https://doi.org/10.33271/nvngu/2023-2/058

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CONTACT TENSIONS UNDER THE SOLE OF RIGID DEEP LAYING FOUNDATIONS AND GROUND ANCHORS

Purpose. To solve the problem of the allocation of contact vertical normal tensions along the sole of a rigid round stamp, located in an elastic isotropic half-space at a certain depth $h \neq 0$. To compare the obtained solution with the well-known classical result for $h = 0$, to check the obtained results for adequacy.

Methodology. Based on the analysis of the decision on the stress-strain state of the base, inside which there is a vertical arbitrary load distributed over the area of the circle, the necessary formulas are obtained to solve the problem. An algorithm for constructing an approximate solution has been developed, the essence of which is to use a combination of the boundary element method and the iteration process by S. N. Klepikov. For a number of depths, approximate solutions of the considered problem are obtained.

Findings. The proposed algorithm for the approximate solution of the problem of indenting a round rigid stamp into the upper boundary of an elastic isotropic half-space has good agreement with the exact solution and can be used to solve contact problems. The outlines of the contact stress diagrams depend on the depth at which they are determined – the greater the depth, the flatter the outlines of the diagrams are, while starting from a certain depth, the diagrams of contact stresses practically coincide. The greater the depth is at which the stamp is located, the more force must be applied to obtain equal displacements of the stamp.

Originality. The obtained research results significantly expand the possibilities of solving various problems of soil mechanics and foundation engineering, make it possible to obtain absolutely new results. In particular, a clear dependence of the contact stresses along the sole of a rigid round stamp on the depth at which it is located was identified. In addition, the presented data allow us to designate an absolutely new direction in the calculation of the foundations of ground anchors, namely, the calculation of their deformations.

Practical value. For engineering practice, it is important that the greater the value of Poisson's ratio of the base is, the greater the contact stresses are, other things being equal.

Keywords: *deep laying foundations, contact tensions, hard stamp, ground anchor, base sinking, isotropic half-space*

Introduction. Deep foundations with round soles are designed to transfer the load to strong soils at very great depths.

These foundations perceive heavy loads, since with a significant depth of their immersion, the protrusion of the soil from under the sole to the day surface is excluded.

At the same time, ground anchors are used for fixing soil slopes, open and underground workings, pits, chimney foundations, masts, towers, other structures and their elements.

Deep foundations and ground anchors are widely used in mining, mine building, civil, transport and hydrotechnical construction [1, 2].

From the point of view of geometry, these two types of structures are identical considering the fact that the ratio of their radius R to the laying depth of their soles h is much more than 10 [1, 3].

When designing such structures, it is very important to know the distribution of vertical normal contact stresses along their sole. In the first approximation, for a theoretical solution of the problem under consideration, it is sufficient to interpret the soil foundation as a linear isotropic elastic medium, and the sole of the deep foundation (or ground anchor) as an absolutely rigid body [4, 5].

The relevance of the work. At present, when calculating stresses in soil foundations, the so-called fundamental solutions and the superposition principle are applied [5–7].

When using one or another fundamental solution, there are:

1. Stresses due to the action of a vertical concentrated force applied to the upper boundary of the half-space (at a depth $h = 0$), where h is a distance from the day surface to the point of application of the force along the vertical (i. e. in the direction of the axis $0z$, Fig. 1).

This fundamental solution is called the Boussinesq problem.

If the force acts in a horizontal direction, then we are dealing with the Cerruti problem.

2. Stresses due to the action of a vertical (and horizontal) concentrated force applied inside the half-space (at a depth $\infty > h \neq 0$).

This fundamental solution is called the Boussinesq problem.

3. Stresses due to the action of a vertical (and horizontal) concentrated force applied inside space (at depth $h = \infty$).

This fundamental solution is called the Kelvin problem.

Also known is the problem of stress distribution at the base of a rigid round stamp located on the upper boundary of an elastic half-space (i. e. at $h = 0$). At the same time, the correct solutions to the problem of a rigid stamp located in the soil half-space (i. e. at $h \neq 0$) soil space (i. e. at $h = \infty$), are practically absent.

The research materials presented in this article are aimed at solving this problem.

Purpose of the work. To obtain a solution to the problem of the distribution of vertical normal stresses along the sole of a rigid round stamp located in an elastic isotropic half-space at a certain depth $h \neq 0$.

The obtained result was compared with the classical solution of the problem known in the literature for depth $h = 0$ and, thus, the obtained results were checked for adequacy.

Materials and research methods. At the first stage of research, we performed an analysis of the well-known solution to the problem of the stress-strain state of the base, inside which there is a vertical arbitrary load distributed over the area of the circle [7].

On this basis, the formulas necessary for solving the problem under consideration were obtained.

Next, an algorithm for constructing an approximate solution to the problem was developed the essence of which is to use a combination of the boundary element method and the iteration process by S. N. Klepikov [8].

Further, for a number of depths, a number of approximate solutions of the problem under consideration were obtained.

Formulation of the problem. Presentation of the main material. Let us consider the problem of determining the contact stresses at the base of a round-shaped, flat, buried ground anchor that has received vertical displacement W_0. The design scheme is shown in Fig. 1.

We formulate the research problem as follows. At a depth h parallel to the horizontal day surface of the base, there is a round, flat stamp with a thickness t. The stamp radius is R. The thickness of the stamp is much less than its radius and the depth at which the stamp is located, i. e. $R \gg t$ and $h \gg t$. The elastic properties of the base are also known – its shear modulus G and Poisson's ratio v. It is required to determine the diagram of contact stresses $q(r)$ and the magnitude of the pull-out (such a design scheme corresponds to a ground anchor) or indentation (such a design scheme corresponds to a deep foundation) concentrated force N. When determining these characteristics, the deformations of the stamp in the radial direction can be neglected, and its bending deformations are equal to zero.

Consider formula (12) from [7]. We have:

- at $0 \le z < h$

$$W^1 = \int_0^\infty B(\alpha,r)\cdot\left\{\begin{matrix} e^{[\alpha\cdot(-h+z)]}\cdot\alpha\cdot h - e^{[\alpha\cdot(-h+z)]}\cdot z\cdot\alpha + \\ +3\cdot e^{[\alpha\cdot(-h+z)]} - 4\cdot e^{[\alpha\cdot(-h+z)]}\cdot\nu + \\ +3\cdot e^{[-\alpha\cdot(h+z)]}\cdot z\cdot\alpha - 12\cdot e^{[-\alpha\cdot(h+z)]}\cdot\nu - \\ -4\cdot e^{[-\alpha\cdot(h+z)]}\cdot\alpha\cdot h\cdot\nu + \\ +3\cdot e^{[-\alpha\cdot(h+z)]}\cdot\alpha\cdot h + 8\cdot e^{[-\alpha\cdot(h+z)]}\cdot\nu^2 + \\ +5\cdot e^{[-\alpha\cdot(h+z)]} + 2\cdot e^{[-\alpha\cdot(h+z)]}\cdot z\cdot\alpha^2\cdot h - \\ -4\cdot e^2\cdot z\cdot\alpha\cdot\nu \end{matrix}\right\}\cdot d\alpha;$$

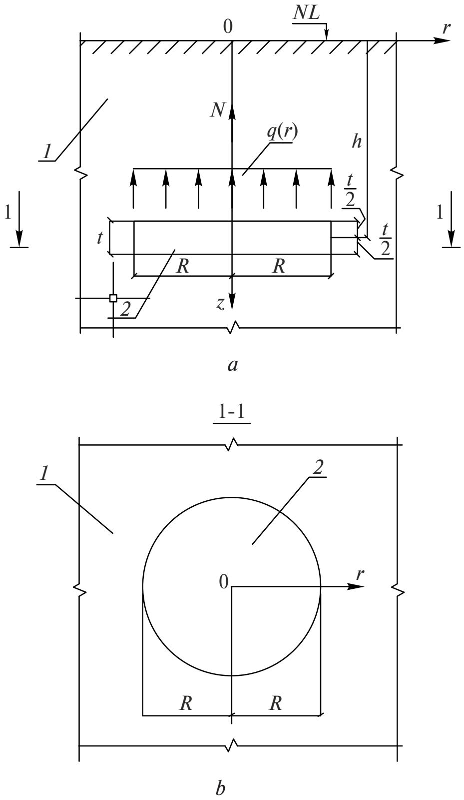

Fig. 1. Calculation scheme of the "base–rigid stamp" system: a – base; b – rigid stamp

- at $h < z \le \infty$ $h < z \le \infty$

$$W^2 = \int_0^\infty B(\alpha,r)\cdot\left\{\begin{matrix} 3\cdot e^{[-\alpha\cdot(-h+z)]} - 4\cdot\nu\cdot e^{[-\alpha\cdot(-h+z)]} + \\ +z\cdot\alpha\cdot e^{[-\alpha\cdot(-h+z)]} - \alpha\cdot h\cdot e^{[-\alpha\cdot(-h+z)]} + \\ +3\cdot e^{[-\alpha\cdot(h+z)]}\cdot z\cdot\alpha - 12\cdot e^{[-\alpha\cdot(h+z)]}\cdot\nu - \\ -4\cdot e^{[-\alpha\cdot(h+z)]}\cdot\alpha\cdot h\cdot\nu + \\ +3\cdot e^{[-\alpha\cdot(h+z)]}\cdot\alpha\cdot h + 8\cdot e^{[-\alpha\cdot(h+z)]}\cdot\nu^2 + \\ +5\cdot e^{[-\alpha\cdot(h+z)]} + 2\cdot e^{[-\alpha\cdot(h+z)]} z\cdot\alpha^2\cdot h - \\ -4\cdot e^{[-\alpha\cdot(h+z)]}\cdot z\cdot\alpha\cdot\nu \end{matrix}\right\} d\alpha,$$

where $B(\alpha,r) = \dfrac{J_0(\alpha\cdot r)\cdot A(\alpha)}{8\cdot G\cdot(1-\nu)}$ and z is a vertical current coordinate.

Next, using the passage to the limit, we find

$$\left.\begin{matrix} W = \lim\limits_{z\to h}(W_1) = \lim\limits_{z\to h}(W_2) = \dfrac{1}{8\cdot(1-\nu)\cdot G}\times \\ \times\int_0^\infty\left[3-4\cdot\nu+\begin{pmatrix} 8\cdot\nu^2+6\cdot\alpha\cdot h-8\cdot\alpha\cdot h\cdot\nu - \\ -12\cdot\nu+5+2\cdot\alpha^2\cdot h^2 \end{pmatrix}\cdot e^{-2\cdot\alpha\cdot h}\right]\times \\ \times A(\alpha)\cdot J_0(\alpha\cdot r)\cdot d\alpha \end{matrix}\right\}, \quad (1)$$

where W is total vertical movement of an absolutely rigid stamp; $W_1$is the same, in plane $z = h - 0$; W_2 is the same, in plane $z = h + 0$; $J_0(x)$is Bessel function of the first kind with zero index and valid argument; $A(\alpha) = \int_0^\infty r\cdot q(r)\cdot J_0(\alpha\cdot r)\cdot dr$; $q(r)$ is some coordinate function r (this function has the physical meaning of a distributed load and the dimension kPa); α is Bessel transform parameter $\alpha \in (0, \infty)$; z is current coordinate [8, 9].

Next, we perform asymptotic estimates of the formula obtained by us (1). For $h \to 0$ we have

$$W_0 = \lim_{h\to 0}(W) = \frac{(1-\nu)}{G}\cdot\int_0^\infty A(\alpha)\cdot J_0(\alpha\cdot r)d\alpha. \quad (2)$$

This estimate is valid when the stamp is located on the upper boundary (i.e. day surface) of the base. It should be noted that formula (2) is usually used to determine the contact stresses in the base located on the border of the soil base of flat rigid stamps.

We accept in (1) $h \to \infty$, we get

$$W_\infty = \lim_{h\to\infty}(W) = \frac{3-4\cdot\nu}{8\cdot G\cdot(1-\nu)}\cdot\int_0^\infty A(\alpha)\cdot J_0(\alpha\cdot r)d\alpha. \quad (3)$$

This estimate is valid in the case when the stamp is located at a great depth (i. e., at a considerable distance from the day surface of the base, in other words, when $h/R \gg 1$). Next, we find the ratio of the movement of the stamp located on the day surface of the base (2) to the movement of the stamp at a considerable depth (3), also Fig. 2

$$W^* = \frac{W_0}{W_\infty} = \frac{8\cdot(1-\nu)^2}{3-4\cdot\nu}. \quad (4)$$

Where W^* is relative displacement.

From formula (4) and Fig. 2, it follows that, other things being equal (i. e., with the same properties of the base, the magnitude and distribution law of the vertical load), depending on the value of Poisson's ratio, sediment ratio at $h \to 0$ and $h \to \infty$ changes by 2–2.7 times, and the greater the Poisson's ratio v is, the less W^* is.

Next, we find the contact pressure along the sole of the rigid stamp. For this, it is necessary to consider the vertical normal stresses $\sigma_z(r, z)$.

From formula (12) on the *18th* page of [7] we have:

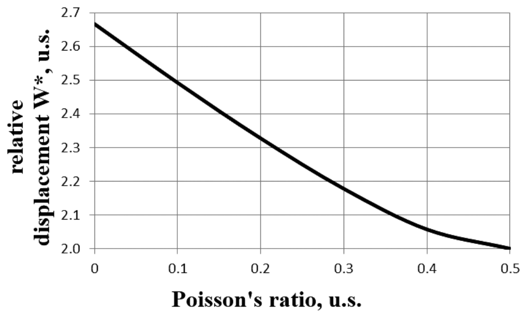

Fig. 2. Dependence of the relative displacement of a rigid stamp W^* on Poisson's ratio of the base ν

- at $0 \le z < h$

$$\sigma_{zz}^{-} = -\int_0^{\infty} B(\alpha, r) \cdot \left\{ \begin{array}{c} 2 \cdot e^{[-\alpha \cdot (h+z)]} + e^{[-\alpha \cdot (h+z)]} \cdot \alpha \cdot h - \\ -2 \cdot e^{[\alpha \cdot (-h+z)]} - e^{[\alpha \cdot (-h+z)]} \cdot \alpha \cdot h + \\ +2 \cdot e^{[-\alpha \cdot (h+z)]} \cdot z \cdot \alpha^2 \cdot h + \\ -e^{[\alpha \cdot (-h+z)]} \cdot z \cdot \alpha + \\ +3 \cdot e^{[-\alpha \cdot (h+z)]} \cdot z \cdot \alpha - \\ -4 \cdot e^{[-\alpha \cdot (h+z)]} \cdot z \cdot \alpha \cdot \nu - \\ -2 \cdot e^{[-\alpha \cdot (h+z)]} \cdot \nu + 2 \cdot e^{[\alpha \cdot (-h+z)]} \cdot \nu \end{array} \right\} \cdot d\alpha;$$

-at $h < z \le \infty$

$$\sigma_{zz}^{+} = -\int_0^{\infty} B(\alpha, r) \cdot \left\{ \begin{array}{c} 2 \cdot e^{[-\alpha \cdot (-h+z)]} - \alpha \cdot h \cdot e^{[-\alpha \cdot (-h+z)]} - \\ -2 \cdot \nu \cdot e^{[-\alpha \cdot (-h+z)]} + z \cdot \alpha \cdot e^{[-\alpha \cdot (-h+z)]} + \\ +2 \cdot e^{[-\alpha \cdot (h+z)]} + e^{[-\alpha \cdot (h+z)]} \alpha \cdot h + \\ +3 \cdot e^{[-\alpha \cdot (h+z)]} \cdot z \cdot \alpha - \\ -4 \cdot e^{[-\alpha \cdot (h+z)]} z \cdot \alpha \cdot \nu + \\ +2 \cdot e^{[-\alpha (h+z)]} \cdot z \cdot \alpha^2 \cdot h - 2 \cdot e^{[-\alpha (h+z)]} \cdot \nu \end{array} \right\} \cdot d\alpha.$$

Where $B(\alpha, r) = \dfrac{A(\alpha) \cdot J_0(\alpha \cdot r) \cdot \alpha}{4 \cdot (1 - \nu)}$.

Then we find the limits

$$\left. \begin{array}{l} \sigma_{zz}^{-} = \lim\limits_{z \to h} (\sigma_{zz}^{1}) = -\dfrac{1}{2 \cdot (1-\nu)} \times \\ \times \displaystyle\int_0^{\infty} \alpha \cdot A(\alpha) \cdot J_0(\alpha \cdot r) \cdot \left(\begin{array}{c} 2 \cdot h \cdot \alpha \cdot e^{-2 \cdot \alpha \cdot h} - 1 + \\ + h^2 \cdot \alpha^2 \cdot e^{-2 \cdot \alpha \cdot h} + \\ + \nu - 2 \cdot h \cdot \alpha \cdot \nu \cdot e^{-2 \cdot \alpha \cdot h} + \\ + e^{-2 \cdot \alpha \cdot h} - \nu \cdot e^{-2 \cdot \alpha \cdot h} \end{array} \right) \cdot d\alpha \end{array} \right\}; \quad (5)$$

$$\left. \begin{array}{l} \sigma_{zz}^{+} = \lim\limits_{z \to h} (\sigma_{zz}^{2}) = -\dfrac{1}{2 \cdot (1-\nu)} \cdot \displaystyle\int_0^{\infty} \alpha \cdot A(\alpha) \cdot J_0(\alpha \cdot r) \times \\ \times \left(\begin{array}{c} -\nu - 2 \cdot h \cdot \alpha \cdot \nu \cdot e^{-2 \cdot \alpha \cdot h} + e^{-2 \cdot \alpha \cdot h} + \\ +1 - \nu \cdot e^{-2 \cdot \alpha \cdot h} + 2 \cdot h \cdot \alpha \cdot e^{-2 \cdot \alpha \cdot h} + \\ + h^2 \cdot \alpha^2 \cdot e^{-2 \cdot \alpha \cdot h} \end{array} \right) \cdot d\alpha \end{array} \right\}. \quad (6)$$

After that, using formulas (5, 6), we find the contact pressure on the sole of the stamp at depth h

$$q(r) = \sigma_z^{-}(r, h) - \sigma_z^{+}(r, h) = \int_0^{\infty} A(\alpha) \cdot J_0(\alpha \cdot r) \cdot \alpha \cdot d\alpha. \quad (7)$$

Formulas (1, 7) make it possible to construct an exact solution to the problem.

To construct an approximate solution, we use the boundary element method [10]. In doing so, we take into account the axial symmetry of the problem. We divide the contact area into n sections, and approximate the contact diagram with a stepped line (Fig. 3).

We find the settlement of the base due to the ring load q_i (Fig. 4), whose width is equal to

$$a = r_{i+1} - r_i. \quad (8)$$

In accordance with the diagrams in Figs. 3 and 4: and using the Bessel preformation, we have

$$\begin{array}{l} A(\alpha) = q_i \cdot \displaystyle\int_{r_i}^{r_{i+1}} J_1(\alpha \cdot r) \cdot r \cdot dr = \\ = \dfrac{q_i}{\alpha} \cdot \left[r_{i+1} \cdot J_1(r_{i+1} \cdot \alpha) - r_i \cdot J_1(r_i \cdot \alpha) \right], \end{array} \quad (9)$$

where q_i is distributed load within the i^{th} ring.

Next, we substitute (9) in (1) and we find the sediment of the foundation at a depth h at the point with coordinate

$$r_j + \frac{a}{2} = \frac{r_{j+1} + r_j}{2} \quad \text{at} \quad q_i = 1.$$

We have

$$\left. \begin{array}{c} B_{ij} = W = \dfrac{1}{8 \cdot (1-\nu) \cdot G} \times \\ \times \displaystyle\int_0^{\infty} \left[3 - 4 \cdot \nu + \left(\begin{array}{c} 8 \cdot \nu^2 + 6 \cdot \alpha \cdot h - 8 \cdot \alpha \cdot h \cdot \nu - \\ -12 \cdot \nu + 5 + 2 \cdot \alpha^2 \cdot h^2 \end{array} \right) \cdot e^{-2 \cdot \alpha \cdot h} \right] \\ \times A(\alpha) \cdot J_0\left(\alpha \cdot \dfrac{r_{j+1} + r_j}{2} \right) d\alpha \end{array} \right\}, \quad (10)$$

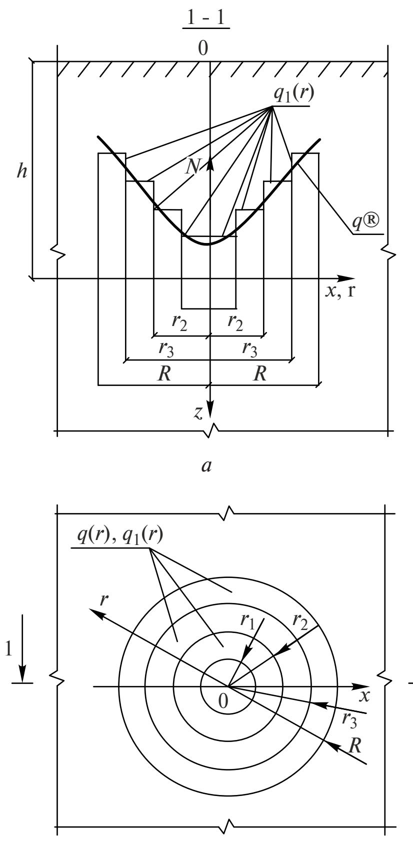

Fig. 3. Actual contact diagram $q(r)$ approximated by a stepped line $q_1(r)$ (scheme):

N is the resulting force acting on the stamp, numerically equal to the sum of the products of each of the contact stresses and the area over which they act

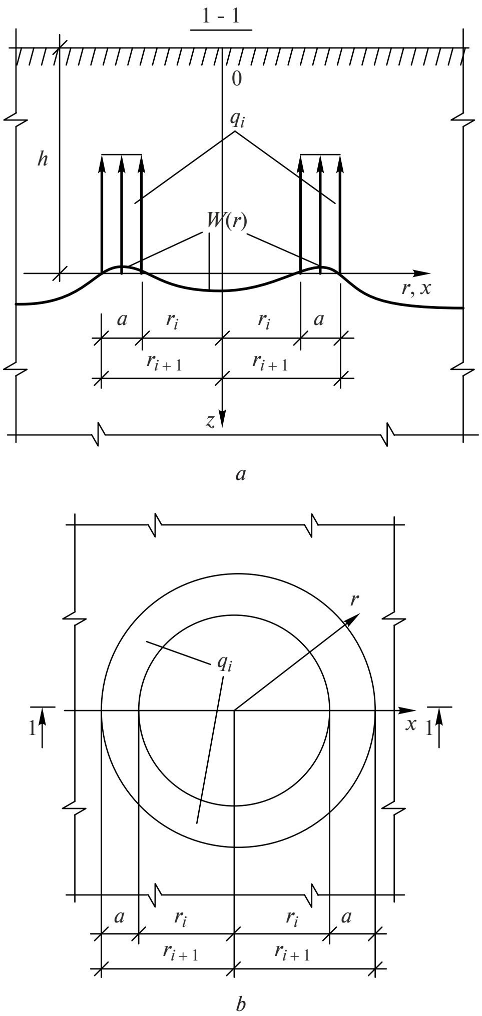

Fig. 4. Scheme for determining the displacements of the base $W(r)$, caused by annular stepped load q_i

where

$$A(\alpha)=\left[\frac{r_{j+1}\cdot J_1(r_{j+1}\cdot\alpha)}{\alpha}-\frac{r_j\cdot J_1(r_j\cdot\alpha)}{\alpha}\right].$$

Next, we find the sediments of the center of each of the rings into which the contact diagram is divided. We get

$$W_i=\sum_{i=1}^{n}B_{ij}\cdot q_i, \qquad (11)$$

where n is the number of ring sections into which the contact diagram is divided.

By the condition of the problem, the stamp is absolutely rigid. Therefore, the displacements at the contact of the stamp with the base are equal to each other and are known in advance. Based on these considerations, we obtain an expression for determining unknown stresses q_i

$$\left|B_{ij}\right|\cdot\vec{q}_i=\vec{W}_i=W_0. \qquad (12)$$

Having determined within each of the concentric rings the value of the load acting within it q_i (Figs. 3 and 4), we find the magnitude of the resulting force N. We get

$$N=2\cdot\pi\cdot\int_0^R q(r)\cdot r\cdot dr\approx\sum_{i=1}^{n}\pi\cdot\left(r_{i+1}^2-r_i^2\right)\cdot q_i. \qquad (13)$$

For a number of values included in the formulas (1–13) of the parameters, we calculated the distribution of contact stresses $q(r)$ and the values of the pulling force N.

To solve the system of equations (12) taking into account condition (13) we used the iteration process. The essence of the process is as follows [10]:

1. First, we need to set the magnitude of the force N. Since the base is linear isotropic, after normalizing the results of solving the problem, the value of the force does not affect their distribution along the coordinate.

2. Next, using the formula $p_{o,i}=\dfrac{N}{\pi\cdot R^2}$ it is necessary to determine the pressure under the sole of the stamp in the initial approximation.

3. After that, using formulas (8), one should calculate the settlements of the centers of each of the annular boundary elements $W_{0,i}$ (Fig. 4). In this case, the load on the base within each of the boundary elements should be taken constant.

4. Next, using the formulas $C_{0,i}=p_{0,i}/W_{0,i}$ it is necessary to calculate the stiffness coefficients within the limits of each of the annular boundary elements.

5. Next, using the formulas $p_{1,i}^*=p_{0,i}\cdot C_{o,i}$ the pressures within each of the annular boundary elements should be calculated.

6. After that, using the formulas $p_{1,i}=p_{1,i}^*\cdot N\Big/\sum_{i=1}^{n}p_{1,i}^*$ the exact pressures within each of the annular boundary elements should be calculated.

7. Further, using paragraphs 3–6, we determine $W_{0,i}$, $C_{1,i}$, $p_{1,i}$.

8. The iteration process is considered completed if the condition

$$\max\left|1-\frac{p_{j-1,i}}{p_{j,1}}\right|\le\varepsilon;\quad i=1,\ldots,n,$$

where $p_{j,i}$ is pressure within the i^{th} boundary element; i is the boundary element number; j is the iteration number; ε is some small predetermined number.

Results of solving the contact problem:

1. First, we find a solution to the problem for $h\to 0$. In this case, the movement of the stamp should be determined using (2), and the contact stresses should be determined using (5).

2. Next, we consider the exact solution of the problem (it is presented in the work [10]). We introduce into consideration an integral of the form

$$-W_0\cdot\frac{2}{\pi}\cdot\int_0^\infty\frac{\sin(\alpha\cdot R)}{\alpha}J_0(\alpha\cdot r)d\alpha=\begin{cases}-W_0 & \text{at}\quad r<R;\\ -\dfrac{2}{\pi}\cdot\arcsin\left(\dfrac{R}{r}\right).\end{cases} \qquad (14)$$

Formula (14) takes into account the direction of the pull-out force acting on the stamp (Fig. 1).

3. Next, from the comparison of (2) and (14), we find

$$A(\alpha)=-W_0\cdot\frac{2}{\pi}\cdot\frac{G}{(1-\nu)}\cdot\frac{\sin(\alpha\cdot R)}{\alpha}\cdot J_0(\alpha\cdot r). \qquad (15)$$

Next, we substitute (15) into (7) and calculate the improper integral thus obtained. We have

$$\begin{aligned}q(r)&=-W_0\cdot\frac{2}{\pi}\cdot\frac{G}{(1-\nu)}\cdot\int_0^\infty\sin(\alpha\cdot R)\cdot J_0(\alpha\cdot r)\cdot d\alpha=\\&=-\frac{2}{\pi}\cdot\frac{W_0\cdot G}{(1-\nu)}\cdot\frac{1}{\sqrt{R^2-r^2}}.\end{aligned} \qquad (16)$$

Next, we normalize by putting in (16)

$$r^*=\frac{r}{R},\quad \xi=\frac{\alpha}{R}\quad\text{and}\quad q^*=\frac{q(r)\cdot\pi\cdot(1-\nu)}{2\cdot W_0\cdot G}.$$

We get

$$q^*(r)=-\int_0^\infty\sin(\xi)\cdot J_0(\xi\cdot r^*)\cdot d\alpha=-\frac{1}{\sqrt{1-(r^*)^2}}. \qquad (17)$$

To solve the problem by the boundary element method, in (12) one should set $W_0 = -1$.

The values of the contact stresses calculated by (12, 17) at the base of the rigid stamp located on the day surface of the soil base (i. e. at $h = 0$) are presented in Fig. 5.

Their comparison allows us to conclude that they almost completely coincide. This, in turn, gives reason to believe that the contact stress diagrams established using the boundary element method for depths $h > 0$ will be close to accurate. When performing calculations at depths $h > 0$ in addition to the dimensionless complexes adopted in (17), one should take

$$h^* = \frac{h}{R}, \qquad (18)$$

where h is the depth at which the contact diagram is determined; R is stamp radius.

The results of determining contact diagrams at different depths are shown in Fig. 6.

Further, using formula (13), the dependence of the pull-out force acting on the stamp was established $N^* = 2 \cdot \pi \cdot \int_0^1 q(r^*) \cdot r^* \cdot dr^*$ from the depth of the stamp h^* (Fig. 7).

After that, we studied the effect of contact stresses of the Poisson's ratio of the base on the distribution (Fig. 8).

Conclusions. The research results allow us to draw the following conclusions:

1. The approximate solution of the problem of indentation of a round rigid stamp into the upper boundary of an elastic isotropic half-space obtained using the algorithm proposed by the authors is in good agreement with the exact solution (Fig. 5). This led to the conclusion that the developed algorithm can be used to solve contact problems.

2. The outlines of the contact stress diagrams depend on the depth at which they are determined – the greater the depth is, the gentler the outlines of the diagram are (Fig. 6). In this case, starting from a certain depth, the diagrams of contact stresses practically coincide.

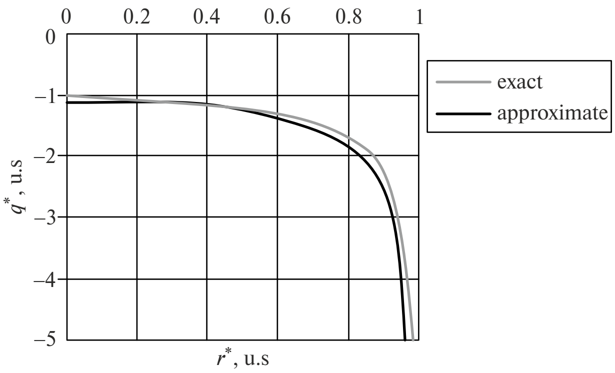

Fig. 5. Exact and approximate solutions of the problem

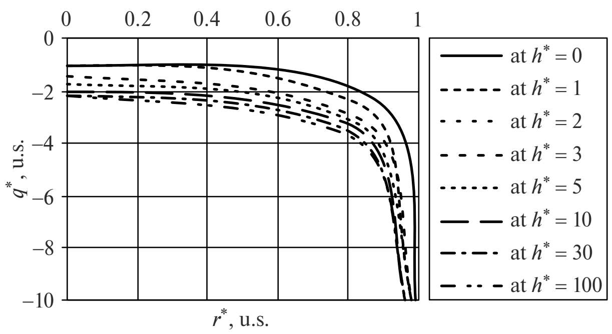

Fig. 6. Diagrams of contact stresses at the base of a rigid stamp at various depths at Poisson's ratio $v = 0.3$

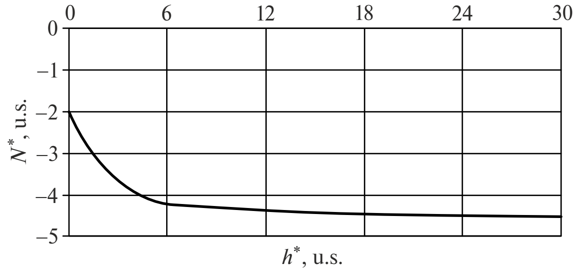

Fig. 7. Dependence of the force pulling out the stamp N^* on the depth of the stamp h^* at a single displacement of the stamp

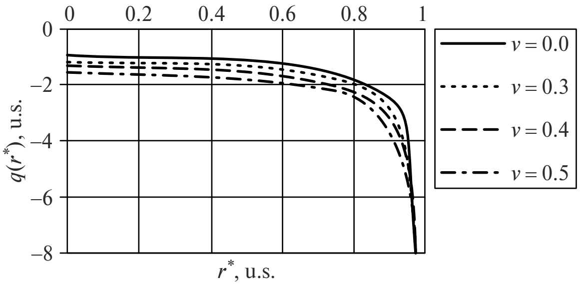

Fig. 8. Diagrams of contact stresses at depth $h^* = 10$ for different values of Poisson's ratio v

3. The greater the depth is at which the stamp is located, the greater the force must be applied to obtain equal stamp movements (Fig. 7).

4. The greater the value of Poisson's ratio of the base is, the greater the contact stresses are, ceteris paribus (Fig. 8). In our opinion, this fact is very important for design practice.

In general, the research materials presented in this article allow us to conclude that the results obtained significantly expand the possibilities of solving various problems of soil mechanics and foundation engineering and make it possible to obtain completely new results.

In particular, a clear dependence of the contact stresses at the sole of a rigid round stamp on the depth at which it is located was revealed. In addition, the presented data make it possible to designate a completely new direction in the calculation of ground anchor base, namely, the calculation of their deformations.

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Контактні напруження під підошвою жорстких фундаментів глибокого закладення і ґрунтових анкерів

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Мета. Вирішення завдання із розподілу контактних вертикальних нормальних напружень по підошві жорсткого круглого штампа, розташованого у пружному ізотропному напівпросторі на деякій глибині $h \neq 0$. Порівняти отримане рішення з відомим класичним результатом при $h = 0$, перевірити отримані результати на адекватність.

Методика. На основі аналізу рішення про напружено-деформований стан основи, усередині якої знаходиться розподілене по площі круга вертикальне довільне навантаження, були отримані необхідні формули для вирішення поставленого завдання. Розроблено алгоритм побудови наближеного рішення, суть якого полягає у використанні комбінації методу граничних елементів і процесу ітерації С. Н. Клепікова. Для ряду глибин отримані наближені рішення даної задачі.

Результати. Запропонований алгоритм наближеного розв'язання задачі про вдавлення круглого жорсткого штампа у верхню межу пружного ізотропного півпростору має гарну відповідність із точним рішенням і може бути використаний для розв'язання контактних задач. Обриси контактних епюр напружень залежать від глибини, на якій вони визначаються – чим більше глибина, тим більш пологими є обриси епюри, при цьому, починаючи з деякої глибини, епюри контактних напружень практично збігаються. Чим більше глибина, на якій розташований штамп, тим більші зусилля слід докласти для отримання рівних переміщень штампа.

Наукова новизна. Отримані результати досліджень значно розширюють можливості вирішення різних завдань механіки ґрунтів і фундаментобудівництва, дають можливість отримати абсолютно нові результати. Зокрема, була виявлена чітка залежність контактних напружень по підошві жорсткого круглого штампа від глибини, на якій він знаходиться. Крім того, представлені дані дозволяють позначити абсолютно новий напрям у розрахунку основ ґрунтових анкерів, а саме – розрахунок їх деформацій.

Практична значимість. Для практики проектування важливий той факт, що чим більше значення коефіцієнта Пуассона основи, тим за інших рівних умов більше контактні напруження.

Ключові слова: *фундамент глибокого закладення, контактні напруження, жорсткий штамп, ґрунтовий анкер, осідання основи, ізотропний полупростір*

The manuscript was submitted 24.03.22.