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MATHEMATICAL MODEL OF AIR FLOW MOVEMENT IN A MOTORIZED FILTER RESPIRATOR

Purpose. Development of a mathematical model of the air flow movement in a motorized filter respirator (hereinafter referred to as MFR), which allows ensuring the control of the fan parameters, taking into account external and internal influences on the duration of the protective action and favourable operating conditions.

Methodology. To describe linear objects of the "input-output" type, it is convenient to use their transfer functions as mathematical models. In this case, to determine the mathematical description of the MFR operation, two tasks need to be solved. The first is related to finding the structure of the mathematical model, and the second involves determining the coefficients of the polynomials in the numerator and denominator of the transfer function that describes the motion of the air flow in the MFR.

Findings. A mathematical model of airflow in a MFR has been developed in the form of a transfer function of the third order; it can be used to develop a pressure control system for the air in the under-mask space in accordance with the user's work mode in order ensure comfortable working conditions. The presented mathematical model of airflow in the MFR differs from the existing approaches by taking into account the influence of the following external and internal parameters of the system on the performance indicators: the user's work mode, atmospheric pressure, filter resistance, pressure drop in air ducts with the effect of air accumulation in the under-mask space based on the "capacity-resistance" principle. Numerical coefficients of the mathematical model of airflow in the air duct of the MFR have been determined, which allow adjusting the number of fan rotations according to the time of operation, the increase in resistance on the filters, and the operating mode.

Originality. A correlation has been established between the external and internal parameters of the MFR: atmospheric pressure, pressure drop in the air duct, filter resistance, and the user's work mode with the effect of air accumulation in the sub-mask space reflected according to the "capacity-resistance" principle.

Practical value. The parameters of the mathematical model have been determined, which can be used when developing a control system for the airflow movement in the MFR: changes in air flow rate in accordance with different conditions of physical exertion of the user when performing professional activities.

Keywords: *motorized filter respirator, breathing resistance, protective efficiency*

Introduction. Personal respiratory protective equipment (hereinafter – RPE) is one of the necessary and important elements of protecting the respiratory organs of a worker who is forced to work in conditions of poor hygiene of the work environment, in which the concentration of harmful substances in the air of the work area exceeds the maximum allowable concentration [1, 2].

The most effective type of RPE among the existing ones is a motorized filter respirator (MFR), because it creates excessive air pressure in the mask space, preventing the suction of harmful substances because of non-hermetic obturation (through gaps between the user's face and the mask) caused by differences in users' facial anthropometry. However, this type of RPE requires solving a number of tasks [3, 4] which allow providing comfortable conditions for its use.

One of such tasks is to ensure effective regulation of supplying the necessary volume of air to the user, taking into account the pressure limitations in the MFR mask [5, 6], based on changes in working conditions, the degree of physical exertion on the user or their emotional state during the performance of professional functions, contamination of MFR filters, and increase in their resistance to airflow [7, 8], climatic conditions (temperature and humidity of the air) in the working area [9], and other parameters that characterize the influence on the quality indicators of RPE and may reduce their protective efficiency and level of comfort during the performance of tasks of varying intensity [10].

There are practical solutions [11, 12] for the described problem, which are based on applying different control modes for supplying air to the mask space by controlling the airflow resistance in the filters and the quantity of airflow specified by the worker's work mode. However, the imperfection of known solutions lies in the construction of stationary models that do not take into account the cyclicality of the process and the change in working modes associated with different loads on the system [13, 14]. Therefore, there is a need to address the current problem of developing a theoretical model of variable airflow in MFR with different input data from the parametric identification of MFR in order to maintain their high efficiency during the specified period of operation.

Literature review. Recently a number of scientific articles have been dedicated to solving the mentioned problem. In particular, the authors of papers [14, 15] presented a breathing control model that describes the volume of air based on lung gas exchange, allowing for the calculation of necessary parameters for changing the settings of a ventilator to provide a desired airflow to the sub-mask space. However, the developed model relates to the operation of lung ventilation assistance systems in medical institutions which require the support of breathing during operations, excluding changes in breathing volume due to the difficulty of the work or filter resistance. At the same time, the authors proposed interesting solutions for controlling the volume of air and the speed of movement in air ducts through a system of several sensors.

In another study, a model was proposed for calculating the protective efficiency of MFR, taking into account the influence of various external factors which may worsen user protection. Additionally, there was a need for monitoring the supply of an appropriate quantity and quality of air to the sub-mask space [16]. Although the proposed model fairly correlates the protection coefficient with the volume of airflow and filter penetration coefficient, it does not allow for control of all parameters that may affect the final result. For example, the temperature of the airflow or the humidity of the air, which can impair the user's comfort and significantly reduce the filter's protective duration.

The analysis of the proposed mathematical model for changes in lung volume due to the frequency of human respiration, developed by the authors of [17], showed the possibility of considering several physical parameters of the production environment, which would allow controlling the necessary number of fan rotations in the MFR. However, this model is limited only to their stationary change, which requires breaking the respirator's working time into certain intervals that are characterized by the stability of input parameters. In contrast, another study attempted to consider a model for evaluating the impact of key parameters of an artificial lung ventilation device on the dynamics of the ventilated respiratory system [18]. However, this model considers a system with a solenoid valve, which is not used in respirators.

The authors of the following work [19] on the development of a ventilation control system proposed a theoretical model for controlling both input and output parameters. Their research idea involved sequentially determining several input indicators, such as the amount of air supply, pressure drop, and protective efficiency, assuming their interdependence, which allows refining unknown coefficients required for calculating the velocity of the airflow. In this development, the authors limited themselves to calculating the constant velocity of the airflow, whereas it can fluctuate due to different user work rates.

The following work [20], which arouses interest, is the development of electrically protective respirators with air purification, which are widely used by healthcare workers. The study proposed and experimentally investigated a model of the airflow movement to the sub-mask space, which allows evaluating the influence of particle size and breathing rate on protective properties. At the same time, the authors noted that this model does not take into account possible air losses due to leaks in the half-mask, valve system, which ultimately results in differences between theoretical and experimental data and requires establishing a coefficient of loss for each particular case.

Purpose. Development of a mathematical model of airflow in the MFR, which allows for control of fan performance parameters, taking into account external and internal influences on the duration of protective action and comfortable operating conditions.

Methods. ***Methods of theoretical research on the movement of airflow in MFR air ducts.*** The schematic diagram of an MFR with forced air supply is shown in Fig. 1 [21].

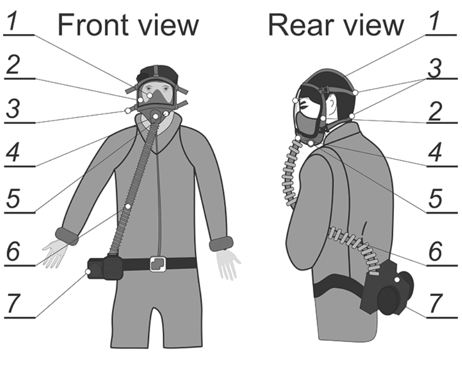

Fig. 1. The general view of an MFR with forced air supply to the user:

1 – a helmet-mask; 2 – an obturator; 3 – head straps; 4, 5 – inhalation/exhalation valve; 6 – corrugated air duct; 7 – air cleaning device [21]

To describe linear input-output objects, it is convenient to use their transfer functions as mathematical models. In this case, two tasks need to be solved to determine the mathematical description of the system. The first task is called structural identification and involves finding the structure of the mathematical model, i. e., determining the orders of the polynomials that appear in the numerator and denominator of the transfer function that describes the airflow in the MFR. The second task, called parametric identification, involves determining the coefficients of the transfer function taking into account the given structure of the mathematical model for the airflow in the MFR.

To mathematically describe the processes that take place in the MFR components, it is convenient to use equilibrium equations that are written for equivalent replacement circuits. According to the theory presented in [22], such replacement is based on the idea of replacing the system being modelled with a series of simple segments, each consisting of a minimum number of concentrated elements.

As the air ducts of the MFR have a closed volume, the airflow rate in each cross-section can differ due to compression. This is conditioned by the physical and chemical properties of the air. Thus, a change in pressure causes a change in the density of air and its mass in a confined volume. The effect of air accumulation occurs not only in the mask but also in the air flow channels.

An analytical description of the processes that occur during air transportation will be performed under certain assumptions. Each air transport channel will be replaced by two elements, one of which has a pressure change characteristic equal to the pressure loss over the length, while the other has zero pressure change and an accumulating capacity of the replaced element. Fig. 2 shows the replacement scheme.

Development of a theoretical model of air flow in MFR air ducts. To develop a mathematical model of the system, we will use the equation of conservation of matter (mass balance)

$$Q_{Min} - Q_{Mout} = V\frac{d\rho}{dt}, \qquad (1)$$

where *r* is air density, kg/m^3; *V* is air volume, m^3; *t* is time, s; Q_{Min}, Q_{Mout} is mass flow rate of air at the inlet and outlet of the object, kg/s.

The pressure in the MFR slightly differs from atmospheric pressure and should not exceed 370 Pa [12]. The temperature is almost constant, so air can be described with the accuracy sufficient for practical calculations using the ideal gas model. The process of air transportation with the help of the fan (as for a compressor) is isothermal.

For the isothermal process, characteristic equation is appropriate

$$pV = RT = K, \qquad (2)$$

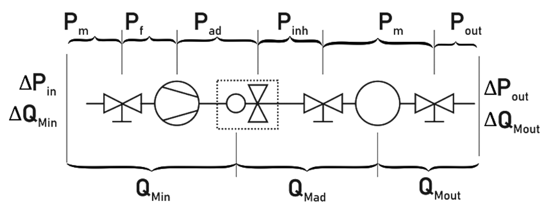

Fig. 2. Object substitution scheme (MFR):

p_{in}, p_{out} – pressure at the inlet and outlet, Pa; p_f – pressure (depression) after the filter, Pa; p_{ad} – pressure in the air duct formed during the operation of the fan, Pa; p_{inh} – pressure before the inhalation valve, Pa; p_m – pressure in the mask, Pa; Δp_{in}, Δp_{out} – changes in air pressure at the inlet and outlet of the system, Pa; Q_{Min}, Q_{Mout} – mass flow rate of air at the inlet and outlet of the system, kg/s; Q_{Mad} – mass flow rate of air in the air duct, kg/s; ΔQ_{Min}, ΔQ_{Mout} – changes in mass flow rate of air at the inlet and outlet of the system, kg/s

where p is the air pressure, Pa; V is the volume of air, m^3; R is the universal gas constant, J/°C · Kmol; T is the temperature of the air, °C.

The specific volume is calculated using the formula

$$\frac{1}{\rho}=\frac{V}{m}.$$

Taking into account equation (2), for the substance density the following expression will be correct

$$\rho=\frac{1}{V}=\frac{p}{K}.$$

If V_s is the volume of the medium in the system, then the amount of gas (air) *ms* in the system will be

$$m_s=\rho\cdot V_s=\frac{p\cdot V_s}{K}. \quad (3)$$

This expression establishes a relationship between the amount of substance in the system m_s and the pressure p.

The change in filling caused by a change in pressure can be found by differentiating expression

$$\frac{dm_s}{dt}=V_C\frac{d\rho}{dt}=\frac{V_s}{K}\frac{dp}{dt}. \quad (4)$$

For small pressure deviations, the ratio of m_s/p can be considered constant. Then, based on (3), we obtain

$$\frac{V_s}{K}\frac{dp}{dt}=\frac{m_s}{p}\frac{dp}{dt}.$$

And expression (1) takes the form

$$Q_{Min}-Q_{Mout}=\frac{m_s}{p}\frac{dp}{dt}.$$

Considering the fact that there is no change in filling in steady-state mode, we get

$$\frac{m_s}{p}=\frac{\overline{m}_s}{\overline{Q}_M}\cdot\frac{\overline{Q}_M}{\overline{p}}=T_i\frac{\overline{Q}_M}{\overline{p}},$$

where $T_i=\frac{\overline{m}_s}{\overline{Q}_M}$ has the dimension of time and can be considered as the constant of time of the isothermal process, s; $\overline{m}_s$ is the mass of air in the system in the steady-state mode (base value), kg; $\overline{Q}_M$ is the mass flow rate of air through the system in the steady-state mode (base value), m^3/s; $\overline{p}$ is the air pressure that corresponds to the steady-state mode (base value), Pa.

The mass of air in the system, as the pressure changes over time, is related by the equation

$$\frac{dm_s}{dt}=\frac{\overline{Q}_M}{\overline{p}}T_i\cdot\frac{dp}{dt}.$$

Given that in the steady-state mode $Q_{Min}=Q_{Mout}=Q_M$ and $p_{in}=\overline{p}$, i.e., there is no change in the filling; linearizing the equation through the small deviation method, we obtain

$$\frac{\overline{Q}_M}{\overline{p}}T_i\cdot\frac{d\Delta p_{in}}{dt}=\Delta Q_{Min}-\Delta Q_{Mout},$$

where Δp_{in} is the change in air pressure at the inlet of the system, Pa; ΔQ_{Min}, ΔQ_{Mout} are the changes in mass flow rate of air at the inlet and outlet, respectively, kg/s.

The balance of mechanical energy (equation of conservation of momentum) is

$$\Delta p_{in}+\Delta p_v-\Delta p_{out}=\Delta p_f+\Delta p_{ad}+\Delta p_{inh}+\Delta p_m,$$

where Δp_{in} is the change in pressure at the inlet, Pa; Δp_{out} is the change in pressure at the outlet, Pa; Δp_v is the change in pressure at the fan, Pa; Δp_f is the change in pressure at the filter, Pa; Δp_{ad} is the change in pressure in the air duct after the fan, Pa; Δp_{inh} is the change in pressure before the inhalation valve, Pa; Δp_m is the change in pressure in the mask, Pa; Δp_{in} and Δp_{out} become zero due to the constant atmospheric pressure.

We obtain

$$\Delta p_v=\Delta p_f+\Delta p_{ad}+\Delta p_{inh}+\Delta p_m+\Delta p_{exh}. \quad (5)$$

Let us examine the components of the equation in detail (5).

Air blower (fan). The change in pressure increase at the fan Δp_v can be determined using the formula [19]

$$\Delta p_v=\Delta p_{ad}-\Delta p_f=-\alpha_v\Delta Q_{Min}+b_v\Delta_n,$$

where α_v is the coefficient of resistance of the air duct with a filter, which we will calculate using the formula

$$\alpha_v=\frac{\partial\Delta p_v}{\partial Q_{Min}};\quad b_B=\frac{\partial\Delta p_v}{\partial n},$$

where ΔQ_{Min} the change in mass air flow rate at the inlet due to the change in fan productivity, kg/s; Δn is the change in rotational speed of the fan blades, s^{-1}; Δp_v is the pressure increase at the fan, Pa; Δp_{ad} is the pressure increase in the air duct, Pa; Δp_f is the pressure increase (decrease) after the filter, Pa.

Coefficients a_v and b_v can be obtained graphically from the fan's aerodynamic characteristic MFR, or from the following relationships

$$\frac{Q_{M1}}{\rho_1}=\frac{Q_{M0}}{\rho_0}\left(\frac{n_1}{n_0}\right);\quad \Delta p_{v1}=\Delta p_{v0}\left(\frac{n_1}{n_0}\right)^2,$$

where Δp_{v0}, Δp_{v1} is pressure drop at the MFR fan at the base and operating points, respectively, Pa; n_0, n_1 are rotational speeds of the MFR fan blades at the base and operating points, respectively, s^{-1}; Q_{M0}, Q_{M1} are mass air flow rates at the MFR fan outlet at the base and operating points, respectively, kg/s; ρ_0, ρ_1 stand for air density corresponding to the pressure at the base and operating points, kg/m$^{3.}$

Filter. The standard pressure drop (pressure drop at the operating point) for existing filters depends on the standard volumetric flow rate (volumetric flow rate at the operating point).

The change in pressure across the filter can be determined by the formula

$$\Delta p_{in}-\Delta p_f=\alpha_f\Delta Q_{Min},$$

taking into account $\Delta p_{in}=0$

$$-\Delta p_f=\alpha_f\Delta Q_{Min},$$

where α_f is the pressure loss coefficient of the MFR filter, 1/(m · s).

Darcy's law [19] for pressure drop due to the passage of air through a porous medium gives an estimate of pressure change with a change in flow rate; in this case, the pressure drop is proportional to the average flow velocity (i.e., $\Delta p\ \alpha\ u_m\ \alpha Q/A$, where u_m, Q, and A are the average flow velocity, volume flow rate, and filter area, respectively), which leads to

$$\Delta p_f=\frac{\overline{p}\cdot\Delta Q_f}{\overline{Q}_M},$$

where ΔQ_f is the change in the mass airflow rate through the filter due to a change in the productivity of the MFR fan kg/s, $\Delta Q_f=\Delta Q_{Min}$.

That is

$$\alpha_f=\frac{\overline{p}}{\overline{Q}_M}.$$

The airflow in the air duct after the fan is described by a system of equations [20], in which the first equation reflects the effect of accumulation due to the air compressibility, and the second equation describes the pressure losses due to the total local resistance of the MFR air duct

$$\begin{cases} \dfrac{1}{a_{ad}}\Delta p_{inh} = \dfrac{\bar{Q}_M}{2\bar{p}} T_i \Delta p_{ad} + \dfrac{1}{a_{ad}}\Delta p_{ad} - \Delta Q_{Min} \\ a_{ad}\dfrac{\bar{Q}_M}{2\bar{p}} T_i \Delta \dot{Q}_{Mad} + \Delta Q_{Mad} = \Delta Q_{Min} - \dfrac{\bar{Q}_M}{\bar{p}} T_i \Delta \dot{p}_{inh} \end{cases}, \quad (6)$$

where $\bar{Q}_M$ is steady-state mass flow rate, kg/s; $\bar{p}$ is pressure in the steady-state mode, Pa; T_i is time constant of the isothermal process, s; ΔQ_{Mad} is change in mass air flow rate in the air duct, kg/s; a_{ad} is the pressure loss coefficient in the air duct, $1/(\text{m} \cdot \text{s})$ [22]

$$a_{ad} = \left(\frac{\lambda \cdot L}{D_{id}} + \sum_{i=1}^{n} \xi_i \right) \cdot \frac{\bar{Q}_M}{\rho \cdot S^2},$$

where L is the pipeline length, m; D_{id} is the internal diameter of the MFR air duct, m; ξ is the dimensionless coefficient of local resistance; ρ is air (gas) density, kg/m^3; S is the cross-sectional area of the MFR air duct, m^2; λ is the Darcy hydraulic friction coefficient for a smooth pipe, which depends on the Reynolds number (Re).

The hydraulic friction coefficient is used in hydraulics and aerodynamics to determine pressure losses in a system's design section. The formula for calculating the Reynolds number through the volume flow of liquid or gas is

$$\text{Re} = (Q \cdot D)/(v \cdot S),$$

where Q is the volumetric flow rate of the fluid, m^3/s; D is the hydraulic diameter, m; v is the kinematic viscosity of the medium, m^2/s; S is the cross-sectional area of channel (duct pipes), m^2.

To calculate the coefficient of hydraulic friction, we use formulas for a hydraulic smooth wall depending on the air flow regime

$$\lambda = \frac{64}{\text{Re}} \quad \text{if} \quad \text{Re} < 2000;$$

$$\lambda = \frac{0.3164}{\text{Re}^{0.25}} \quad \text{if} \quad \text{Re} > 4000.$$

When transitioning from laminar to turbulent flow, the friction coefficient is difficult to calculate, so it is estimated by linear interpolation between the friction coefficients at Reynolds numbers of 2000 and 4000. This relationship for the friction coefficient was chosen because it can be easily modified to account for rough pipes.

The change in pressure at the inhalation valve can be described by the equation

$$\Delta p_{inh} - \Delta p_M = a_{inh} \Delta Q_{Mad},$$

where a_{inh} is the pressure loss coefficient at the inhalation valve, $1/(\text{m} \cdot \text{s})$

$$a_{inh} = \frac{2K_{inh} \cdot \bar{Q}_M}{\bar{S}_{inh}^2}; \quad K_{inh} = \frac{\xi_{inh}}{2\rho},$$

where K_{inh} is the resistance coefficient of the inhalation valve; $\bar{S}_{inh}$ is the cross-sectional area of the inhalation valve in the steady-state mode, m^2; ξ_{inh} is the local resistance coefficient of the inhalation valve which depends on the design of the valve.

There are several components that determine changes in air pressure in the MFR mask:

- an air blower (fan), which provides an airflow in response to the pressure values in the mask (discussed above). The airflow from the fan enters the mask through an air duct;
- the lungs, which add or remove air in accordance with the user's breathing. The breathing process is not considered in the current model and depends on the user and their current workload (physical and/or mental);
- mask seal leakage. It is not considered in the current model and largely depends on the quality of the user's facial seal;
- an exhalation valve, which prevents external air from entering when the pressure in the mask is below a certain set value. When this value is exceeded, the valve will open with a volumetric flow rate determined by the formula

$$\Delta p_M - \Delta p_{out} = a_{exh} \Delta Q_{Mout}.$$

Taking into account $\Delta p_{out} = 0$

$$\Delta p_M = a_{exh} \Delta Q_{Mout}, \quad (7)$$

where a_{exh} is the pressure loss coefficient at the exhalation valve, $1/(\text{m} \cdot \text{s})$.

The coefficient a_{exh}

$$a_{exh} = \frac{2K_{exh} \cdot \bar{Q}_M}{\bar{S}_{exh}^2}; \quad K_{exh} = \frac{\xi_{exh}}{2\rho},$$

where K_{exh} is the exhalation valve resistance coefficient; $\bar{S}_{exh}$ is the cross-sectional area of the exhalation valve in the steadstate mode, m^2; ξ_{exh} is the local resistance coefficient of the exhalation valve, which depends on the design of the MFR valve.

For the design of the MFR mask, considering the submask space as an accumulated volume of the air that is inhaled by the user, while assuming that there is no resistance to the air flow in the mask, we can write the equation as follows

$$\frac{\bar{Q}_M}{\bar{p}_M} T_i \Delta \dot{p}_M = \Delta Q_{Mad} - \Delta Q_{Mout}. \quad (8)$$

Thus, taking into account equations (6–8), we obtain a system of equations that describes the air flow in the MFR air duct

$$\begin{cases} -\Delta p_f = a_f \Delta Q_{Min} \\ \Delta p_{ad} - \Delta p_f = -a_v \Delta Q_{Min} + b_v \Delta n \\ \dfrac{1}{a_{ad}}\Delta p_{inh} = \dfrac{\bar{Q}_M}{2\bar{p}} T_i \Delta \dot{p}_{ad} + \dfrac{1}{a_{ad}}\Delta p_{ad} - \Delta Q_{Min} \\ a_{ad}\dfrac{\bar{Q}_M}{2\bar{p}} T_i \Delta \dot{Q}_{Mad} + \Delta Q_{Mad} = \Delta Q_{Min} - \dfrac{\bar{Q}_M}{\bar{p}} T_i \Delta \dot{p}_{ad} \\ \Delta p_{inh} - \Delta p_M = a_{inh} \Delta Q_{Mad} \\ \dfrac{\bar{Q}_M}{\bar{p}} T_i \Delta \dot{p}_{ad} = \Delta Q_{Mad} - \Delta Q_{Mout} \\ \Delta p_M = a_{exh} \Delta Q_{Mout} \end{cases}. \quad (9)$$

After making the appropriate substitutions and transformations in the system of equations (9) with respect to the air pressure in the MFR mask, we obtain an equation of the form

$$a_3 \Delta \dddot{p}_M + a_2 \Delta \ddot{p}_M + a_1 z \Delta \dot{p}_M + a_0 \Delta p_M = b_1 \Delta \dot{n} + b_0 \Delta n. \quad (10)$$

The coefficients in equation (10) in general are determined by

$$z = \frac{\bar{Q}_M}{\bar{p}} T_i; \quad a_3 = z^3 \frac{a_{ad}(a_f + a_v)(a_{ad} + 2a_{inh})}{4};$$

$$a_2 = z^2 \frac{a_{ad}(a_f + a_v)(a_{ad} + 2a_{inh} + 4a_{exh})}{4a_{exh}} + \frac{(a_f + a_v + 1)}{(a_f + a_v)} \cdot \frac{(a_f + a_v)(a_{ad} + 2a_{inh})}{2} =$$

$$= z^2 \frac{a_{ad}(a_f + a_v)(a_{ad} + 2a_{inh} + 4a_{exh}) + 2a_{exh}(a_f + a_v + 1)(a_{ad} + 2a_{inh})}{4a_{exh}};$$

$$a_1 = z\left(a_{inh} + \frac{a_{ad}(a_f + a_v)}{2a_{exh}} + \frac{(a_f + a_v + a_{ad})}{(a_f + a_v)} \cdot \frac{(a_f + a_v)(a_{ad} + 2a_{inh} + 4a_{exh})}{2a_{exh}} \right) =$$

$$= z\left(a_{inh} + \frac{a_{ad}(a_f + a_v)}{2a_{exh}} + \frac{(a_f + a_v + a_{ad})(a_{ad} + 2a_{inh} + 4a_{exh})}{2a_{exh}} \right) =$$

$$= z\left(\frac{2a_{inh}a_{exh} + 2a_{ad}a_f + 2a_{ad}a_v + 2a_{inh}a_f + 4a_{exh}a_f}{2a_{exh}} + \frac{2a_{inh}a_v + 4a_{exh}a_v + a_{ad}^2 + 2a_{inh}a_{ad} + 4a_{exh}a_{ad}}{2a_{exh}} \right) =$$

$$= z\frac{2a_{exh}(a_{inh} + 2a_f + 2a_v + 2a_{ad}) + a_{ad}(a_{ad} + 2a_f + 2a_v + 2a_{inh}) + 2a_{inh}(a_f + a_v)}{2a_{exh}};$$

$$a_0 = \frac{(a_{inh} + a_{exh})}{a_{exh}} + \frac{(a_f + a_v)}{a_{exh}} = \frac{a_{inh} + a_{exh} + a_f + a_v}{a_{exh}}; \quad b_1 = z\frac{b_v}{2};$$

$$b_0 = b_v - \frac{a_{ad} \cdot b_v}{(a_f + a_v)} = \frac{a_f b_v + a_v b_v - a_{ad} b_v}{a_f + a_v} = \frac{b_v(a_f + a_v - a_{ad})}{a_f + a_v}.$$

To obtain the transfer function for pressure change in the mask due to changes in the rotation speed of the MFR fan, we will use the recommendations [20]

$$a_3 \Delta\dddot{p}_M + a_2 \Delta\ddot{p}_M + a_1 \Delta\dot{p}_M + a_0 \Delta p_M = b_1 \Delta\dot{n} + b_0 \Delta n;$$

$$W_{\Delta n \to \Delta p_M}(p) = \frac{\Delta p_M(p)}{\Delta n(p)} = \frac{b_1 p + b_0}{a_3 p^3 + a_2 p^2 + a_1 p + a_0}.$$

The obtained mathematical model in the form of a transfer function, which has a *3rd* order, can be used for further parametric identification of the control object for air flow movement in the air duct of the MFR.

Research results. Using the information about the design features, technical characteristics of the MFR components, and the physical properties of the air which is inhaled by the user of the MFR, we obtained numerical coefficients of the mathematical model for the air flow movement in the air duct of the MFR (Table).

The mathematical model for the air flow movement in the air duct of the MFR will have the following form

$$W_{\Delta n \to \Delta p_M}(p) = \frac{0.0000028405p + 0.75422785}{0.0124972p^3 + 1.20p^2 + 8.43p + 7.06}.$$

The mathematical modelling of air flow movement in the air duct of the MFR was performed in the Simulink environment of Matlab software. The modelling was performed under the condition that the initial pressure inside the MFR was equal to atmospheric pressure, there were no air leaks due to gaps, and no breathing (the mask was worn on a mannequin). The results of the modelling at different rotation speeds of the MFR air duct are presented in Fig. 3. The results of the modelling at different degrees of filter contamination in the system are presented in Fig. 4.

The obtained data demonstrate the possibility of controlling the amount of air entering the sub-mask space, taking into account the filter contamination and operating time with a change in operating modes that are defined by corresponding numerical coefficients.

Results. The mathematical model for the airflow movement in the MFR pipeline with mechanical air cleaning is developed on the basis of analysis of the properties of the structural components and the device as a whole, as well as the peculiarities of its operation when used. The model will be used for developing a control system based on several parameters: pressure drop, airflow rate, taking into account the mode of operation and the degree of filter contamination with consideration of protective action time. In the development of the mathematical model for the airflow movement in the MFR pipeline, the peculiarities inherent in the transportation processes of gaseous substances, including air, were taken into account. For the mathematical description of air transport zones, the "capacity-resistance" substitution method was used to mathematically describe the zones of air transportation, which reflects the accumulation effect due to the compressibility of air, as well as pressure loss because of the total local resistance of the air duct. This effect is particularly important to consider when choosing or using centrifugal fans which consist of two main parts: the turbine (impeller) and the spiral-shaped casing (housing). The impeller of such a fan is a hollow cylinder, and during its rotation, the air that enters between the blades moves radially from the centre being compressed.

Under the action of centrifugal force, the air is pushed into the spiral-shaped casing of the fan housing and then directed towards the outlet (injection), which significantly affects the amount of air flow, especially when the battery charge drops, and requires appropriate adjustment and control [19, 20].

One of the main parameters of the model for air flow in the pipeline is the pressure inside the MFR under different operating conditions. For the MFR, there is a strict requirement for

Table

Numerical coefficients of the mathematical model for the air flow movement in the air duct of the MFR

a_3	a_2	a_1	a_0	b_1	b_0
0.0124972	1.20	8.43	7.06	0.0000028405	0.75422785

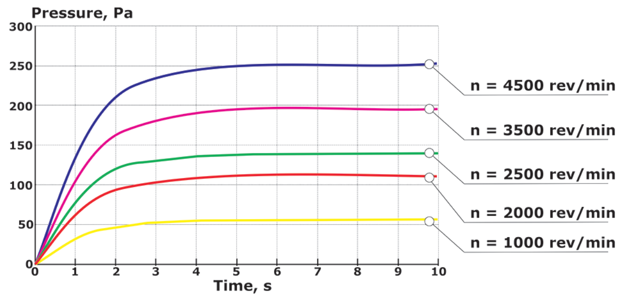

Fig. 3. Dependence of pressure in the mask at different speeds of rotation of the air duct wheel of the MFR

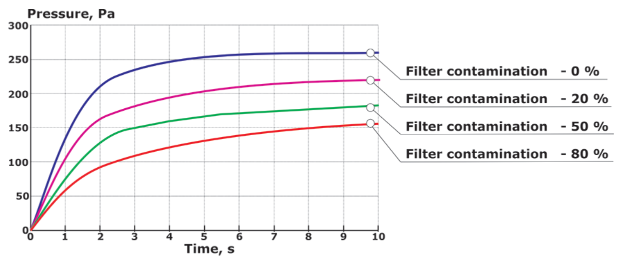

Fig. 4. Dependence of pressure in the mask at different degrees of contamination of the MFR filters

constant maintenance of excess pressure inside the mask to prevent the entry of external contaminated air. When developing a system for controlling air flow in the MFR based on the presented model, it is necessary to take into account the possibility of identifying when this condition is violated, which is one of the important reasons for stopping the operation of the respirator.

The developed model provides that an increase in the resistance of the filters may result in a decrease in the air supply, which will require an increase in the number of fan rotations on the one hand, and an increase in the resistance of the air flow through the filter, on the other. The contradiction which arises can be eliminated by using several filters, one of which is a spare one. In case of a significant increase in the resistance of the air flow, the system for controlling the MFR parameters will either switch the flow to another route or indicate the need to replace the filter [21, 22]. The same approach is implemented in case of significant accumulation of moisture inside the filter, which not only increases the resistance but also reduces the protective efficiency of the MFR.

Further research will be related to improving the current model by taking into account the influence of air humidity on protective efficiency and selecting the necessary number of fan revolutions to maintain the required level of protection for respiratory protective equipment.

Conclusions.

1. The mathematical model of the air flow movement in the MFR has been developed in the form of a transfer function, which is a third-order equation; it can be used to develop a control system for air pressure in the mask according to the user's working conditions to ensure compliance with regulatory requirements for similar RPEs. The unique feature of this model compared to existing calculation algorithms is the relationship established between external and internal system parameters, such as atmospheric pressure, pressure drop, filter resistance. These parameters take into account the zones of air transport with the effect of air accumulation in the ducts, considering the property of air compression, which is reflected through the "capacity-resistance" principle.

2. The numerical coefficients of the mathematical model of the air flow in the MFR air duct have been established, which allow for the process of adjusting the fan rotation speed according to the operating time, pressure drop across the filters, and the operating mode.

3. A relationship has been established between the external and internal parameters of the MFR: atmospheric pressure, pressure drop in the air duct, filter resistance, and user operation mode, taking into account the accumulation of air in the sub-mask space, which is reflected through the principle of "capacity-resistance".

4. The parameters of the mathematical model have been established, which can be used when developing a control system for the air flow movement of in the MFR, including changes in air flow rate in accordance with different conditions of physical exertion by the user during professional activity.

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Математична модель руху повітряного потоку в моторованому фільтрувальному респіраторі

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Мета. Розробка математичної моделі руху повітряного потоку в моторованому фільтрувальному респіраторі (далі – МФР), що дозволяє забезпечити керування параметрами роботи вентилятора, з урахуванням зовнішніх і внутрішніх впливів на термін захисної дії та комфортні умови експлуатації.

Методика. Для опису лінійних об'єктів виду «вхід–вихід» зручно в якості математичних моделей використовувати їх передавальні функції. У цьому випадку для визначення математичного опису роботи МФР необхідно вирішити два завдання. Перше, пов'язане зі знаходження структури математичної моделі, а друге – полягає у визначенні коефіцієнтів поліномів, що стоять у чисельнику та знаменнику передавальної функції, яка описує рух повітряного потоку у МФР.

Результати. Розроблена математична модель руху повітряного потоку у МФР у вигляді передавальної функції, що має 3-й порядок і використана для розробки системи керування тиском повітря в підмасковому просторі відповідно до режиму праці користувача з метою забезпечення комфортних умов праці. Відмінністю представленої математичної моделі руху повітряного потоку у МФР від існуючих підходів є врахування впливу на експлуатаційні показники зовнішніх і внутрішніх параметрів системи: режиму праці користувача, атмосферного тиску, опору фільтрів, перепаду тиску в повітропроводах з ефектом накопичення повітря в підмасковому просторі на основі принципу «ємність–опір». Визначені числові коефіцієнти математичної моделі руху повітряного потоку в повітропроводі МФР, що дозволяють здійснювати корегування кількості обертів вентилятора відповідно до часу експлуатації, зростання опору на фільтрах, режиму роботи.

Наукова новизна. Встановлено взаємозв'язок між зовнішніми та внутрішніми параметрами МФР: атмосферним тиском, перепадом тиску в повітропроводі, опором фільтрів, режимом роботи користувача з ефектом накопичення повітря в підмасковому просторі за принципом «ємність–опір».

Практична значимість. Встановлені параметри математичної моделі, що можуть бути використані при розробці системи керування рухом повітряного потоку у МФР: зміна витрати повітря у відповідності до різних умов фізичного навантаження користувача під час виконання професійної діяльності.

Ключові слова: *моторований фільтрувальний респіратор, опір дихання, захисна ефективність*

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