https://doi.org/10.33271/nvngu/2024-2/140

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DETERMINING THE PARAMETERS OF THE FUNCTIONING FOR A NONLINEAR BALLISTIC SYSTEM IN A REAL EXTERNAL ENVIRONMENT

Purpose. Development of an approximate nonlinear model for the solution of the external ballistics problem with determination of the nonlinear system parameters and development of a methodology for calculating the aerodynamic characteristics of the cargo, located on the external suspension of unmanned aerial vehicle (UAV) in order to increase the efficiency of their delivery to the specified landing target by means of asymptotic approach with given parameters of the studied system and external load.

Methodology. The development of an effective model was carried out using analytical and numerical research algorithms based on a nonlinear system of differential equations in a general form with time-varying coefficients. In order to obtain a solution to the nonlinear problem of external ballistics in a two-dimensional formulation, the assumption of a significant influence of the projection of the velocity function on the ordinate axis in relation to the component on the abscissa axis is introduced. The problem is reduced to the solution of a related system of differential equations with variable coefficients along the corresponding coordinates using the asymptotic approach for a small parameter of the coefficient of frontal aerodynamic resistance. Applied mathematical analysis and modeling have been used for the problem formulation considering studied environmental parameters.

Findings. Analytical dependences of the nonlinear problem of ballistics and application of finite-element analysis (FEA) with respect to the cargo motion from the UAV in the presence of the initial speed and wind load in the plane of motion are proposed. It is shown that the obtained analytical solution is correlated with the direct numerical calculation of the basic differential equation with respect to the ordinate axis.

Originality. A mathematical nonlinear model of the dynamic process is proposed, assuming the prevailing influence of the speed function along the ordinate axis compared to the function along the abscissa axis. To obtain an approximate analytical solution of the basic nonlinear system of differential equations with variable coefficients the asymptotic perturbation method is applied. The dependence for the axial displacement function is presented considering actual time-flight parameter.

Practical value. The obtained analytical dependencies for estimating the time and distance reaching the target with the initial speed of movement and the presence of wind load can be used in applied problems of mathematical physics and engineering calculations of functional dependencies and control of the cargo delivery process and target reaching from an UAV. The obtained analytical results and the solution algorithm can be integrated into applied problems of mathematical physics and engineering calculations, particularly the development of ballistic system control algorithms.

Keywords: *ballistics, FEA, mathematical model, dynamics, UAV, perturbation, aerodynamic resistance*

Introduction. Nowadays, unmanned aerial vehicle (UAV) have been involved in logistics tasks, cargo transportation in various ways. In connection with the lack of risk of expensive manned aircraft loss, the use of UAVs becomes relevant.

The use of air vehicles for the cargo delivery in the conditions of man-made disasters consequences liquidation and the determination of the characteristics of the complex dynamic systems components, taking into account the aerodynamic parameters and the influence of the external environment, is relevant from the point of view of improving their operational characteristics. One of the effective methods for delivering cargo to a given target is the free-falling method of dropping from low heights and flight speeds of the carrier [1, 2].

A significant number of publications in the field of research is related to the creation of numerical and analytical mathematical models built based on the theory of external ballistics [3]. The peculiarity of the aerodynamic load is that its value significantly depends on the shape of the cargo flowing around the wind stream.

One of the most important problems of unmanned aircraft vehicles usage UAV is the delivery of cargo by dropping from external rigging devices to a strictly predefined target point [1, 2].

However, during the rigging and subsequent dropping of cargo with the same masses while the parameters and environmental conditions are the same, but with different ballistic characteristics, there are significant deviations of the experimental data and analytical results [3]. The ballistic model's quality of the dynamics process according to the predefined goal depends on the geometric parameters of the cargo and its shape, associated with the characteristics of aerodynamic resistance, initial velocity, and wind loading.

To determine the aerodynamic drag forces that act on a body during a fall, it is necessary to indirectly consider the shape. However, considering the heterogeneity of the environment, non-laminar flow etc., the analytical determination of air resistance can introduce a high degree of integral error. The force of air resistance cannot be expressed by a simple and accurate formula obtained based on theoretical conclusions. Air resistance is usually determined experimentally. However, the use of numerical methods of modeling and analysis of the obtained results allows one to increase the accuracy of the obtained data and provide multi-iteration modeling of the n^{th} number of geometric parameters of the body under different initial conditions. When modeling, it is possible to determine the coefficient of aerodynamic resistance, as a value proportional to the speed pressure and the force of air friction along the aerodynamic surfaces using turbulent flow models. Excluding shape influence, there must be mentioned some parameter, which facilitates the function of ballistics quality influenced by environment and initial conditions.

For the purposes of cargo delivery using UAV, special onboard technical equipment is used, which includes navigation, ground flight support and external suspension devices, etc. Accelerometers, angular velocity sensors, pressure sensors, magnetometers, GPS are most often used on UAV's board, which allow obtaining objective online information. At the same time, most UAV in recent years have been modernized from radio-controlled to semi-fully autonomous control systems.

Therefore, the task of determining the cargo drop point, considering real flight parameters with time-varying characteristics, is relevant for the development of ballistic system control algorithms.

Literature review. To ensure accurate dropping of the cargo at the given point, it is necessary to solve the aiming problem, which in general consists in bringing the UAV to the calculated point, which dropping from is ensured that the cargo reaches the given site. The cargo drop point must be calculated taking into account the actual flight parameters that change.

In the methodological approach considered in paper [4], the point of dropping the cargo by parachute from UAV is calculated considering the actual flight parameters, ballistic characteristics of the cargo, the direction and velocity of the averaged wind, the method for determining which is proposed. However, as practical tests have shown, the ballistic characteristics do not fully reflect the real properties of the cargo along the trajectory. As a result, the use of defined ballistic characteristics can lead to a change in the trajectory of the cargo.

As it is stated in [4], the authors have developed methodical approach to solving the problem of the cargo delivery by parachute from UAV, taking into account required longitudinal and lateral coordinates, determination of the release moment by comparing the current longitudinal and lateral coordinates of the UAV to the specified target point, which ensures that the cargo will reach the specified point. The position of the cargo drop point in space can be specified by coordinates.

There are a significant number of publications in the denoted research area that are associated with the application of both the linear [5, 6] theory of ballistics, which allows assessing dynamic process's main parameters, and the nonlinear ones [7, 8], in which research results involve the use of special functions, that complicates engineering calculations to a certain extent.

Thus, in [5], a table is provided of the Lambert function, with described calculation method, which significantly simplifies the determination of the flight range of a particle in a gaseous medium with linear resistance to motion and can be concerned as background for the engineering task in external ballistics formulation. Furthermore, in [7] there is proposed a technique of approximate integration of ballistics equations, that allows one, without changing the physical nature of the forces acting on the particle, to achieve any accuracy of calculations of the parameters of the particle's movement in each area. Dividing the obtained time range into equal intervals simplifies calculations using proposed method. Averaging obtained velocity values accelerates the convergence of calculations and prevents manifestations of instability of the solutions. The accuracy of the solutions increases with an increase in the number of intervals in time ordinates, an increase in the degree of polynomials and the number of their refinements, but it is shown that three intervals are sufficient.

However, there are absent solved tasks of external ballistics, which include environmental variables and possible dynamic influence of side forces in the free-falling object, such as dropped cargo.

Purpose. As mentioned above, the purpose of the paper is development of an approximate nonlinear model for the solution of the external ballistics problem with determination of the nonlinear system parameters and development of a methodology for calculating the aerodynamic characteristics of the cargo, located on the external suspension of UAV in order to increase the efficiency of their delivery to the specified landing target by means of asymptotic approach with given parameters of the studied system and external load.

Methods. In order to obtain a solution to the nonlinear problem of external ballistics in a two-dimensional formulation, the assumption of a significant influence of the projection of the velocity function on the ordinate axis in relation to the component on the abscissa axis should be created. The problem is reduced to the solution of a related system of nonlinear differential equations with variable coefficients along the corresponding coordinates using the asymptotic approach for a small parameter of the coefficient of frontal aerodynamic resistance. Development of an effective model by means of analytical and numerical research algorithms [9, 10], based on a multidimensional nonlinear system of differential equations, generally with time-varying coefficients [11, 12], is a rather complex problem of dynamic process management [13, 14]. The results in [15, 16] should be noted, where modeling of the system behavior with a closed control loop is based on a nonlinear approach [17].

Results. To develop an approximate analytical-numerical approach to solving the nonlinear problem of external ballistics and develop a calculation algorithm, the calculation scheme of cargo motion influenced by the external forces at the point of the trajectory in the one-dimension view, which are presented in (Fig. 1), is considered.

According to the paper [18], the solution of the problem is based on a mathematical model, which is described by a nonlinear inhomogeneous differential equation with variable coefficients of the Ricatti equation [17], the general solution of which requires a partial solution, obtaining which is a rather difficult problem

$$\begin{cases}\dot{V}_x(t)+\varepsilon\cdot b_0\cdot V_x(t)\cdot V_y(t)=\varnothing_0\\ \dot{V}_y(t)=\varepsilon b_0 V_y^2(t)-g\end{cases}.$$

Flight dynamics includes several variables, like flying object trajectories, as well as issues of stability and controllability during its movement. The study of trajectory movement tasks is carried out under the assumption that a flying object is a material point that moves under the action of applied forces. While studying the stability and controllability of a flying object, it is considered as a material body moving under the action of moments of this force system.

Most dynamics problem occur nonlinear properties under certain parameters of external disturbance. In this paper, a flat nonlinear system of forces is considered, taking into account the nonlinear components of the motion of a material point with time-varying parameters. According to Newton's second

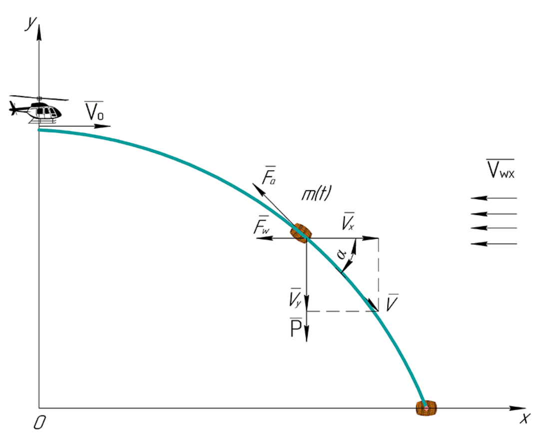

Fig. 1. Scheme of object motion and acting forces along the trajectory at instant point

law, the basic system of differential equations of the external ballistics of the dynamic process under study follows from the projections acting on the mass forces on the axis of the coordinate system *X* and *Y*.

To obtain an approximate analytical solution of nonlinear differential equations with variable coefficients of the investigated system, the asymptotic perturbation method with a small parameter is used ε [19, 20]

$$V_y(t) = Y_0(t) + \varepsilon Y_1(t).$$

Substituting expression (1) into equation (17) [18], we obtain the following

$$\dot{Y}_0(t) + \varepsilon \dot{Y}_1(t) = \varepsilon b_0 \cdot \left[Y_0(t) + \varepsilon Y_1(t) \right] - g.$$

Multiple coordinate systems are generally required to obtain and understand the dynamic behavior of an air vehicle and its associated cargo.

Equating the coefficients with the same degrees of the development parameter ε, we obtain a system of differential equations for determining the required functions $Y_0(t)$ and $Y_1(t)$

$$\varepsilon: \quad \dot{Y}(t) = -g; \quad dy(t) = -gdt; \tag{1}$$

$$V_0(t) = -gt + C_2;$$

$$\varepsilon': \quad \dot{Y}_1(t) = b_0 Y_0(t);$$

$$\dot{Y}_1(t) = b_0 \cdot \left[-gt + C_2 \right] = b_0 \left[g^2 t^2 - 2gtC_2 + C_2^2 \right]. \tag{2}$$

The result of integration of the equation's both parts (6) results in a function $Y_1(t)$

$$Y_1(t) = b_0 \left[g^2 \frac{t^3}{3} - \frac{2gC_2 t^2}{2} + C_2^2 t \right] + d_1.$$

Taking into account (1, 2), the velocity function in the projection onto the ordinate axis turns into the following form

$$V_y(t) = -gt + C_2 + \varepsilon \cdot \left\{ b_0 \left[g^2 \frac{t^3}{3} - gC_2 t^2 + C_2^2 t + d_1 \right] \right\}. \tag{3}$$

Either

$$V_y(t) = -gt + \varepsilon b_0 g^2 \frac{t^3}{3} - \varepsilon g C_2 t^2 + \varepsilon C_2^2 t + C_3,$$

where

$$C_3 = \varepsilon d_1 + C_2.$$

Under condition $V_y(0) = 0$, we get a constant $C_3 = 0$. Taking above mentioned into account, dependence (3) turns into

$$V_y(t) = -gt + \varepsilon b_0 g^2 \frac{t^3}{3} + \varepsilon g C_2 t^2 - \varepsilon C_2^2.$$

An independent constant C_2 is determined from the condition: $V_y(0) = Y_0(0) + \varepsilon Y_1(0) = 0$, which follows: $Y_0(0) = 0 + C_2$ and $C_2 = 0$. Thus, the velocity function projection on the ordinate axis is obtained in the form

$$V_y(t) = -gt + \varepsilon b_0 g^2 \frac{t^3}{3}.$$

The corresponding displacement projection function on the ordinate axis is calculated by the formula

$$S_y(t) = \int V_y(t) dt = -g \frac{t^2}{2} + \varepsilon b_0 g^2 \frac{t^4}{12} + C^*.$$

Under the condition $S_y(0) = -h$ the desired constant equals to $C^* = -h$.

The function of the corresponding projection displacement appears as follows

$$S_y(t) = -h - g \frac{t^2}{2} + \varepsilon b_0 g^2 \frac{t^4}{12}. \tag{4}$$

In two approximations of the asymptotic solution with retention of a small parameter of the second order represents $V_y(t) = V_0(t) + \varepsilon V_1(t) + \varepsilon^2 V_2(t) + \dots$ velocity projection function $V_y(t)$ turns into the form

$$V_y(t) = -gt + K_0 g \left(\frac{t^3}{3} - \frac{2}{45} K_0 t^5 \right). \tag{5}$$

Under condition (11), the equation for obtaining the velocity function in the direction of the X-axis is determined from the nonlinear equation (4) [18]

$$\frac{dV_x(t)}{dt} + A(t) V_x(t) V_y(t) = \Phi_0. \tag{6}$$

Take

$$A(t) = \varepsilon b_0 \cdot \left[-gt + \varepsilon b_0 g^2 \frac{t^3}{3} \right]. \tag{7}$$

Equation (6) is an inhomogeneous differential equation with time-varying coefficients, the solution of which is obtained by the standard procedure for solving inhomogeneous equations of the first order from the desired function

$$V_x(t) = u_0(t) + u^*(t), \tag{8}$$

where $u_0(t)$ and $u^*(t)$ are the general solutions of a homogeneous and a particular inhomogeneous differential equation, respectively.

General solution of a homogeneous differential equation $u_0(t)$ is obtained by the following steps

$$\frac{dV_x(t)}{dt} + A(t) \cdot V_x(t) = 0; \tag{9}$$

$$\frac{du_0(t)}{dt} = -A(t) \cdot u_0(t); \quad \frac{du_0(t)}{u_0 t} = -A(t) dt;$$

$$\int \frac{du_0(t)}{u_0 t} = -\int A(t) dt + C_4; \quad \ln \left| u_0(t) \right| = -\int A(t) dt + C_4;$$

$$u_0(t) = e^{\left[-\int A(t) dt + C \right]} = e^{C} \cdot e^{-\int A(t) dt} = C_5 \cdot e^{-\int A(t) dt},$$

where $C = e$

Applying the procedure of the arbitrary constants variation method, the inhomogeneous equation partial solution results (6)

$$u^*(t) = C_5(t) \cdot e^{-\int A(t) dt}; \tag{10}$$

$$\dot{u}^*(t) = C_5(t) \cdot e^{-\int A(t) dt} + C_5(-A(t)) \cdot e^{-\int A(t) dt}. \tag{11}$$

Considering (9, 10) after simplification and transformation, equation (7) takes the form

$$C_5(t) = \int \varnothing_0 \cdot e^{\int A(t) dt} dt + C_6. \tag{12}$$

Substitution (11) to (9) gives the partial solution $u^*(t)$ in the form of

$$u^*(t) = \left[\int \varnothing_0 \cdot e^{\int A(t) dt} dt + C_6 \right] \cdot e^{-\int A(t) dt}.$$

Considering (8) and (12) in equation (6), we obtain the dependence for the velocity projection function on the abscissa

$$V_x(t) = C_5 \cdot e^{-\int A(t) dt} + C_6 \cdot e^{-\int A(t) dt} + e^{-\int A(t) dt} \cdot \varnothing_0 \cdot \int e^{\int A(t) dt} dt =$$
$$= (C_5 + C_6) \cdot e^{-\int A(t) dt} + e^{-\int A(t) dt} \cdot \varnothing_0 \cdot \int e^{\int A(t) dt} dt,$$

from which

$$V_x(t) = e^{-\int A(t) dt} \left[d_2 + \varnothing_0 \cdot \int e^{\int A(t) dt} dt \right],$$

where $d_2 = c_5 + c_6$.

Constant d_2 is determined by the conditions of the external environment parameters ratio. In case $V_x(0) = V_0 \mp V_{wx}$

$$V_x(0) = e \cdot \left[d_2 + \varnothing_0 \cdot 0 \right] = V_0 \mp V_{wx},$$

from which

$$d_2 = V_0 \mp V_{wx}.$$

The resulting relation for the velocity projection function $V_x(t)$ turns into

$$V_x(t) = e^{-\int A(t)dt} \cdot \left[(V_0 \mp V_{wx}) + \varnothing_0 \cdot \int e^{\int A(t)dt} dt \right].$$

The corresponding displacement function $S_x(t)$ is determined from the dependence

$$S_x(t) = \int \left\{ e^{-\int A(t)dt} \cdot \left[(V_0 \mp V_{wx}) + \varnothing_0 \cdot \int e^{\int A(t)dt} dt \right] \right\} dt + d_3.$$

Subject to $S_x(0) = 0$ and after the conversions

$$S_x(0) = \left\{ (V_0 \mp V_{wx}) \cdot t + \varnothing_0 \cdot \frac{t^2}{2} + d_3 \right\},$$

where $d_3 = 0$.

The final dependency for the motion function $S_x(t)$ represents in the form

$$S_x(t) = \int \left\{ I_1(t) \cdot \left[(V_0 \mp V_{wx}) + \varnothing_0 \cdot \int I_2(t) dt \right] \right\} dt,$$

which $I_1(t) = e^{-\int A(t)dt}$; $I_2(t) = e^{\int A(t)dt}$.

Determining the dependence of the time on which the cargo reaches its target t_1 accepts, according to formula (13), the condition under which $S_y(t_t) = 0$.

Under this condition, we obtain an equation of the form

$$t_t^4 - \frac{6}{\varepsilon b_0 g} \cdot t_t^{\,2} - \frac{12h}{\varepsilon b_0 g^2} = 0.$$

After entering the designations $p = \frac{6}{\varepsilon b_0 g}$, $q = \frac{12h}{\varepsilon b_0 g^2}$ it will be reduced to the form

$$t_t^4 - p \cdot t_t^2 - q = 0.$$

The dependence of the time to reach the target on the height of the release from the UAV, aerodynamic characteristics and wind load is determined by the formula

$$t_t = \left[\frac{3}{k_0} \cdot \left(1 + \sqrt{1 + \frac{4hK_0}{3} / g} \right) \right]^{1/2},$$

where $K_0 = \varepsilon b_0 g$ – coefficient of the cargo "aerodynamic quality".

Fig. 2 shows the dependence of the falling time on the constant parameters of the studied system and the height of the release.

It should be noted that the accuracy of achieving the target also depends on the influence of the wind load projection along the lateral coordinate Z, which can be calculated similarly to the case of the wind load component by coordinate *X*. The general effect of the side load is calculated as $\sqrt{S_x^{*2} + S_z^{*2}}$

$$\hat{S} = \sqrt{S_x^{*2} + S_z^{*2}} = S_x^* \left(1 + \frac{S_z^{*2}}{S_x^{*2}} \right)^{0.5} = S_x^* (1 + a^4)^{0.5},$$

where $a = \frac{V_z^*}{V_x^*}$.

Fig. 3 shows the qualitative dependence of the vertical displacement on the K_0 parameter and the time of fall.

A comparison of the proposed approximate solution with the direct numerical integration of the nonlinear differential equation (5) [18] is shown in Fig. 4.

As depicted in Fig. 2, uneven surface of the falling time function influenced by K_0 and the surface parameters distribution are subject to numerical simulation initial data at the given moment of time.

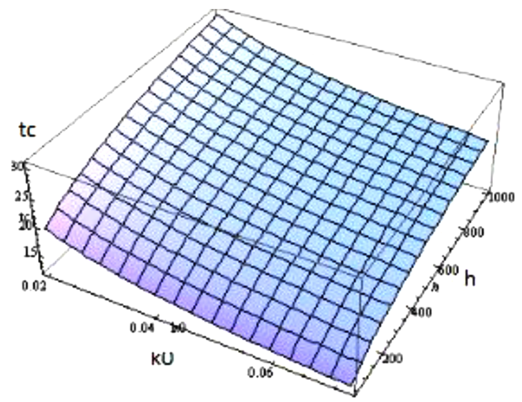

Fig. 2. Three-dimensional surface of the dependence of the falling time on the parameters of the studied system and the release height

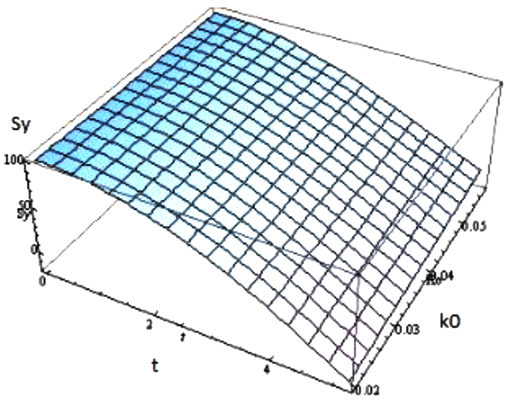

Fig. 3. Dependence of the vertical movement on the K_0 parameter and the time of fall

For initial numerical integration, graphical three-dimensional surface representation of the K_0 influence (Fig. 3) describes the effective time in range up to 6 s.

The results of the comparison of approximate analytical and direct numerical solutions show that according to the first approximation of the nonlinear component of the differential equation for the vertical velocity function, the maximum difference is 25 %, according to the second asymptotic approximation – 11 %.

Conclusion. A mathematical model and an approximate analytical approach with the use of computer algebra of the nonlinear problem of the cargo external ballistics in the presence of initial velocity are proposed. This allows determining the parameters of the dynamic system functioning under conditions of atmospheric pressure and wind load. At the same time, it is possible to consider the side force, associated with the angular rotation of the falling body (Magnus effect). The results of the numerical analysis in two asymptotic approximations to the nonlinear component of the solution of the basic

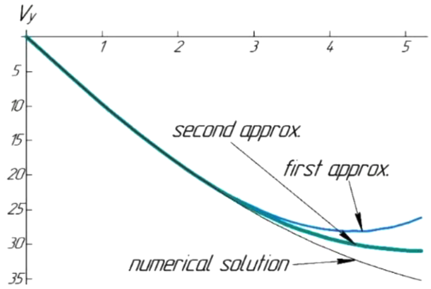

Fig. 4. Results of comparing the approximate solution with direct numerical integration

nonlinear differential equation with respect to the velocity function in the projection on the vertical axis coincides with the direct numerical calculation enough, from the point of practical application. To solve the investigated problem in the case when the properties of the system and the environment depend on time, it is possible to use a hybrid asymptotic approach based on perturbation and phase integrals methods.

The obtained analytical results and the solution algorithm can be used in applied problems of mathematical physics and engineering calculations, in particular, the development of ballistic system control algorithms.

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Визначення параметрів функціонування нелінійної балістичної системи у реальному зовнішньому середовищі

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Мета. Розробка наближеної нелінійної моделі розв'язку задачі зовнішньої балістики з визначенням параметрів нелінійної системи та методики розрахунку аеродинамічних характеристик вантажу, розташованого на зовнішніх підвісних пристроях безпілотного літального апарату (БПЛА) для підвищення ефективності їх доставки у визначену точку приземлення шляхом асимптотичного підходу із заданими параметрами досліджуваної системи й зовнішнього навантаження.

Методика. Створення ефективної моделі виконувалось із застосуванням аналітико-чисельних алгоритмів дослідження, що базується на нелінійній системі диференціальних рівнянь у загальному вигляді зі змінними за часом коефіцієнтами. Для вирішення нелінійної задачі зовнішньої балістики у двовимірній постановці уведене припущення щодо істотного впливу проекції функції швидкості на вісь ординат відносно компоненти на вісь абсцис. Вирішення задачі зведено до розв'язку системи диференціальних рівнянь зі змінними коефіцієнтами за відповідними координатами з використанням асимптотичного підходу для малого параметра коефіцієнта аеродинамічного опору. Формулювання задачі виконане з урахуванням досліджуваних параметрів середовища, прикладного математичного аналізу й моделювання.

Результати. Запропоновані аналітичні залежності нелінійної задачі балістики й застосування методу скінченних елементів (МСЕ) для визначення параметрів руху вантажу із БПЛА за наявності початкової швидкості та вітрового навантаження у площині руху. Визначено, що отриманий аналітичний розв'язок співвідноситься із прямим чисельним розрахунком основного диференціального рівняння відносно осі ординат.

Наукова новизна. Запропонована математична нелінійна модель динамічного процесу, що передбачає переважаючий вплив функції швидкості по осі ординат відносно функції по осі абсцис. Для отримання наближеного аналітичного розв'язку основної нелінійної системи диференціальних рівнянь зі змінними коефіцієнтами застосовано метод асимптотичного збурення. Залежність для функції переміщення за координатою абсцис представляється з урахуванням реальних параметрів польоту зі змінними у часі характеристикам.

Практична значимість. Здобуті аналітичні залежності оцінки часу й відстані досягнення точки приземлення при початковій швидкості руху та наявності вітрового навантаження можуть бути використані у прикладних задачах математичної фізики, інженерних розрахунках, керування процесом доставки вантажу й досягнення точки приземлення із використанням БПЛА. Отримані аналітичні результати та алгоритм розв'язання можуть бути інтегровані у прикладні задачі математичної фізики та інженерних розрахунків, зокрема розробки алгоритмів керування балістичними системами.

Ключові слова: *балістика, метод скінчених елементів, математична модель, динаміка, БПЛА, збурення, аеродинамічний опір*

The manuscript was submitted 20.12.23.