical analysis or the occurrence of oscillations are observed on the diagrams, suggesting changes in the dynamics of the system (changes in parameters or conditions).

Conclusions. The conducted research allows the following conclusions:

1. The application of traditional methods based on the Gaussian stochastic processes is impractical for analyzing data on the chemical composition of cast iron, as they do not account for the presence of chaotic processes in the investigated time series.



a



b

Fig. 6. Bifurcation diagrams:

a – analysis of bifurcation zones for the time period from 2011-01-01 01:04:00.0 to 2011-05-12 11:56:00.0; b – analysis of bifurcation zones for the time period from 2012-01-01 12:03:00.0 to 2012-04-24 06:03:00.0

2. Experimental studies of time series data on the chemical composition of cast iron allowed for reconstructing a strange attractor of the dynamic system, geometrically represented as a fractal object in phase space. The external appearance of the geometric shape of this time series clearly shows a single core, indicating a unified region of trajectory attraction of the dynamic system.

3. Based on the computed fractal dimension of the attractor, the embedding space dimension was determined, allowing for the estimation of the maximum forecasting period.

4. For visual assessment of the dynamic properties of the investigated nonlinear system, a spectrogram of the largest Lyapunov exponent was constructed, revealing the peculiarities of its behavior.

5. During bifurcation analysis, bifurcation zones and transitions between different states of the system were identified, indicating changes in raw materials or production technology.

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Дослідження стохастичних властивостей часових рядів даних про хімічний аналіз чавуну

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Мета. Забезпечення процедури ідентифікації хаотичних процесів у динамічній системі та перевірка часових рядів, що описують хімічний склад чавуну на випуску доменної печі, з метою ідентифікації нелінійності досліджуваної системи й наявності в ній хаотичних процесів.

Методика. Визначення унікальних характеристик атрактора динамічної хаотичної системи на основі часового ряду значень хімічного складу чавуну виконано методами нелінійної динаміки й теорії динамічного хаосу, такими як метод автокореляційної функції, кореляційної та фрактальної розмірності.

Результати. Методами нелінійної динаміки й теорії динамічного хаосу досліджено характер поведінки часових рядів даних про хімічний склад чавуну на випуску доменної печі. Виявлена наявність у досліджуваній динамічній системі хаотичних процесів, які мають фрактальну структуру, що зумовлює неефективність застосування традиційних методів аналізу, які ґрунтуються на гаусових властивостях стохастичних процесів.

Наукова новизна. Уперше обґрунтована можливість і доцільність застосування методів теорії хаосу для аналізу та прогнозування часових рядів даних про хімічний склад чавуну на випуску доменної печі. Також уперше виконана ідентифікація нелінійності досліджуваної динамічної системи й виявлена в ній наявність хаотичних процесів шляхом визначення унікальних характеристик дивного атрактора системи за аналізованими часовими рядами, таких як розмірність вкладення, часова затримка та значення старшого показника Ляпунова.

Практична значимість. Отримані результати відкривають можливість більш ефективного та якісного аналізу поведінки досліджуваної динамічної системи шляхом розробки нових інструментів оцінки та прогнозу, адекватних характеру досліджуваних процесів.

Ключові слова: *нелінійна динаміка, динамічний хаос, дивний атрактор, фрактальні властивості часових рядів, спектрограма*

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