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ON THE ISSUE OF LOAD'S EXTERNAL BALLISTICS UNDER LOW-SPEED TRANSPORTATION

Purpose. Solution of the three-dimensional nonlinear problem of external ballistics and development of an approximate mathematical model of the dynamic process for cargo falling from low-speed aircraft, which makes it possible to obtain an analytical solution, which is possible in combination with a geometric representation of the dynamic process using computer algebra.

Methodology. Utilizing a blend of analytical and numerical research algorithms, an innovative model was devised, grounded in a nonlinear system of differential equations characterized by time-varying coefficients. The applied three-dimensional approach to the dynamic problem with an initial velocity from a UAV, in the presence of frontal and side wind loading, made it possible to use the nonlinear theory of external ballistics. This streamlining of the problem involved solving a related system of differential equations with variable coefficients along corresponding coordinates, leveraging an asymptotic approach. Furthermore, the formulation of the problem incorporated applied mathematical analysis and modeling, accommodating various pertinent environmental parameters.

Findings. The creation of mathematical models and algorithms for calculating the parameters of the dynamic process of falling loads from low-speed aircraft within the framework of the theory of nonlinear external ballistics is an urgent problem both from the point of view of the development of the dynamic theory of the specified class of systems, and the creation of effective calculation algorithms with the possibility of practical application. As a result of the research, characteristic valuations of the influence of variable coefficients on the results of estimated accuracy of landing as a derivative of time were obtained. The obtained analytical and graphical dependences with the provision of appropriate assessments of the applied approach make it possible to establish the correlation of methods and results.

Originality. The relevance of scientific research in the field of nonlinear external ballistics is based both on the internal trends of the development of this science and on the urgent needs of modern industry. In this paper, an approximate analytical solution of the nonlinear problem of external ballistics is proposed, which is subject to the applied conditions of motion. The resulting dependencies made it possible to establish the relationship between the parameters and find the degree of their influence on the landing time function.

Practical value. The derived analytical findings and solution methodology can be integrated into practical applications within the realms of mathematical physics and engineering computations. This is particularly pertinent in the advancement of control algorithms for ballistic systems.

Keywords: *analytical solution, ballistics, nonlinear system, aerodynamic pressure, wind load*

Introduction. Unmanned Aerial Vehicles (UAVs) are utilized for a multitude of industrial, military, and commercial purposes [1]. Their deployment in place of manned aircraft for these tasks minimizes the risk to human lives while simultaneously keeping operational costs low: manned aircraft require significantly more fuel, higher rental costs for planes or helicopters, and greater logistical support. UAVs offer several distinct advantages, including the ability to access natural and hazardous areas that are difficult or impossible for manned aircraft to reach, high mobility to perform air missions, and the capability to deliver cargo to specified targets according to pre-programmed control instructions [2, 3]. Advances in modern technology and trends in the development of pertinent scientific fields, such as nonlinear mechanics, control theory, miniaturization, and the computerization of complex systems, have enabled the creation of highly effective UAVs in terms of tactical and technical characteristics (TS), operational potential, and cost-effectiveness.

The utilization of air vehicles for cargo delivery to specific targets can be approached in two ways: either by halting over the target or following a traditional drop trajectory. In the former, the load behaves like a free-falling body, whereas in the latter, the cargo, moving as one unit with the air vehicle, follows a defined curve during descent. At low speeds (up to 50 m/s) and altitudes (up to 200 m), one of the main challenges in achieving delivery accuracy is wind presence. The vertical component of air resistance decelerates the fall, while the horizontal component steepens the trajectory compared to a vacuum parabola, causing a lag. The load's fall speed increases until air resistance, which grows with the square of the speed, balances the load's weight.

Modeling the complete flight dynamics of an air vehicle, coupled with environmental factors, involves a multidimensional and highly complex nonlinear system of equations, necessitating the development of advanced guidance algorithms. Therefore, it is more practical to consider low-order nonlinear equations to model behavior within a closed control loop system.

Despite their numerous advantages, UAVs face several operational limitations stemming from critical challenges. These challenges include ensuring flight autonomy, dealing with adverse weather conditions, creating accurate programmable movement trajectories, extending the service life of energy sources, handling limited payload capacity, and maintaining optimal speed to reach targets. The precision of cargo delivery to the designated destination is a crucial parameter that significantly influences the overall performance of the UAV.

Achieving this precision requires a robust algorithm to accurately calculate dynamic characteristics.

There exists a classification system for UAVs based on various features and intended purposes. For instance, paper [4] explores several design types: fixed-wing UAVs, single-rotor helicopters, hybrid designs, and multicopter systems, each deemed effective based on their control mechanisms and high positioning accuracy.

The UAV control system is typically composed of three distinct subsystems: the target recognition and position estimation subsystem, the cargo drop planning subsystem, and the guidance and control subsystem. The target recognition and position estimation subsystem is responsible for determining the fixed coordinates of the ground target. The cargo drop planning subsystem calculates the optimal path from the UAV's current position to the ideal drop point, where the UAV should release its cargo. The guidance and control subsystem then calculates the guidance algorithm along the line of sight of the cargo's landing point. The UAV operator, who receives set values from the target recognition and position estimation subsystem, handles the low-level control of the UAV. This operator also transmits wind parameters to the cargo drop planning and guidance and control subsystems. UAVs can be remotely controlled using various electronic devices such as microprocessors and sensors [5].

Fig. 1 illustrates the general architecture of a UAV, highlighting the communication channels between the satellite and the ground control system. This architecture enables seamless coordination and execution of the UAV's mission, ensuring that all subsystems work together harmoniously to achieve the desired operational outcomes. The integration of advanced technologies and control algorithms within this architecture underscores the sophistication and capability of modern UAV systems.

Literature review. In general, the problem of accurate cargo delivery consists in bringing the aircraft to a calculated point in space, when dropping from which it is ensured that the free-falling cargo hits the target. The paper [6] describes a military high-precision system of aircraft using a controlled parachute to direct the cargo to the target.

To accurately capture and comprehend the dynamic behavior of an air vehicle and its associated cargo, multiple coordinate systems are typically needed. These systems are interconverted through two fundamental operations: rotation and translation.

The objective of this study is to mathematically simulate and analyze the dynamic characteristics of a load considering the velocity of its carrier. This involves accounting for the influence of external environmental factors and identifying the elements that impact the operational characteristics and the parameters of the system's dynamic control. For instance, the authors of the study [7] present the results of modeling in which a probabilistic operation planner is used for unguided landings with parachutes. Optimization of the movement distance required for cargo delivery using parachute landings is considered in [8].

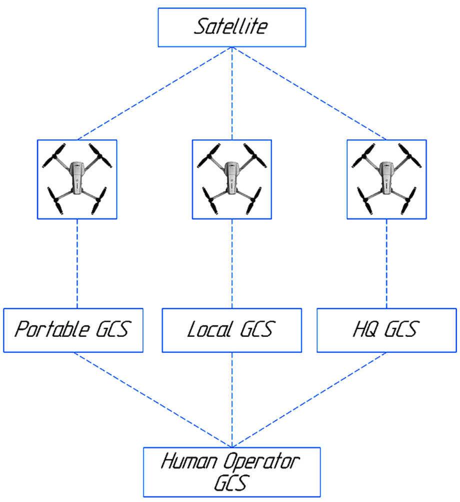

Fig. 1. Architecture of the UAV system

It should be noted that these studies are intended, mainly, for manned aircraft of high speed and significant heights of dropping cargo. From the point of view of mathematical modeling of the dynamic process of such systems and the solution of the corresponding related nonlinear system of differential equations, as a rule, direct numerical, hybrid analytical-numerical or combined algorithms are applied with the use of design experience and experimental data.

Purpose. Statement of the three-dimensional nonlinear problem of external ballistics and an approximate mathematicalr model of the dynamic process for falling cargo from low-speed aircraft, which makes it possible to obtain an analytical solution and, in combination with a geometric representation of the dynamic process using computer algebra, to obtain analytical and graphical dependencies with the provision of appropriate assessments of the applied approach are proposed.

Methods. In the paper [9], a methodical approach to solving the problem of aiming during the delivery of cargo by parachute from transport UAVs was developed. The solution of the aiming problem is reduced to the calculation of the required longitudinal and lateral coordinates, determination of the moment of reset by comparing them with the current longitudinal and lateral coordinates (distances) of the UAV to the specified point of fall, which ensures the hitting of the free-falling cargo at the specified point [10, 11].

The creation of mathematical models and algorithms for calculating the parameters of the dynamic process of falling loads from low-speed aircraft within the framework of the theory of nonlinear external ballistics is an urgent problem both from the point of view of the development of the dynamic theory of the specified class of systems, and the creation of effective calculation algorithms with the possibility of practical application [12, 13].

Results. The problem of determining the parameters of external ballistics and the functioning of real structures of UAVs and cargo in the presence of an initial speed in the conditions of atmospheric pressure, frontal and side wind load is connected with the need to integrate a spatial system of nonlinear differential equations, which is a rather difficult task when using direct numerical approaches, especially if it is necessary to obtain estimates of the efficiency of the calculation algorithm. It should be noted that exact analytical solutions of the specified problem, which are reduced to a system of nonlinear differential equations, can be obtained only in exceptional cases [14, 15].

In [16], the author gives the equation of motion of a particle falling in a constant gravitational field with a resistive medium

$$\bar{F} = \bar{F}_g + \bar{F}_r,$$

where F_g is the force of gravity; F_r is the braking force in a resistive environment.

This dependence can be rewritten as

$$\bar{F} = m\bar{g} + \bar{F}_r(V).$$

At the same time, it is enough to assume that $F_r(V)$ is proportional to some extent to the speed. This type of approximation is considered by the author as

$$\bar{F} = m\bar{g} - mkV^n \frac{\bar{V}}{V},$$

where k is a positive constant that determines the braking force; V is a unit vector in the direction $\frac{\bar{V}}{V}$, where V is the

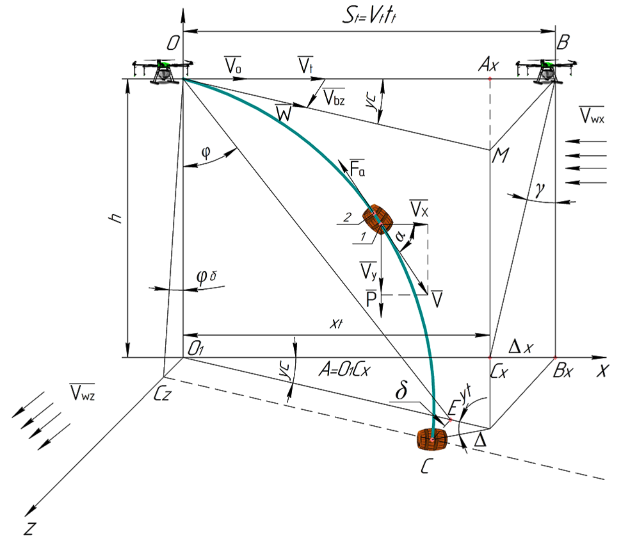

Fig. 2. Coordinate system and cargo loading scheme at the point of the trajectory

wind velocity relative to the direction of the velocity vector of the falling load.

Equations of motion can then be developed in a polynomial form that allows the application of the Carleman technique.

In the paper, mass m_0, moving with an initial speed V_0 from an UAV of height h, in the presence of frontal V_{wx} and side V_{wx} wind loading, based on the nonlinear theory of external ballistics, is considered [17]. The geometry of the three-dimensional coordinate system and the diagram of the cargo load at the point of the trajectory are presented in Fig. 2.

In external ballistics, various forces and moments caused by air resistance play an important role in determining where the payload will end up.

To determine the aerodynamic drag forces acting on a body during descent, it is essential to consider its shape indirectly. Given the environmental heterogeneity, such as turbulence and non-laminar flow, analytically determining air resistance can lead to significant integral errors. A straightforward, accurate formula for air resistance derived from theoretical conclusions is elusive. Typically, air resistance is measured experimentally by testing the body in a wind tunnel. However, employing numerical modeling methods and analyzing the results enhance the accuracy of the data obtained and allow for iterative modeling of various geometric parameters under different initial conditions. Through modeling, one can determine the coefficient of aerodynamic resistance as a value proportional to speed pressure and the force of air friction along aerodynamic surfaces using turbulent flow models.

The static stability of a load refers to its ability to return to a specified flight path after being released. To determine the static stability, one must identify the positions of the load's center of mass and center of pressure. During free fall, wind forces and the moment arm created by the distance between the center of mass and the center of pressure induce rotational motion around the center of mass within the planes of available degrees of freedom.

The static stability margin of a load is calculated as the ratio of the distance from the center of mass to the center of pressure to the diameter of the load's body (assuming it has an axis of symmetry). Typically, a high static stability margin (≥ 1) results in more stable rectilinear motion and greater resistance to external forces that could alter the trajectory.

A spatial nonlinear system of forces is considered, taking into account the nonlinear components of the movement of a material point [18]. According to Newton's second law, the basic system of differential equations of the external ballistics of the dynamic process under study follows from the projections of the forces acting on the mass m_0, on the axis of the coordinate system x, y and z respectively

$$\sum x: \frac{d}{dt}\left[m_0(V_x+V_0\mp V_{wx})\right] = -F_t\sin\alpha - F_{wx} + R_x(t); \qquad (1)$$

$$\sum y: \frac{d}{dt}\left[m_0V_y(t)\right] = F_t\cos\alpha - m_0g + R_y(t); \qquad (2)$$

$$\sum z: \frac{d}{dt}\left[m_0(V_Z\mp V_{wz})\right] = -F_t sin - F_{wz} + R_z(t). \qquad (3)$$

The main parameters of the studied system are determined by the following formulas [10]

$$F_t = C_x\frac{S_m\rho}{2}V^2(t); \quad F_{wx} = C_{wx}\frac{S_{mx}\rho}{2}V_{wx}^2;$$

$$F_{wz} = C_{wz}\frac{S_{mz}\rho}{2}V_{wz}^2,$$

where F_t, F_w are forces of aerodynamic resistance of cargo with a stabilizer, including $\bar{F}_{st}$ and wind load, respectively; C_x is the drag coefficient (determined from experimental data in the direction of movement); C_{wx}, C_{wz} – the coefficients of lateral resistance, which is calculated from the given initial conditions in the direction of the X and Z axes; S_m, S_{mx}, S_{mz} – the areas of the middle of the body along the axis of rotation of the body and in orthogonal directions; ρ – air density; $R(t)$ – control function.

Aerodynamic drag is the resistance of an object moving in a medium such as air or water. Aerodynamic resistance does not depend on the weight of the cargo. As mentioned above, in external ballistics, various forces and moments caused by air resistance play an important role in determining where the payload will end up. Drag modeling is quite a serious problem, as it is very difficult to analytically model the aerodynamic coefficients C_x, C_{wx}, C_{wz}, and often almost impossible due to the complex behavior of the air flow around the irregularly shaped cargo. According to Fig. 2, the angle of the direction of the velocity of the body is determined by the dependencies in the XOY plane

$$\sin\alpha = \frac{V_x}{V}; \quad \cos\alpha = \frac{V_y}{V}; \quad \sin = \frac{V_z}{V}.$$

For the formula cargo velocity

$$V(t) = \sqrt{V_x^2+V_y^2+V_z^2} = V_y\sqrt{1+\frac{V_x^2}{V_y^2}+\frac{V_z^2}{V_y^2}} \approx V_y(t).$$

Inequalities are accepted

$$\frac{V_x^2}{V_y^2}<1; \quad \frac{V_z^2}{V_y^2}<1.$$

Entering to simplify the entry definition $\frac{d_x}{d_t} = \dot{x}$ equation (1) is obtained in the form

$$m_0\dot{V}_x(t) = -F_t\sin x - F_{wx} + R_x(t), \qquad (4)$$

or

$$\dot{V}_x(t) = \frac{F_t}{m_0}\frac{V_x}{V} - \frac{F_{wx}}{m_0} + \frac{R_x(t)}{m_0}. \qquad (5)$$

Introducing the definition for further simplification

$$\dot{V}_x(t) = -\frac{C_x}{2}\frac{S_m\rho}{m_0}V_x(t)V_y(t) - \frac{C_{wx}}{2}\frac{S_m^*\rho}{m_0}V_{wx}^2; \qquad (6)$$

$$\dot{V}_x(t) = -\varepsilon b_0V_x(t)V_y(t) - \bar{F}_{wX} + \bar{R}_x, \qquad (7)$$

where

$$b_0 = \frac{S_m \cdot \rho}{m_0}; \quad \varepsilon = \frac{C_x}{2}. \tag{8}$$

Taking into account the new definitions (8), the equation (7) is obtained in the form

$$\dot{V}_x(t) + \varepsilon b_0 V_x(t) V_y(t) = -\bar{F}_{wX} + \bar{R}_x, \tag{9}$$

where

$$\bar{F}_{wX} = \frac{C_{wx}}{2} \cdot \frac{S_{wm}\rho}{m_0}. \tag{10}$$

Finally, the differential equation of the problem in the projection onto the horizontal axis *X* has the form

$$\dot{V}_x(t) + \varepsilon b_0(t) V_y(t) = \varnothing(t), \tag{11}$$

where

$$\varnothing(t) = -\bar{F}_w + \bar{R}_x. \tag{12}$$

Applying new dimensionless parameters

$$X = \frac{x}{h}; \quad Y = \frac{y}{h}; \quad Z = \frac{z}{h}; \quad T = \frac{t}{t_0}, \tag{13}$$

where $t_0 = \sqrt{\frac{2h}{g}}$ is time of free fall of the load in a non-atmospheric environment.

From the system [1–3] we obtain

$$\sum X : \frac{d}{dT}\left[V_X(T)\right] = -\bar{F}_t \sin\alpha - \bar{F}_{wX} + \bar{R}_x(T); \tag{14}$$

$$\sum Y : \frac{d}{dT}\left[V_y(T)\right] = \bar{F}_t \cos\alpha + 2 - \bar{R}_y(T); \tag{15}$$

$$\sum Z : \frac{d}{dT}\left[V_Z(T)\right] = -\bar{F}_t \sin - \bar{F}_{wX} + \bar{R}_z(T). \tag{16}$$

The possibility of applying the approximate analytical approach to the solution of the simplified system of differential equations (14–16) is considered in this study. This approach offers a valuable method for tackling complex dynamic processes. The nature of the dynamic process, particularly due to the presence of the control function $\bar{R}(T)$, can be investigated further on the basis of the proposed calculation algorithm. This allows for a deeper understanding of how the control function influences the overall system behavior.

Taking into account the introduced definitions and initial conditions, the system of differential equations (14–16) is transformed into a system of related nonlinear differential equations. This transformation is crucial as it provides a more comprehensive framework for analysis. The main equation in this system, from the point of view of the possibility of direct analytical integration, is the nonlinear Riccati differential equation with respect to the ordinate axis. This particular equation is of significant importance because it encapsulates the core dynamics of the system.

The nonlinear Riccati differential equation can be further simplified and reduced to a linear differential equation of the second order. This reduction is a pivotal step, as it transforms a complex, nonlinear problem into a more manageable linear one, facilitating the application of established analytical methods. The ability to reduce the equation to a second-order linear differential equation provides a clearer pathway for obtaining precise solutions, which are essential for both theoretical insights and practical applications.

By transforming the original system of differential equations into a related set of nonlinear equations, the approach highlights the interdependencies between various factors influencing the system. The resulting solutions not only enhance our understanding of the system dynamics but also improve the accuracy and reliability of predictions and control strategies.

$$\sum X : \frac{d}{dT}\left[V_X(T)\right] = -Bo V_X(T) V_Y(T) - \bar{F}_{wX}; \tag{17}$$

$$\sum Y : \frac{d}{dT}\left[V_Y(T)\right] = -Bo\left[V_Y(T)\right]^2 + 2; \tag{18}$$

$$\sum Z : \frac{d}{dT}\left[(V_Z(T)\right] = -Bo V_Z(T) V_Y(T) - \bar{F}_{wz}. \tag{19}$$

Where we enter the notation

$$Bo = \varepsilon \cdot b_0 \cdot h; \quad \bar{F}_{wX} = \frac{t_0^2}{h} F_{wx} = \tilde{n}_{wx} \cdot \frac{S_{mx} \cdot \rho}{2} \cdot V_{wx}^2 \frac{t_0^2}{h};$$

$$\bar{F}_{wz} = \frac{t_0^2}{h} F_{wz} = \tilde{n}_{wz} \cdot \frac{S_{mz} \cdot \rho}{2} \frac{t_0^2}{h} \cdot V_{wx}^2 \frac{t_0^2}{h}.$$

To reduce equation (18) to a linear differential equation of the second order, the definition is introduced

$$q_0 = 2, \quad q_i = -\text{Bo}, \tag{20}$$

where q_i ($i = 0, 1$) – are the coefficients of the corresponding components of equation (23), and the substitution

$$u(T) = V_y(T) q_i. \tag{21}$$

Which brings the initial equation (18) to the form

$$\frac{d}{dT}\left[u(T)\right] = \left[u(T)\right]^2 + q_0 q_i. \tag{22}$$

Representing the function $u(T)$ as

$$u(T) = -\frac{dV(T)/dT}{V(T)}. \tag{23}$$

We obtain, taking into account (22), equation (18) in the form of a second-order linear differential equation

$$\frac{d^2V(T)}{dT^2} - Q_0 V(T) = 0. \tag{24}$$

Which allows for constant coefficients an exact analytical solution

$$u(T) = C_1 \,\text{sh}[\lambda_0 T] + C_2 \,\text{ch}[\lambda_0 T], \tag{25}$$

where $\lambda_0 = Q_0^{0.5} = (2B_0)^{0.5}$.

The function $u(T)$ is obtained by dependence (23)

$$u(T) = -\lambda_0 \frac{\left[C_1 \,\text{ch}(\lambda_0 T) + C_2 \,\text{sh}(\lambda_0 T)\right]}{\left[C_1 \,\text{sh}(\lambda_0 T) + C_2 \,\text{ch}(\lambda_0 T)\right]}. \tag{26}$$

Under the condition

$$V_y(0) = u(0) = 0, \quad C_1 = 0.$$

Then the sought function $V_y(0)$ has the form

$$V_y(T) = \frac{u(T)}{q_i} = -\frac{u(T)}{B_0} = -\left(\frac{\lambda_0}{B_0}\right) \text{tg}h\left[\lambda_0 T\right]. \tag{27}$$

The limits of the existence of the equation were determined by analyzing it using a given set of initial conditions. The initial horizontal speed is equal to the speed of the UAV, and the initial vertical speed is 0 m/s. Horizontal motion was found to decelerate continuously from V_0 to a stopping velocity of 0 m/s. Therefore, the horizontal speed is limited from 0 to V_0. It was also found that the vertical velocity continuously decelerates non-linearly until the load either hits the ground or stops accelerating due to drag (terminal velocity). Depending on which model order was chosen, different degrees of accuracy can be obtained for the approximation.

Finally, we have obtained the dependence, (27) in analytical form

$$V_y(T) = -\left(\frac{2}{B_0}\right)^{0.5} \operatorname{tg}h\left[(2B_0)^{0.5}T\right]. \quad (28)$$

The projection function of displacement in the direction of the ordinate axis is defined in the form

$$S_y(T) = \left(\frac{1}{B_0}\right)\ln\left\{\operatorname{ch}\left[(2B_0)^{0.5}T\right]\right\} + d_1. \quad (29)$$

Under the condition $S_y(0) = -1$, $d_1 = -1$, we determine the time to achieve the goal

$$T_t = Gr^{0.5}\operatorname{arch}\left[\exp\left(\frac{2}{Gr}\right)\right], \quad (30)$$

where $Gr = \frac{1}{2B_0}$ is the "quality" parameter of the studied system [11], which can serve as a criterion for the application of the proposed analytical approach to solving the problem of external ballistics under conditions of atmospheric pressure according to the inequality $T_t > 1$

$$Gr^{0.5}\operatorname{arch}\left[\exp\left(\frac{2}{Gr}\right) > 1\right]. \quad (31)$$

The criterion for applying the analytical approach to the Gr parameter is established by the graphical representation (Fig. 3) of the function (31)

$$y = \operatorname{arch}\left[\exp(B_0)\right] = \frac{1}{2B_0^{0.5}} = \operatorname{arch}\left[\exp(2Gr)\right] = \frac{1}{Gr^{0.5}}.$$

Flight dynamics encompass the trajectories of flying objects as well as the stability and controllability issues encountered during their movement. The study of trajectory tasks assumes that a flying object acts as a material point, moving under the influence of applied forces. When analyzing the stability and controllability of a flying object, it is treated as a material body influenced by the moments of these forces.

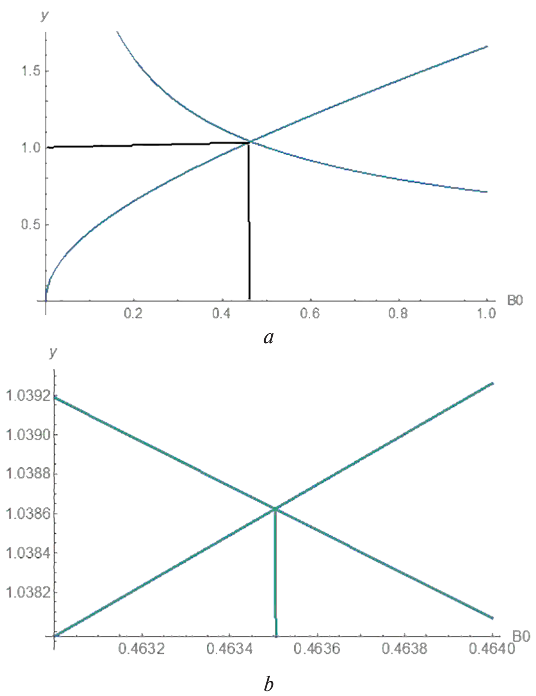

Fig. 3. Criterion for applying an analytical approach to the solution of the problem of external ballistics according to the B_0 parameter

Most mechanical systems exhibit nonlinear properties under certain external disturbance parameters. Thus, examining a flat nonlinear system of forces, considering the nonlinear components of a material point's motion with time-varying parameters should result in the external ballistics arguments.

As evidenced by the graphical representation of the function $y = y(B_0^*)$ (Fig. 3), the criteria for applying the proposed approach to solving the research problem are values at which $B_0^* = 0.475$, $Gr^* = 1.05$.

Under the condition of equation (1), after substituting (30) into (17), we obtain an equation in the form

$$\frac{d}{dT}\left[V_X(T)\right] + G_0(T)V_X(T) = -P_x, \quad (32)$$

where parameters

$$G_0(T) = B_0\left[\left(\frac{2}{B_0}\right)^{0.5} \operatorname{tg}h\left[(2B_0)^{0.5}T\right]\right];$$

$$P_x = B_{wX}(V_{wX} + V_{0X})^2; \quad B_{wX} = \tilde{n}_{wX}\frac{S_{mX}\cdot\rho}{2}\frac{t_0^2}{h}V_{wX}^2 t_0^2,$$

where S_{mX} is the area of the middle in the direction X.

Equation (32) is a non-homogeneous differential equation with variable coefficients with respect to the desired function $V_x(T)$, the solution of which is obtained by a standard procedure using the method of variation of arbitrary constants to obtain a partial solution.

It should be noted that the displacement projection function in the direction of the Z axis is obtained by a similar algorithm. The total distance from the cargo drop point to the target is given by the dependence $S = \left[S_x^2 + S_y^2 + S_z^2\right]^{0.5}$.

Carrying out field tests with load dropping will allow one to identify and classify possible errors in order to evaluate the quality of the developed control algorithm and introduce the necessary corrections. At the same time, possible sources of errors can be grouped into sources of errors affecting the quality of a separate on-board subsystem of the UAV. Where the payload drop error is the distance from where the payload landed to the target, the drop point error is the distance from the calculated drop point to the position where the UAV actually released the payload, and the drop velocity error is the speed the UAV had when releasing the payload, minus the speed he was supposed to have when releasing the cargo, etc.

A comparison of the analytical solution with direct numerical integration and the asymptotic solution [5] of the basic differential equation with respect to the ordinate axis for selected parameters of the system under study is presented in Fig. 4.

A comparison of an approximate analytical solution with a direct numerical calculation for given system parameters is

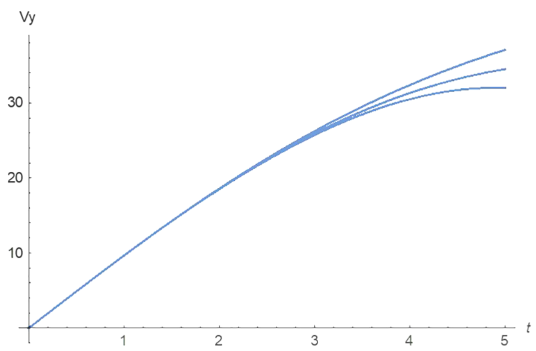

Fig. 4. Comparison of the direct numerical calculation for nonlinear basic differential equation with respect to ordinate axis with asymptotic solutions with two and three approximations

presented in Fig. 4. According to the target achievement time the errors of the asymptotic solution are 11.9 and 6 %: in two and three approximations respectively.

Thus, the nonlinear external ballistic model is closer to the model of a real physical process, and the numerical solution will allow better predicting the trajectory of the fall of the cargo given its parameters.

Conclusions. The mathematical model of the researched process, created on the basis of the proposed approach, is formulated as a system of coupled nonlinear differential equations. The core of this system is a first-order nonlinear Riccati differential equation related to the function of the projection of the velocity of the falling cargo onto the ordinate axis. This first-order nonlinear equation is subsequently transformed into a second-order linear differential equation. This transformation facilitates the derivation of an analytical solution, and when combined with a geometric representation of the dynamic process using computer algebra, it allows for the extraction of analytical and graphical dependencies. These dependencies provide appropriate estimates of the efficacy of the applied approach.

The proposed model offers an approximate analytical solution to the nonlinear problem of external ballistics for low-speed aircraft under specific movement conditions. A critical factor in determining the landing point of the load is aerodynamic drag, which significantly influences the load's trajectory by exerting the most considerable force at the beginning of the drop.

To obtain a comprehensive understanding of the process, the mathematical model incorporates various parameters that affect the dynamic behavior of the falling load. These parameters include the initial velocity, altitude, angle of release, and the characteristics of the surrounding environment such as air density and wind speed. By adjusting these parameters, the model can simulate different scenarios, providing valuable insights into the behavior of the load under varying conditions.

The model's primary dependencies and solution approach are highly versatile, enabling their application in direct numerical research methods. This versatility extends to practical applications, where the model can be used for the further analytical analysis of dynamics and control problems related to falling loads from unmanned aerial vehicles (UAVs). These applications are particularly relevant when dealing with time-varying parameters of the investigated system and external environmental conditions.

The model's adaptability to different conditions and its integration with computer algebra systems make it a valuable asset for both theoretical research and practical applications in fields such as military logistics, emergency response, and aerospace engineering.

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До питання зовнішньої балістики падаючих вантажів з літальних апаратів малої швидкості

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Мета. Вирішення тривимірної нелінійної проблеми зовнішньої балістики й формулювання наближеної математичної моделі динамічного процесу падіння вантажу з повітряних суден за низької швидкості, для отримання аналітичного розв'язку, що можливе при поєднанні із геометричним зображенням динамічного процесу за допомогою комп'ютерної алгебри.

Методика. Використання комбінації аналітичних і числових алгоритмів досліджень застосовано для отримання інноваційної моделі, ґрунтуючись на нелінійній системі диференціальних рівнянь зі змінними за часом коефіцієнтами. Застосований тривимірний підхід до динамічної задачі із наявною початковою швидкістю безпілотного літального апарату за умови наявності фронтального й бічного вітрового навантаження надало змогу застосувати нелінійну теорію зовнішньої балістики. Зазначене спрощення проблеми включало вирішення відповідної системи диференціальних рівнянь змінних коефіцієнтів уздовж відповідних координат за допомогою асимптотичного підходу. Крім того, формулювання проблеми враховує прикладний математичний аналіз і моделювання з урахуванням різних актуальних параметрів зовнішнього середовища.

Результати. Створення математичних моделей і алгоритмів для обчислення параметрів динамічного процесу падіння вантажів із повітряних суден за низької швидкості в межах теорії нелінійної зовнішньої балістики є актуальною проблемою як із точки зору розвитку динамічної теорії вказаного класу систем, так і створення ефективних обчислювальних алгоритмів з можливістю практичного застосування. У результаті досліджень отримані характеристичні оцінки впливу змінних коефіцієнтів на результати оціночної точності приземлення як похідної часу. Отримані аналітичні та графічні залежності з наданням відповідних оцінок застосованого підходу надають змогу встановити кореляцію методів і результатів.

Наукова новизна. Актуальність наукових досліджень у галузі нелінійної зовнішньої балістики ґрунтується як на внутрішніх тенденціях розвитку цієї науки, так і на нагальних потребах сучасної промисловості. У роботі запропоновано приблизний аналітичний розв'язок нелінійної проблеми зовнішньої балістики, що визначається умовами руху. Отримані залежності надали змогу встановити взаємозв'язок між параметрами та встановити ступінь їх впливу на функцію часу приземлення.

Практична значимість. Отримані аналітичні результати й методологія розв'язання можуть бути інтегровані у практичні застосунки математичної фізики та інженерних обчислень. Це особливо актуально для розвитку алгоритмів управління балістичними системами.

Ключові слова: *аналітичний розв'язок, балістика, нелінійна система, аеродинамічний тиск, вітрове навантаження*

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