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## OPTIMIZATION MATHEMATICAL MODEL OF A CONTACT AIR COOLER FOR A MINE TURBOCOMPRESSOR

**Purpose.** Establishing the dependencies of rational parameters of turbocompressor contact air coolers on the operating mode and initial conditions.

**Methodology.** The methods of analytical research, mathematical modeling, physical modeling, and mathematical statistics were used in the study.

**Findings.** As a result of the research, an optimization mathematical model of the mine turbocompressor air cooler was developed, which allows establishing its rational parameters depending on the initial conditions and operating modes of the turbocompressor. The adequacy of the theoretical studies was proved on a specially designed experimental setup. The obtained dependencies make it possible to minimize the theoretical flow rate at the inlet to the uncooled section of the turbocompressor, which reduces the specific energy consumption for the compressed air production.

**Originality.** For the first time, a method has been developed for determining the rational parameters of contact air coolers when the initial temperature of water, air, and air pressure changes, which allows developing a methodology for the constructive calculation of the contact cooling system of mine turbocompressors.

**Practical value.** Compressed air is widely used in all industries. It is one of the most common energy carriers in industrial enterprises, and the devices associated with its distribution and processing are an energy-intensive complex industrial energy system; the level of its perfection depends on the performance of technological processes that use compressed air. Compressed air is widely used in the mining industry (ore mining and fuel production). Compressed air is produced by turbocompressors. To increase the efficiency of the compressor, compressed air coolers are used. A significant weakness of the standard compressor cooling system is the rapid contamination of the heat exchange surfaces of air coolers with scale layers, which leads to a decrease in their efficiency and an increase in the specific energy consumption for compressed air production. This disadvantage is not found in the Venturi tube – centered droplet separator contact air coolers. As a result of the study, the dependencies were obtained and used to develop a methodology for the constructive calculation of contact air coolers for a mine turbocompressor.

**Keywords:** *mathematical model, Venturi tube, contact air cooler, cooling, compressed air, turbocompressor*

**Introduction.** Compressed air is one of the most common energy sources in industrial enterprises, and the devices associated with its distribution and processing are an energy-intensive complex industrial energy system; the level of its perfection depends on the parameters of technological processes in which compressed air is used [1].

No industry can do without the use of compressed air, which is an affordable source of both raw materials and energy. As the consumption of compressed air has increased in recent years, centrifugal compressors or turbo compressors have become widely used, as they have a higher capacity than volumetric machines. Turbo compressors are the most powerful turbo machines. These machines are designed to increase air pressure and transport it through pipelines [2].

Today, most of the compressor systems used in Ukraine are technically outdated, as they were manufactured in the 70s and 80s of the twentieth century. Their energy efficiency is very low, which increases the cost of energy transported by these systems or increases the price of products manufactured with their help. The operating efficiency of turbochargers depends on a number of external and internal factors: environmental temperature, pressure and temperature of the suction air, air temperature after cooling in intermediate air coolers and pressure loss in them, as well as leakage in the flow path, etc. The turbocharger's performance is most affected by an increase in temperature between stages. An air compressor increases the pressure of the incoming air by reducing its volume. The exhaust air from the compressor is heated [3].

A requirement for the normal operation of turbochargers is the cooling of compressed air, which significantly reduces the

required power. Air cooling (intermediate and final cooling in compressor units) is mainly carried out by shell-and-tube devices. Such devices have a number of significant disadvantages, the most important of which are: insufficient cooling of compressed air, formation of deposits on the working surfaces of the devices and increased content of compressed air vapor and oil. These shortcomings have a negative impact on the quality of compressed air and increase the danger of using pneumatic energy (the possibility of hydraulic shocks, explosions and fires, and internal ice formation in pneumatic lines).

**Literature review.** Gas coolers are usually installed after each stage to bring the compression process closer to isothermal. Gas coolers are divided into built-in and remote ones by location. Integrated gas coolers are located inside the compressor casing next to the stages. External gas coolers are installed in their own housings outside the compressor casing as close to it as possible. The disadvantages of compressor units with integrated coolers due to the need to form a sufficient cooling surface are the increased axial size of the machine, especially the radial size.

Increasing the gas path reduces aerodynamic efficiency. One of the major obstacles to the use of water jacket cooling is the difficulty of dismantling the water jacket and descaling the water channels. This type of internally cooled turbocompressor is characterized by an increase in size, due to the desire to improve air cooling by increasing the size of the guide vanes, and the design of the turbocompressor is also becoming more complex.

Most turbocompressor manufacturers use external gas coolers with rolled copper tubes. The advantages of such gas coolers are a high degree of uniformity and easy access for inspection and cleaning. The most commonly used heat ex-

change surfaces are tubular, plate and finned heat exchangers. Gas coolers are structurally divided into: shell-and-tube, U-shaped, elemental, coil, tube-in-tube, radiator and sectional. Coil and U-shaped heat exchangers have smooth tubes, while others can be finned and smooth-tube ones.

The most commonly used coolers are plate-and-fin coolers consisting of unified elements (sections). The design of turbocompressor gas coolers depends on the operating conditions and gas pressure. At low pressures (up to 3.5 MPa), shell-and-tube, plate-and-fin, and elemental coolers are mostly used, while for higher pressures, shell-and-tube, tube-and-pipe, and U-shaped coolers are used. Coil heat exchangers are used at different pressures, but in low-capacity compressors [4].

The advantages of "pipe-in-pipe" coolers are high heat transfer coefficient, the ability to install additional sections if necessary, and an increased heat transfer surface. The disadvantages of this type of heat exchanger are its large volume, low heat output, high specific metal capacity (the highest of all types of equipment), and the difficulty of removing solid salt deposits from the intertube spaces.

A coil heat exchanger is a cylindrical single-row or multi-row coil immersed in a vessel filled with a working fluid. In most cases, steam or hot water is supplied through the coil to heat the working medium and cold water is supplied to cool it. By heating or cooling the working medium with the help of coils, the temperature of the working medium can be increased (decreased) by 30–50 °C. The main disadvantages of this type of heat exchanger are the low heat transfer coefficient of the outer surface of the coil at low medium velocity and the difficulty of cleaning the heat transfer surface.

The coil heat exchanger is quite simple to manufacture, low in cost, easy to install inside the equipment, easy to inspect and repair, compact, but the heat transfer surface is small, and it can be under a small thermal load. To reduce the hydraulic resistance when the medium moves inside the heat exchanger, the coil is shortened and connected to a common manifold at the inlet of the coolant to the coil and at the outlet. The radiator-type gas cooler, which is widely used for air-cooled compressors, is a bundle of several rows of pipes with continuous fins. The bundle is transversely washed by the air flow from the fan at a speed of 7–10 m/s. Compared to shell-and-tube and plate heat exchangers, they are more compact and use less metal per unit heat transfer surface area, and the stacked heat exchangers are easy to clean from deposits on both sides of the heat transfer plates.

Fundamental research was carried out to create calculations and design methods for heat exchangers [5]. It was found that the highest heat transfer coefficients are those of plate-and-fin heat exchangers. The studies showed that 20X13 steel can be used to produce durable and corrosion-resistant heat exchangers by heat treatment of brazed heat exchangers. A methodology for the calculation and design of a heat exchanger device for a mine compressor unit has been developed.

Methods for increasing the durability and reliability of heat exchangers have been developed. Tests have shown that the developed heat exchanger has high wear resistance in environments with abrasive particles, in neutral (water, air, steam, nitrogen, oil products, gaseous hydrocarbons) and alkaline environments, as well as in solutions of some organic acids (oxalic, acetic), and in other liquid and gaseous environments. The disadvantages of this heat exchanger are the inability to operate at high pressures and reduced efficiency due to contamination.

Paper [6] examined combined cooling (evaporative cooling to saturation in compression sections and remote cooling in one to three refrigerators) and compared it to other types of cooling. The analysis shows that combined cooling has advantages over both remote and evaporative cooling. This is due to the fact that the air temperature at the discharge of the compression section is much lower with combined cooling than with remote cooling. In addition, the resistance of remote re-

frigerators is simultaneously reduced by reducing the specific volume of compressed air as the temperatures decrease.

There was also an increase in the pressure head of the sections and the entire compressor due to a decrease in the relative air velocity at the outlet of the impeller and a corresponding increase in the circumferential component of the absolute velocity. The use of combined cooling in a centrifugal air compressor can increase its performance by 5–15 % compared to a machine with a remote refrigerator, depending on the temperature and humidity of the suction air. At the same time, energy consumption for air compression is reduced by 8–10 %. An inspection of the evaporative-cooled compressors revealed that there were no signs of erosion or corrosion on the moving and fixed elements of the flow part.

However, during the operation of some compressors with periodic or prolonged condensate supply to the flow part, corrosion of the pivot pins of the guide blade and the bearings of the flap axis of the check valve installed on the compressor discharge was observed. In addition, corrosion of the casing connector was detected during the opening of one of the compressors.

Paper [7] considered the use of water injection at the compressor inlet to cool air using an air thermopressor. The operation of an air heat press is based on the process of thermo-gas-dynamic compression (thermopressure). This process is characterized by an increase in pressure due to the instantaneous evaporation of the liquid introduced into the gas stream, which is accelerated to a speed close to the speed of sound. At the same time, heat from the gas is transferred to the evaporation of water, resulting in a decrease in its temperature. The disadvantages of this cooling method are the increased moisture content of the air leaving the compressor, and this water condenses in the pipeline, increasing its corrosion. Thorough pre-treatment of the injected water is required to eliminate the possibility of scale formation in the turbocompressor working spaces.

Article [8] discusses the improvement of the efficiency of the cooling system of compression units by intensifying the process of heat exchange between the cooled (compressed air) and the coolant (water). This is primarily realized by preventing scale and sludge deposits in the coolers. Today, water hardness mitigation is achieved by chemical means. The chemical method for preventing scale formation is effective, but it requires constant costs, is highly polluting and harmful to the health of the service personnel. The formation of scale can be prevented by electromagnetic treatment of the circulating cooling water. Technical solutions have also been proposed that involve the following changes in the existing cooling system of compressor units:

- an air cooling chamber in the form of a heat exchanger will be installed between the filter and the first stage of the compressor, the coolant will be the cooled water from the cooling tower;
- the cooling tower spray nozzles will be replaced with a new design of ejection spray nozzle;
- an electromagnetic water treatment device will be installed on the cooled water pipeline leaving the cooling tower.

Analytical studies showed that the implementation of the proposed technical solutions at the compressor units of mining enterprises will reduce the energy consumption of the air compression process in the cylinder by an average of 4–5 %, which will increase the compressor's productivity by up to 8 %, depending on the operating conditions. It will also help to eliminate emergencies and increase the productivity of compressor units, which will reduce the cost of compressed air production.

Paper [9] substantiates the fact that the use of contact air coolers with compressed air cooling and dehumidification is more efficient. This method combines the advantages of internal and external evaporative methods, but does not have their disadvantages. Water in the amount of 1.6 kg per 1 kg of air is injected into the compressor air duct after each compression section and cools the air to a temperature of 135 to 35 °C. This ensures evaporation-free operation of the cooler with compressed air drying.

The water is separated in special drip traps that are installed in front of the next section. With this type of cooling, water can only be heated to the wet bulb temperature under the given conditions (air pressure, temperature and humidity). The main advantage of this method is the high intensity of heat transfer, as there are no separating surfaces that are prone to scale contamination. The disadvantage is a somewhat more complicated water supply scheme compared to the traditional one.

The contact air cooler comprises a mixing device in the form of a venturi tube, a drip water separator and a float level control. The cold water is fed into the mixing device, where it is sprayed by a stream of heated air and mixed with it. The resulting air-water mixture undergoes intensive heat and mass transfer. In the separator, the dripping water is thrown to the periphery by centrifugal force and, with some air, enters the free space through a pocket. There, the air velocity decreases to below the droplet velocity, and then it merges with the main stream of dry cooled air. The water separated in the separator flows by gravity to the level regulator, which ensures that it is discharged to the cooling tower for cooling with the support of a hydraulic gate.

Paper [10] established the rational parameters of a contact air cooler for the nominal operating mode under normal initial conditions.

**Problem aspects to be solved.** As a result of the analytical studies, it was found that the rational parameters of turbocharger contact coolers are justified for the nominal mode under normal initial conditions, so additional research is needed to determine the parameters depending on the operating mode and initial conditions other than normal. Such parameters include the cross-section of the Venturi tube neck, which determines the velocity of the medium in the neck and the size of the water grinding and, accordingly, the degree of compressed air cooling.

**Purpose.** The work is aimed at establishing the dependencies of rational parameters of turbocharger contact coolers on the operating mode and initial conditions.

**Methods.** The methods used in the study were analytical research, mathematical modelling, physical modelling and mathematical statistics.

**Problem statement.** Cooling of the turbocompressor is the main condition for its normal operation. In addition, by cooling the compressed air before the uncooled section of the turbocompressor, the theoretical flow rate of the section is reduced, which results in a reduction in specific energy consumption. Fig. 1 shows a diagram of a turbocompressor with 6 compression stages and intermediate air cooling after each section. Cooling is provided by a contact cooler Ven-

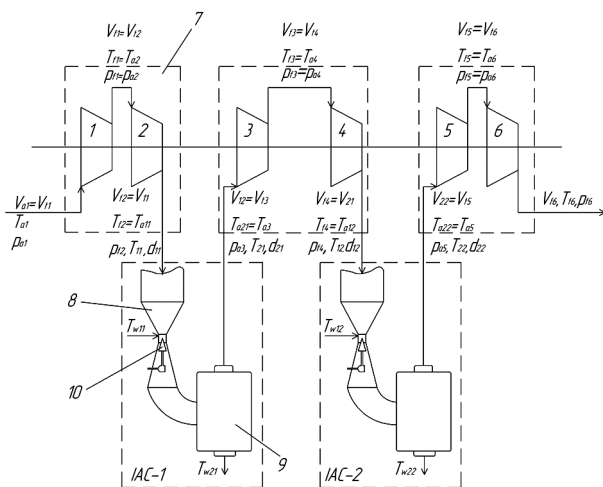


Fig. 1. Diagram of compressed air cooling between the compressor turbocompressor stages by a contact air cooler of the Venturi tube-centrifugal separator-drip trap type:

1–6 – turbocompressor stage; 7 – turbocompressor section; 8 – Venturi tube; 9 – centrifugal separator-drip trap; 10 – control cone

turi pipe – centrifugal separator-drip trap. Since the cross-section of the Venturi tube neck has the greatest impact on the efficiency of the contact cooling system [9], it is advisable to use a venturi tube with a regulating neck using a special cone to minimise specific energy consumption (Fig. 2). The cross-section of the air cooler mixer determines the temperature and pressure of the air at the outlet of the apparatus.

To determine the rational cross-section of the Venturi tube neck, we first investigate the objective function

$$V_m = V_{a1} \cdot \frac{p_{a1}}{T_{a1}} \cdot \frac{T_{an}}{p_{an}}, \quad (1)$$

where  $n$  is the number of the stage ( $n = 3$ , for IAC-1,  $n = 5$ , for IAC-2);  $T_{an}$  – air temperature at the inlet to the  $n$  stage, K;  $p_{an}$  – air pressure at the inlet to  $n$  stages, Pa;  $p_{a1}$  – initial air pressure at the inlet to the turbocompressor, Pa;  $T_{a1}$  – initial air temperature at the inlet to the turbocompressor, K;  $V_{a1}$  – compressor supply,  $m^3/s$ .

Consider the effect of the air velocity in the Venturi tube neck  $v_n$  on the compressor supply after the air cooler.

Analysing dependence (1), we can conclude that  $V_{a1} \cdot \frac{p_{a1}}{T_{a1}} = \text{const}$ , so  $V_m$  will depend on  $T_{in}$  and  $p_{in}$ . Obviously, the theoretical compressor flow rate will be determined by the nature of the change in  $T_{an}$  and  $p_{an}$ . Thus, when  $T_{an}$  decreases or  $p_{in}$  increases the theoretical flow rate decreases.

Let us analyse how  $T_{an}$  and  $i_{an}$  depend on the neck cross section.

Take into account the fact that the air pressure at the inlet of the  $n$  stage (at the outlet of the air cooler) is described by the following dependence

$$p_{an} = p_{f(n-1)} - \Delta p_{fi}, \quad (2)$$

where  $p_{f(n-1)}$  is the air pressure at the outlet of the previous stage, Pa;  $i$  – the number of the air cooler ( $i = 1$ , for intermediate air cooler IAC-1,  $i = 2$ , for IAC-2);  $\Delta p_{fi}$  – hydraulic resistance of the air cooler, Pa.

The hydraulic resistance of the apparatus to air is

$$\Delta p_{fi} = 0.5(\zeta_h v_{hi}^2 (\rho_{a2i} + 0.63 \rho_{wi} m_i^{0.7}) + \zeta_{sj} \rho_{d2i} U_{cj}^2), \quad (3)$$

where  $\zeta_h$  is hydraulic resistance coefficient of a “dry” Venturi pipe;  $v_{hi}$  – gas velocity in the mixer head, m/s;  $\rho_{a2i}$  – air density at the outlet of the mixer,  $kg/m^3$ ;  $\rho_{wi}$  – density of the liquid,  $kg/m^3$ ;  $m_i$  – specific irrigation,  $m^3/m^3$ .

Air velocity in the centrifugal space of the separator is

$$U_{si} = \frac{4V_{li}}{\pi D_{si}^2},$$

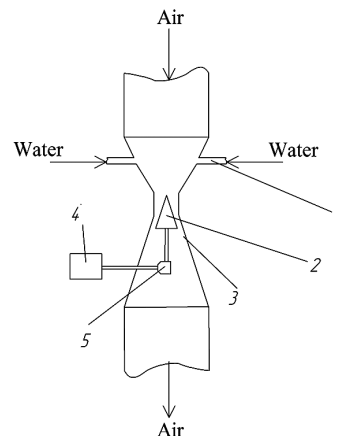


Fig. 2. Venturi tube design with adjustable neck cross-section:

1 – nozzle; 2 – regulating cone; 3 – venturi pipe body; 4 – electric motor; 5 – linear drive

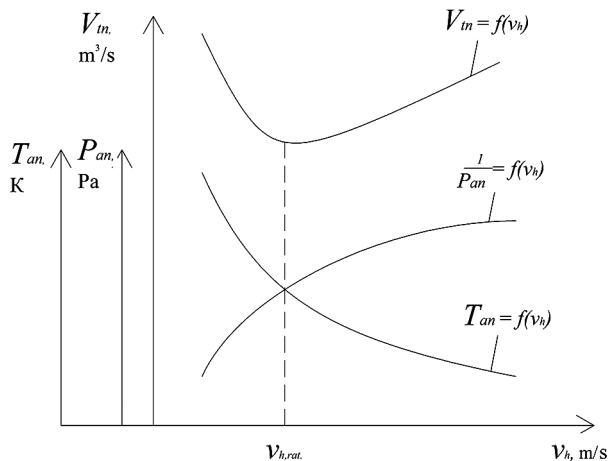


Fig. 3. Dependence of  $V_{in}$ ,  $T_{an}$  and  $p_{an}$  on  $v_{hi}$

where  $D_{si}$  is the diameter of the centrifugal space of the separator, m;  $V_{li}$  – volume flow rate of air at the inlet to the air cooler,  $m^3/s$ .

Air temperature at the inlet to the stage is

$$T_{a2i} = t_{a2i} + 273. \quad (4)$$

Gas temperature at the outlet of the cooler is

$$t_{a2i} = \frac{I_{2i} - r_0 d_{2i}}{c_{ph} + c_s d_{2i}}, \quad (5)$$

where  $r_0$  is latent heat of vapour formation corresponding to the enthalpy reference point, J/kg;  $c_s$  – average specific heat of steam, J/(kg · K);  $c_{pai}$  – specific heat capacity of air, J/(kg · K).

The enthalpy of the air at the outlet of the apparatus is

$$I_{2i} = c_{ph} t_{m2i} + (r_0 + c_s t_{m2i}) d_{m2i}. \quad (6)$$

The final air temperature (at the outlet of the heat exchanger) according to the wetted thermometer (at the boundary of two layers of saturated and unsaturated gas) is

$$t_{m2i} = i_{wlj} + (t_{m1} - t_{wlj}) \cdot Km_i, \quad (7)$$

where  $t_{wlj}$  is the initial temperature of the cooling water;  $t_{m1}$  – the initial temperature of the air according to the wet thermometer (at the boundary of two boundary layers of saturated and unsaturated gas).

The enthalpy of the air is

$$\begin{aligned} I_{1i} &= at_{m1i} + (r_0 + c_s t_{m1i}) d_{m1i} = \\ &= 1005 t_{m1i} + (2500 \cdot 10^3 + 1884 t_{m1i}) d_{m1i}. \end{aligned}$$

Absolute moisture content is

$$d_{m1i} = \frac{0.622 p_{m1i}}{p_{a1i} - p_{m1i}}.$$

The partial pressure of steam corresponding to the saturation temperature  $t_{m1}$  is determined by the formula

$$\begin{aligned} \lg p_{m1i} &= \frac{8.12 t_{m1i} + 156}{t_{m1i} + 236}; \\ p_{m1i} &= 133.32 \cdot 10^{\lg p_{m1i}}. \end{aligned}$$

Heat and mass transfer intensity ratio is

$$Km_i = 3.9 \text{Re}_{ci}^{-0.1} Bm_{1i}^{-0.45} LD_i^{-0.01}, \quad (8)$$

where  $LD_i$  is the parametric similarity number.

The combined Reynolds-Froude number taking into account the acceleration in the field of centrifugal forces is

$$\text{Re}_{ci} = \frac{v_{hi}^3 D_{si}}{2U_{si}^2 v_{li}}, \quad (9)$$

kinematic viscosity of air at temperatures from  $-20$  to  $+140$  °C

$$\begin{aligned} v_{li} &= (0.101 t_{ali} + 13.7) \frac{p_0}{p_{ali}} \cdot 10^{-6}; \\ p_0 &= 0.98 \cdot 10^5. \end{aligned}$$

And from  $140$  to  $400$  °C

$$v_{li} = (0.1455 t_{ali} + 6.7) \frac{p_0}{p_{ali}} \cdot 10^{-6}.$$

Similarity number (criterion) of thermal equivalents is

$$Bm_{1i} = Bm_i + 1.$$

The ratio of heat equivalents is given

$$Bm_i = \frac{Bw_{ni}}{1 + Ke_i}.$$

Irrigation ratio is

$$B_{ni} = \frac{G_{wi}}{G_{ai}},$$

where  $G_{wi}$  is mass flow rate of water, kg/s;  $G_{ai}$  – mass flow rate of air, kg/s.

The ratio of heat equivalents is

$$Bw_{ni} = \frac{G_{wi} c_{pwi}}{G_{ai} c_{pai}}.$$

Evaporation ratio is

$$Ke_i = \frac{r_0 (d_{mRli} - d_{mli})}{c_{pai}}.$$

Calculated absolute moisture content makes

$$d_{mRli} = \frac{0.622 p_{mRli}}{p_{a1i} - p_{mRli}}.$$

Calculated partial pressure is

$$\begin{aligned} \lg p_{mRli} &= \frac{8.12 t_{mRli} + 156}{t_{mRli} + 236}; \\ p_{mRli} &= 133.32 \cdot 10^{\lg p_{mRli}}. \end{aligned}$$

Calculated temperature is

$$t_{mRli} = t_{m1i} + 1.$$

Absolute moisture content of the air at the final gas temperature according to the wetted thermometer  $t_{m2}$  is

$$d_{m2i} = \frac{0.622 p_{m2i}}{p_{a1i} - p_{m2i}}, \quad (10)$$

if  $\Delta t < 0.5$

The partial pressure of steam at  $t_{m2i}$  is

$$\begin{aligned} \lg p_{m2i} &= \frac{8.12 t_{m2i} + 156}{t_{m2i} + 236}; \\ p_{m2i} &= 133.32 \cdot 10^{\lg p_{m2i}}. \end{aligned} \quad (11)$$

Absolute moisture content at the outlet of the machine is

$$d_{2i} = d_{m2i} - \Delta d_{0i} (2\Delta_i - 1). \quad (12)$$

The highest possible concentration pressure is

$$\Delta d_{0i} = d_{m1i} - d_{1i}.$$

Absolute moisture content of the air is

$$d_{1i} = \frac{I_{1i} + c_{pai} t_{a1i}}{r_0 + c_s t_{a1i}},$$

where  $t_{ai}$  is air temperature according to a dry thermometer at the input to the apparatus, °C.

Heat transfer intensity ratio is

$$\Delta_r = \frac{\Delta t_{mi}}{\Delta t_{m0i}}. \quad (13)$$

The highest possible temperature head is

$$\Delta t_{m0i} = t_{m1i} - t_{w1j}.$$

Arithmetic mean temperature head is

$$\Delta t_{mi} = \frac{t_{m1i} - t_{m2i}}{2} - \frac{t_{w1j} + t_{w2j}}{2}. \quad (14)$$

Final water temperature makes

$$t_{w2i} = t_{w1i} \pm \frac{Q_i}{c_{pwi} G_{wi}}, \quad (15)$$

where  $c_{pwi}$  is specific isobaric heat capacity of a liquid, J/(kg · K).

Heat flow from one medium to another is

$$Q_i = G_{ai}(I_{2i} - I_{1i}). \quad (16)$$

Mass air flow rate is

$$G_{ai} = \frac{V_{ai} \cdot p_{a1}}{R \cdot t_{a1}},$$

where  $R$  is gas constant of dry air, J/(kg · K).

Having analysed formulas (1–16), taking into account the fact that the neck cross-section uniquely determines the air velocity in the neck, we can conclude that with an increase in the head velocity,  $T_{an}$  will decrease, and  $1/p_{am}$  will increase. The variation of  $V_m$ ,  $T_{an}$  and  $p_{an}$  from  $v_{hi}$  is shown in Fig. 3.

Thus, at a certain value of  $v_{hi}$  the objective function (1) has a minimum.

To find out how  $V_m$  will change when  $v_{hi}$ , we study  $V_m$  as a function  $v_{hi}$ . To do this, we first express  $V_m$  as a function of  $v_{hi}$ , using formulas (1–16). We get the following expression

$$V_m = V_{a1} \cdot \frac{p_{a1}}{T_{a1}} \times \frac{c_{pai} \cdot W + [r_0 + c_s \cdot W] \cdot B - r_0 \cdot Z + 273}{c_{pai} + c_s \cdot Z} \times \frac{1}{p_{an} - 0.5(v_{hi}^2 \cdot J + N)}, \quad (17)$$

where  $W = t_{w1i} + H \cdot v_{hi}^{-0.3}$ ;

$$Z = B - \Delta d_{0i} \cdot \left[ \frac{t_{m1i} + H \cdot v_{hi}^{-0.3} - t_{w1i}}{\Delta t_{m0i}} + \frac{G_{ai} \cdot [c_{pai} \cdot W + (r_0 + s \cdot W) \cdot B - I_{1i}]}{c_{pai} \cdot G_{wi}} - 1 \right];$$

$$B = \frac{0.622 \cdot 133.32 \cdot 10^{\frac{8.12 \cdot W + 156}{W + 236}}}{p_{an} - 133.32 \cdot 10^{\frac{8.12 \cdot W + 156}{W + 236}}};$$

$$H = (t_{m1i} - t_{w1i}) \cdot \left[ 3.9 \cdot \left( \frac{R_{si}}{U_{si}^2 v_{1i}} \right)^{-0.1} B m_{1i}^{-0.45} \cdot L D_i^{-0.01} \right];$$

$$J = \zeta_h \cdot \rho_{a2i} + \zeta_{wi} \cdot m_i \cdot \rho_{wi};$$

$$N = \zeta_{si} \cdot \rho_{a2} \cdot U_{si}^2.$$

To find the minimum of the objective function, we take the derivative of  $V_m$  with respect to  $v_{hi}$  and set it to zero.

$$\frac{dV_m}{dv_{hi}} = \frac{V_{a1} \cdot p_{a1} \cdot \left[ \frac{r_0 \cdot M + E}{c_{pai} - c_s \cdot D} + \frac{c_s \cdot M \cdot P}{c_{pai} - c_s \cdot D^2} \right] + T_{a1} \cdot (p_{an} - 0.5N - 0.5 \cdot J \cdot v_{hi}^2)}{T_{a1} \cdot (p_{an} - 0.5N - 0.5 \cdot J \cdot v_{hi}^2)} + \frac{J \cdot v_{hi} \cdot V_{a1} \cdot p_{a1} \cdot \left( \frac{P}{c_{pai} - c_s \cdot D} + 273 \right)}{T_{a1} \cdot (0.5 \cdot N - p_{an} + 0.5 \cdot J \cdot v_{hi}^2)^2} = 0, \quad (18)$$

where

$$M = \frac{\Delta d_{0i} \left( \frac{G_{ai} \cdot E}{G_{wi} \cdot c_{pwi}} - \frac{0.3 \cdot H}{v_{hi}^{1.3}} \right)}{\Delta t_{m0i}} + \frac{11,055.57 \cdot 10^{2Y} \cdot C}{(p_{an} - 133.32 \cdot 10^Y)^2} + \frac{82.93 \cdot 10^Y \cdot C}{p_{an} - 133.32 \cdot 10^Y};$$

$$Y = \frac{8.12 \cdot t_{w1i} + \frac{8.12 \cdot H}{v_{hi}^{0.3}} + 156}{t_{w1i} + \frac{8.12 \cdot a}{v_{hi}^{0.3}} + 236};$$

$$E = -\frac{0.3 \cdot H \cdot c_{pai}}{v_h^{1.3}} - \frac{82.93 \cdot 10^Y \cdot C \cdot \left[ r_0 + c_s \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} \right) \right]}{p_{an} - 133.32 \cdot 10^Y} - \frac{24.88 \cdot 10^Y \cdot H \cdot c_s}{v_{hi}^{1.3} \cdot (p_{an} - 133.32 \cdot 10^Y)};$$

$$C = \ln 10 \cdot \left[ \frac{2.44 \cdot H}{v_{hi}^{1.3} \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} + 236 \right)} - \frac{0.3 \cdot H \cdot \left( 8.12 \cdot t_{w1i} + \frac{8.12 \cdot H}{v_{hi}^{0.3}} + 156 \right)}{v_{hi}^{1.3} \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} + 236 \right)^2} \right];$$

$$D = \Delta d_{0i} \cdot \left[ \frac{c_{pa} \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} \right)}{G_{wi} \cdot c_{pwi}} + \frac{t_{m1i} - t_{w1i} + \frac{H}{v_{hi}^{0.3}} + G_{ai} \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} \right)}{\Delta t_{m0i}} - I_1 + \frac{82.93 \cdot 10^Y \cdot \left[ r_0 + c_s \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} \right) \right]}{p_{an} - 133.32 \cdot 10^Y} \right] + \frac{G_{ai} \cdot \left[ c_{pai} \cdot W + (r_0 + s \cdot W) \cdot B - I_{1i} \right]}{G_{wi} \cdot c_{pwi}} - 1 - \frac{82.93 \cdot 10^Y}{p_{a1i} + 133.32 \cdot 10^Y};$$

$$P = c_{pai} \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} \right) + r_0 \cdot D + \frac{82.93 \cdot 10^Y \cdot \left[ r_0 + c_s \cdot \left( t_{w1i} + \frac{H}{v_{hi}^{0.3}} \right) \right]}{p_{an} - 133.32 \cdot 10^Y}.$$

Let us find the root of equation (18) using numerical methods with the root function in Mathcad. The value corresponds to the extreme point of the curve  $V_m = f(v_{hi})$ . The study shows that there is a minimum at the extreme point. Thus, at the obtained speed value, the theoretical flow rate of the non-



cooling section of the turbocompressor will be minimal, so we can say that this speed value will be rational, since at the minimum air flow rate, minimal energy consumption will be observed at the outlet of the air cooler.

Then the rational value of the cross-section of the Venturi pipe can be found using the dependence

$$S_{hi} = \frac{V_m}{V_{hi}}. \quad (19)$$

**Results.** To verify the adequacy of the model, experiments were carried out on a specially designed laboratory setup (Fig. 4).

The adequacy of the mathematical model was checked using Mathcad software for each series of experimental studies.

Laboratory tests of the contact cooler are carried out on a special test facility (Fig. 4). After the compressor, the compressed air enters the air heater 1, where it is heated to the required temperature. After that, it is mixed with cold water supplied from the tank 11 in the mixing device, which includes a venturi pipe 2 and distance pipes 3, and is shot, and then the mixture of water and air enters the elbow separator-droplet collector 4. Venturi pipe 2 is made in the form of a textolite insert. All other elements of the separator and mixing device are made of high-pressure polyethylene. This design allows for high-precision experiments, as it makes it possible to place thermocouples in the media streams immediately after separation. The use of non-metallic materials also reduces heat loss to the environment and the thermal inertia of the unit. The water and air streams are finally separated in the moisture separator 5. The heated water flows to the collection tank 16, while the cooled air flows to the measuring manifold 7 and then to the atmosphere. The air temperature is controlled by changing the voltage across the air heater using a laboratory autotransformer. Valve 6 maintains the required operating air pressure. Valve 15 controls the air flow rate. The air flow rate is determined by means of a measuring manifold 7 with a venturi flow device and a differential liquid manometer 9. The flow rate of water supplied from the tank 11 is regulated by the needle valve 12.

The parameters to be recorded in the studies included: the pressure drop across the flow measuring device  $h_d$ , pressure  $p_{a1}$  and initial (after the air heater) air temperature  $T_{a1}$ , initial wet-thermometer air temperature  $T_1$ , initial water temperature  $T_{w1}$ , final air temperature  $T_{a2}$ , final wet-thermometer air temperature  $T_2$ , and final water temperature  $T_{w2}$ , the volume of water that passed through the installation during the experiment  $V$  and the duration of the experiment  $\tau$ .

To verify the adequacy of the mathematical model, the experiments were conducted in 3 stages. In each stage, a series of experiments were conducted in which one of the following parameters was changed: initial air temperature  $t_{a1}$  (from 105 to

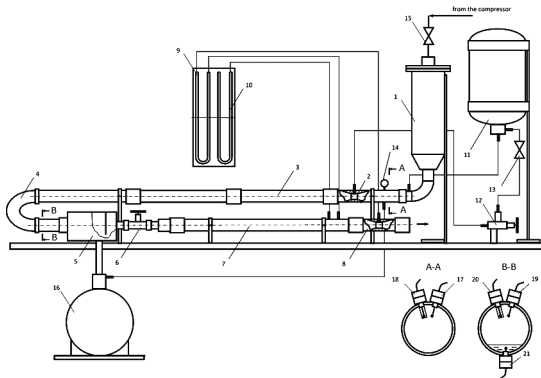


Fig. 4. Diagram of a laboratory contact air cooler:

1 – air heater; 2 – mixer – Venturi pipe; 3 – distance pipes; 4 – droplet separator; 5 – moisture separator; 6, 13, 15 – valve; 7 – measuring manifold; 8 – flow meter – Venturi pipe; 9 – differential pressure gauge; 10 – liquid pressure gauge; 11, 16 – tank; 12 – needle valve; 14 – spring pressure gauge; 17–20 – thermocouples

165 °C), inlet air pressure  $p_{a1}$  (from 0.2 to 0.4 MPa), water temperature  $t_{w1}$  (from 10 to 30 °C). All other parameters were taken for the nominal operation of the turbocompressor. For each of the parameters, the diameter of the venturi pipe head  $d_h$  was varied from 0.013 to 0.02 m in increments of 0.001 m.

All the experiments involved the following operations. The compressor was switched on. The air flow and pressure were set using valves 15 and 6, respectively. The initial air temperature was regulated by a laboratory autotransformer. The water flow rate was set with the needle valve 12. The water supply tap 13 was opened and the stopwatch was switched on at the same time. The readings of the differential manometer and potentiometers were recorded after temperature stabilisation was achieved in each mode, which was determined by the unchanged readings of the potentiometers for 30 s. Then the water supply tap 13 was turned off and the stopwatch was switched off. Water was drained from tanks 11 and 16 and its volume was determined using a measuring cylinder.

The dependences of the volumetric air flow rate at the outlet of the air cooler on the cross-section of the Venturi tube head (Figs. 5–7) were plotted when the initial temperature of water, air, and air pressure at the inlet to the air cooler changed.

**Conclusions.** The most efficient air cooler for a mine turbocompressor is a contact device Venturi tube – centrifugal

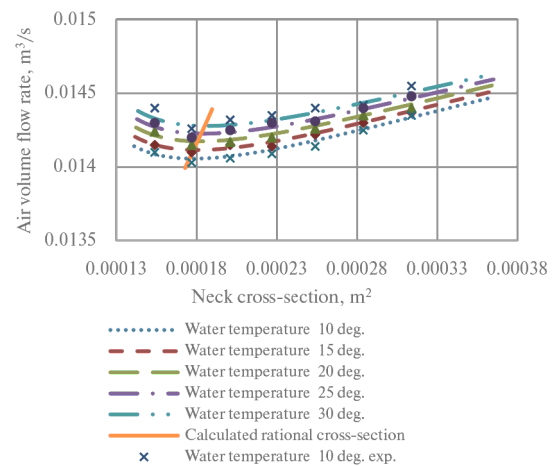


Fig. 5. Dependence of the air volume flow rate at the outlet of the air cooler on the cross-section of the Venturi tube head at a change in water temperature ( $p_{a1} = 0.2$  MPa;  $G_w = 0.05$  kg/s;  $G_a = 0.0188$  kg/s;  $t_{a1} = 135$  °C;  $t_1 = 37.5$  °C)

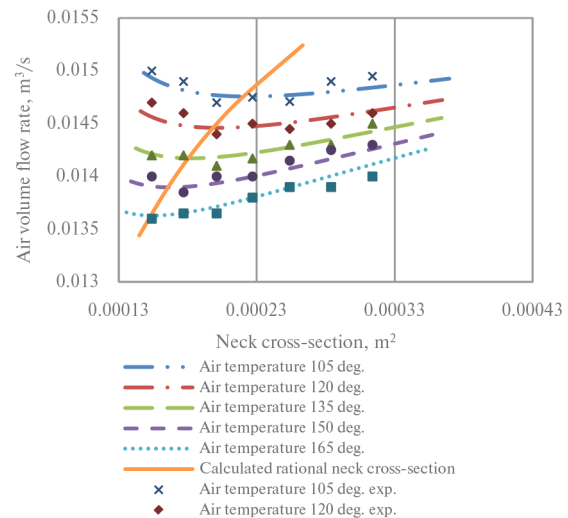


Fig. 6. Dependence of the air volume flow rate at the outlet of the air cooler on the cross-section of the Venturi tube neck at a change in the initial air temperature ( $p_{a1} = 0.2$  MPa;  $t_{w1} = 2$  °C;  $G_w = 0.05$  kg/s;  $G_a = 0.0188$  kg/s;  $t_1 = 3.5$  °C)

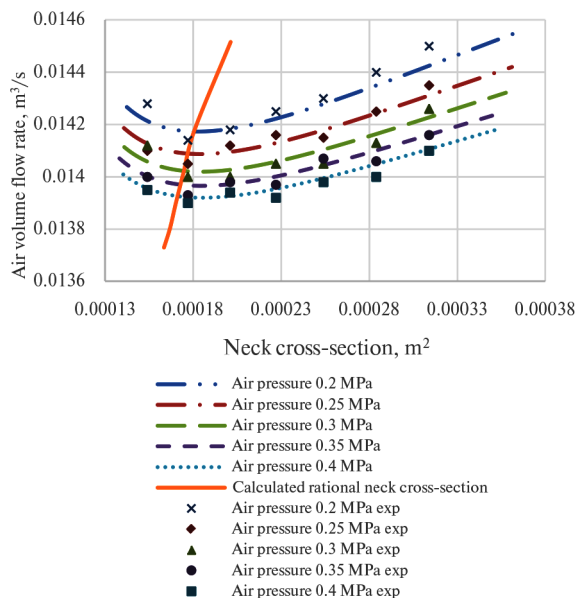


Fig. 7. Dependence of the air volume flow rate at the outlet of the air cooler on the air velocity in the neck of the Venturi pipe at a change in the initial air pressure ( $t_{w1} = 20\text{ }^{\circ}\text{C}$ ;  $G_w = 0.05\text{ kg/s}$ ;  $G_a = 0.0188\text{ kg/s}$ ;  $t_{a1} = 135\text{ }^{\circ}\text{C}$ ;  $t_1 = 37.5\text{ }^{\circ}\text{C}$ )

separator operating in the compressed air drying mode. So far, rational parametric design of such devices has been substantiated only for normal initial conditions and the nominal mode of the turbocompressor. To maintain the efficient operation of the turbocompressor in the entire range of initial conditions and modes, it is necessary to use a venturi tube with an adjustable neck cross-section as a mixer.

As a result of the research, an optimisation mathematical model of the mine turbocompressor air cooler was developed, which allows one to establish its rational parameters depending on the initial conditions and operating modes of the turbocompressor. The adequacy of the mathematical model has been proved by laboratory studies on a specially designed experimental setup. The discrepancy between the calculated and experimental data does not exceed 12%. The dependences of the neck cross-section of the air cooler Venturi tube on the initial conditions and the turbocompressor operating mode were determined.

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## Оптимізаційна математична модель контактного повітроохолоджувача шахтного турбокомпресора

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**Мета.** Встановлення залежностей раціональних параметрів контактних повітроохолоджувачів турбокомпресорів від режиму роботи й початкових умов.

**Методика.** У роботі були використані методи аналітичних досліджень, математичного моделювання, фізичного моделювання й математичної статистики.

**Результати.** У результаті проведених досліджень розроблена оптимізаційна математична модель повітроохолоджувача шахтного турбокомпресора, що дозволяє встановити його раціональні параметри в залежності від початкових умов і режимів роботи турбокомпресору. На спеціально розробленій дослідній установці доведена адекватність теоретичних досліджень. Отримані залежності дозволяють мінімізувати теоретичну подачу на вхід до неохолоджуваної секції турбокомпресора, що дозволяє зменшити питомі енергозатрати на виробництво стиснутого повітря.

**Наукова новизна.** Уперше розроблено метод визначення раціональних параметрів контактних повітроохолоджувачів при зміні початкової температури води, повітря й тиску повітря, що дозволяє розробити методику конструктивного розрахунку апаратів контактної системи охолодження шахтних турбокомпресорів.

**Практична значимість.** Стиснене повітря широко застосовується у всіх галузях промисловості. Воно є одним із найпоширеніших енергоносіїв на промислових підприємствах, а пристрої, що пов'язані з його розподілом і обробкою, є енергоємною складною промисловою енергетичною системою, від рівня її досконалості залежать показники технологічних процесів, у яких використовується стиснене повітря. Широке застосування стиснене повітря має в гірничорудній промисловості (рудовидобувній та паливобудувній). Для отримання стисненого повітря використовують турбокомпресори. Для підвищення ефективності турбокомпресора використовують охолоджувачі стисненого повітря. Суттєвим недоліком шахтної системи охолодження турбокомпресорів є швидке забруднення теплообмінних поверхонь повітроохолоджувачів накипними відкладеннями, що призводить до зниження їх ефективності за зростання питомих енергозатрат на виробництво стиснутого повітря. Цього недоліку немає в контактних повітроохолоджувачах труба Вентурі – відцентровий сепаратор-краплеуловлювач. У результаті досліджень отримані залежності, що використані при розробці методики конструктивного розрахунку контактних повітроохолоджувачів шахтного турбокомпресора.

**Ключові слова:** математична модель, труба Вентурі, контактний повітроохолоджувач, охолодження, стиснене повітря, турбокомпресор

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