MODELING OF INVESTMENT IMPACTS ON INDUSTRIAL ENTERPRISE PROFITS

**Purpose.** To build economic and mathematical model of impacts of investment from internal sources on the profit of an industrial enterprise.

**Methodology.** An economic and mathematical model of enterprise operations is built to enable development of methods for analyzing the impact of the internal investment amount on the profit of an industrial enterprise through considering specific features of its operation. The research involves methodological principles of economic cybernetics, namely the representation of the enterprise as a multipolar object with an unknown structure. Application of the systemic approach enables comprehensive investigation of the process of the amount of internal investment impacting profits of this enterprise. The use of structural synthesis underlies determination of the type of economic and mathematical model without considering its parameters.

**Findings.** Mathematical modeling of operation of an industrial enterprise as a complex object enables numerical determination of the impact of the internal investment amount on the profit of an industrial enterprise. This approach makes it possible to use regression models to obtain an analytical dependence of the enterprise profit on the size of its internal investment. Theoretical research results in the determined sequence of building an economic and mathematical model of internal investment amount impacts on profits. It is proved expedient to divide the process of building a model of an industrial enterprise operation into two stages: structural synthesis and identification of model parameters.

**Originality.** For the first time, an economic and mathematical model of operation of an industrial enterprise in the form of a “black box” has been used to analyze the impact of internal investment of an industrial enterprise on its profits.

**Practical value.** Analysis of results of economic and mathematical modeling of the PJSC “PryvHZK” operation proves expediency of estimating the statistical dependence of the profit on the amount of internal investment. It is recommended to introduce the developed system–logical scheme in the investment practice of enterprises.

**Keywords:** economic and mathematical modeling, black box, profit, investment, regression model, industrial enterprise

---

In Ukraine, the current economic situation, namely the sectoral structure of investments, cannot be considered optimal because it constantly reproduces the sectoral structure of the economy as a whole which is anything but perfect. To overcome negative economic trends, investment activities of enterprises are of paramount importance. The conducted analysis demonstrates that at present the main direction of investment efficiency enhancement consists in increasing impacts of investment on growth of enterprises’ profits. Naturally, a saved part of the enterprises’ incomes forms a source of investment. It is clear that solution of this problem is impossible without economic analysis of activities of enterprises through using modern means of mathematical modeling.

**Literature review.** Under the influence of scientific and technological progress in the formation of material and technical base of production, the role of research, qualifications, knowledge and experience of scientists increases. Researchers–economists pay considerable attention to investment activities of enterprises. Thus, L. O. Satalinka investigates characteristics of the main forms of investment resources [1]. V. V. Kostetskyi, A. M. Butov investigate and identify the main sources of forming investment resources of an enterprise [2]. Peculiarities of using efficiency indicators for estimating extraordinary investments are studied by S. O. Safarovon, N. A. Karavan [3]. V. A. Zamlynskyi offers methodological tools to determine efficiency of investment processes at the enterprise [4]. Statistical methods that allow mathematically substantiated conclusions make the basis for such studies. Works by O. V. Yaremenko [5], S. A. Matiukh [6], S. M. Ivanova [7] and others deal with efficiency of investments and estimation methods in economics. However, now an increas-
ing number of works on investment economy are relying on digital economy as a result of deeper application of modern information technology. Thus, L. V. Kryuk [8], P. I. Miroshnychenko [9], M. M. Shiyun [10] and others analyze activities of enterprises in conditions of investment injections in the form of their economic and mathematical modeling.

In general, the whole process of implementing an investment project is a set of very time-consuming measures that require detailed and accurate calculation. Unsolved aspects of the problem. Concurrently, insufficient attention is still paid to analysis of investment impacts on growth of profits of enterprises based on digital economy for obtaining economic and mathematical modeling results.

The article aims to build an economic and mathematical model of internal investment impacts on the profit of an industrial enterprise.

Results. Given the fact that the enterprise is a complex economic object, the first step to solve the problem involves building an economic and mathematical model of enterprise operation.

The country’s economic system includes a variety of enterprises that differ in both quality and quantity. The potential of the enterprise is an integrated assessment of current and future capabilities of the economic system to transform some resources into profits. The elements of the potential of the enterprise include everything related to the functioning and development of the enterprise. The decision on the investment project is based on the assessment and comparison of the volume of expected investments and future revenues. The general logic of the analysis in this case is as follows – it is necessary to somehow compare the amount of investment required with the projected profit.

It should be emphasized that enterprises have a rather complex structure due to the nature of the tasks it performs. Given the fact that the company is an open system, its entry receives funds which are then distributed according to needs.

The complexity and multi-vector investment in the enterprise can be streamlined through the formation of an appropriate economic and mathematical model.

The enterprise model is understood as dependence that binds input and output variables and can be written in the form of [11]

\[ y = f(x), \quad (1) \]

where \( x \) is the input variable, dollar equivalents; \( y \) is the output variable, dollar equivalents.

Obviously, the input variable can be treated as total costs and the output variable is the enterprise income, the difference between them is the enterprise profit

\[ P = y - x. \quad (2) \]

In general, the functional dependence (1) is unknown, so it is considered expedient to treat the enterprise as a “black box”, the internal content of which is unknown, and its model is a multipolar object of an unknown structure [12, 13].

Sources of investment are naturally represented by all types of funds in material, financial and other forms used to attract necessary investment assets in order to ensure the effective investment activity of the enterprise. Let us conduct a study assuming that the source of investment is internal money of the enterprise [14]. Concerning the problem in question, part of the enterprise investments is used as investment at the expense of its own sources to increase profits of the enterprise [14].

Input variable of the enterprise as a “black box” consists of the internal money (\( x \)), some of which is formed at the expense of investment (\( \delta y \)). As a result, according to (1), the output variable will look like

\[ \hat{y} = f(x + \delta y). \quad (3) \]

Given that part of the original variable (3) is used as an investment of the enterprise, this variable takes the form

\[ \hat{y} = f(x + \delta y) - \delta y. \quad (4) \]

Considering (3), equation (4) looks like

\[ \hat{y} = f(x + \delta y) - \delta y. \quad (5) \]

Taking advantage of the fact that the amount of the investment is considerably smaller than internal sources of the enterprise, i.e.

\[ \delta y \ll y, \quad (6) \]

formula (3) can be linearized by expanding it in Taylor series [3]

\[ f(x + \delta y) \approx f(x) + f(x)\delta y. \quad (7) \]

According to (7), formula (5) will look like

\[ \hat{y} = f(x) + f(x)\delta y - \delta y. \quad (8) \]

Then taking into account the fact that part of internal sources is used for investing in the enterprise, its profit will be as follows

\[ P_1 = \hat{y} - x. \quad (9) \]

Considering (8), formula (9) will look like

\[ \hat{y} = f(x) + f(x)\delta y - x. \quad (10) \]

According to (2), formula (10) is written as follows

\[ P_1 = P + f(x)\delta y - x. \quad (11) \]

The analysis of (11) shows that the total profit of the enterprise consists of two summands (subject to their positivity). The first part is related to operating activities, and the addend – to investment activities.

Analysis of the added value in (11) shows that it depends on the derivative of functional dependence (1), i.e. on a “just noticeable” characteristic of the “black box”, which is associated with the rate of change in the value of internal investment sources of the enterprise from the value of costs. It is clear that the enterprise profit from the second addend of (11) will occur provided that

\[ f'(x) > 1. \quad (12) \]

Further on, for research convenience, it is advisable to write (11) in the dimensionless form

\[ \frac{P_1 - P}{P} = \frac{f(x)\delta y - x}{P}. \quad (13) \]

The left part of (13) determines the relative increase in profits due to the relative increase of investment in the enterprise.

To apply (13), additional data on the enterprise is required to build its model. Building a model of industrial enterprise operation can be divided into two stages. At the first stage called structural synthesis, the type of dependence (1) is determined without considering its parameters. In general, the structure means a type of components the object consists of and the relationship between them. There are many different structures of objects. Linearity, the static character, determinacy are structural categories. Thus, at the stage of structural synthesis, a type and characteristics of the model are determined, while its parameters are determined at the second stage. The second stage is defined as identification of model parameters and is associated with determination of numerical values of the parameters.

In our case, for research convenience, it is advisable to choose a linear static deterministic structure of model (1) of the enterprise [15]

\[ f(x) = a + bx, \quad (14) \]

where \( a \) and \( b \) are parameters.

Then, considering that
formula (13) takes the following form

\[ \frac{P_0 - P}{P} = (b - 1) \frac{dy}{P}. \]  

(16)

According to (12), formula (16) will determine the relative profit of the enterprise if the following condition is met

\[ b > 1. \]  

(17)

Fig.1 presents graphs of function (16) for different values of parameter \( b \) considering (6 and 17).

The conducted research studies, as an example, will be used in the model analysis of investment impacts on profits of the enterprise through building a regression model of economic activities of the enterprise.

It should be noted that when building a regression model, the main thing is to determine the influencing factor and the resultant feature. The logic of the statement and the proposed method for solving the problem give grounds to determine the influencing factor — the company’s own funds, the resultant sign — the corresponding profit. Table 1 presents statistical information on the economic activity of the industrial enterprise of PJSC “PivdHZK” for 2009–2020. The indicators on the basis of which the regression model is built and the corresponding analysis are carried out are defined.

<table>
<thead>
<tr>
<th>Years</th>
<th>Internal money (x)</th>
<th>Profit (y)</th>
<th>Regression of ( y ) on ( x )</th>
<th>Limit</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>2,279</td>
<td>2,884</td>
<td>3,608</td>
<td>2,480</td>
<td>1.265</td>
</tr>
<tr>
<td>2010</td>
<td>2,315</td>
<td>4,599</td>
<td>3,674</td>
<td>2,515</td>
<td>1.986</td>
</tr>
<tr>
<td>2011</td>
<td>2,202</td>
<td>2,128</td>
<td>3,465</td>
<td>2,402</td>
<td>0.967</td>
</tr>
<tr>
<td>2012</td>
<td>3,626</td>
<td>5,765</td>
<td>6,084</td>
<td>3,826</td>
<td>1.590</td>
</tr>
<tr>
<td>2013</td>
<td>5,906</td>
<td>1,077</td>
<td>10,279</td>
<td>6,107</td>
<td>1.823</td>
</tr>
<tr>
<td>2014</td>
<td>5,853</td>
<td>8,243</td>
<td>10,180</td>
<td>6,054</td>
<td>1.408</td>
</tr>
<tr>
<td>2015</td>
<td>5,903</td>
<td>9,203</td>
<td>10,271</td>
<td>6,103</td>
<td>1.559</td>
</tr>
<tr>
<td>2016</td>
<td>4,764</td>
<td>11,536</td>
<td>8,176</td>
<td>4,964</td>
<td>2.422</td>
</tr>
<tr>
<td>2017</td>
<td>7,081</td>
<td>10,567</td>
<td>12,437</td>
<td>7,281</td>
<td>1.492</td>
</tr>
<tr>
<td>2018</td>
<td>8,702</td>
<td>14,605</td>
<td>15,418</td>
<td>8,902</td>
<td>1.678</td>
</tr>
<tr>
<td>2019</td>
<td>9,222</td>
<td>15,657</td>
<td>16,374</td>
<td>9,422</td>
<td>1.698</td>
</tr>
<tr>
<td>2020</td>
<td>7,306</td>
<td>16,836</td>
<td>12,851</td>
<td>7,506</td>
<td>2.304</td>
</tr>
</tbody>
</table>

Table presents indicators of economic activities of the PJSC “PivdHZK” (Southern ore mining and beneficiation plant) during 2009–2020.

In the future, it is necessary to build an economic and mathematical model of PJSC “PivdHZK”. The structure of such a model is given by equation (14). To identify the parameters of this model it is necessary to use the statistical material given in the table. Given that the structure of the model is linear and static, it is advisable to use a non-adaptive identification algorithm. To determine the parameters included in equation (14), we use the nonadaptive method to substitute the data in Table. The result is a system of 12 equations with two unknowns \( a, b \). It is clear that this system of linear equations has no solution. In modern conditions, more and more research on the economics of investment is based on the digital economy as a result of deeper use of modern information technology. However, the problem of non-adaptive identification comes down to the need to solve this system. If we generalize the concept of solution, then such a system of linear equations can be solved by the method of least squares (MNC). To do this, we form the function of the total residual in the form of the sum of the squares of the residuals of each of equations (14)

\[ Q(a, b) = \sum_{i=1}^{12} (a + b \cdot x_i - y_i)^2. \]  

(18)

Function (18) is nonnegative and equal to zero when the right and left parts of the system equations coincide. In this case, the closer the right parts of the system are to the left, the smaller the value of the residual function is (18). This gives reasons to consider the following parameter values to be the solution of the system

\[ (a^*, b^*), \]  

at which the residual function will be minimal, i.e.

\[ Q(a^*, b^*) = \min_{a, b} Q(a, b). \]

Thus, the task of identifying a static system is reduced to the task of minimizing some specially constructed function of many variables. In general, solving such a problem is quite difficult. In our case, when the structure of the model is linear and static, the minimization problem (18) is reduced to the solution of a system of linear algebraic equations, which is solved quite simply by standard methods. The simple form of the function (18) makes it possible to solve the minimization problem (18) by equating to zero the partial derivatives of the function (18), i.e.:

\[ Q_a(a, b) = 0, \quad Q_b(a, b) = 0. \]
Since function (14) is linear with respect to the parameters, (18) is a quadratic function, which determines the linearity of the system of equations (20). Calculating the partial derivatives in (20), we obtain a system of two equations
\[ \begin{align*}
    12 \cdot a + b & + \sum_{i=1}^{12} x_i - \sum_{i=1}^{12} y_i = 0 \\
    a + \sum_{i=1}^{12} x_i + b \cdot \sum_{i=1}^{12} x_i^2 - \sum_{i=1}^{12} x_i \cdot y_i = 0
\end{align*} \] (21)
After the obvious transformations we obtain a system of two linear algebraic equations
\[ \begin{align*}
    a + X \cdot b = \overline{y} \\
    X \cdot a + X^2 \cdot b = \overline{x} \overline{y}
\end{align*} \] (22)
where
\[ \begin{align*}
    X &= \frac{1}{12} \sum_{i=1}^{12} x_i; \\
    \overline{y} &= \frac{1}{12} \sum_{i=1}^{12} y_i; \\
    \overline{x^2} &= \frac{1}{12} \sum_{i=1}^{12} x_i^2; \\
    \overline{xy} &= \frac{1}{12} \sum_{i=1}^{12} x_i y_i.
\end{align*} \]
According to the table, we have
\[ \begin{align*}
    X &= 5430; \\
    \overline{y} &= 9399; \\
    \overline{x^2} &= 34,959,739; \\
    \overline{xy} &= 61,103,015.
\end{align*} \] (23)
The solution of the system of equations (22) taking into account (23) gave the following values
\[ a = -585; \quad b = 1.839. \] (24)
According to (24), the regression equation (14) will take the form
\[ y = -585 + 1.839 \cdot x. \] (25)
The coefficient of determination, according to (25), was calculated by the formula
\[ R^2 = 1 - \frac{\sum_{i=1}^{12} (y_i - 585 - 1.839 x_i)^2}{\sum_{i=1}^{12} (y_i - \overline{y})^2}, \]
and amounted to the value
\[ R^2 = 0.987. \] (26)
The significance of the model (25) was tested using the Fisher test. Fisher’s criterion, taking into account (26), was the value
\[ F = \frac{R^2}{1-R^2} \frac{n-m}{m-1} = 0.987 \frac{12-2}{2-1} = 759. \] (27)
The critical value of the Fisher test, according to [17], is
\[ F_{\alpha}(n-m, m-1) = F_{\alpha}(0.05, 10, 1) = 4.97, \] (28)
where \( \alpha \) is the level of significance; \( n \) is the number of statistical data; \( m \) is the number of parameters in the regression equation.
\[ F = 55.4 > F_{\alpha}(0.05; 10; 1) = 4.97, \]
then the obtained equation (18) at the level of certainty \( 1 - \alpha = 0.95 \) is significant [13].

Fig. 2 presents a correlation field of variables “internal investment costs” and the graph of regression equation (25). According to (25), parameter \( b \) satisfies condition (17) because
\[ b = 1.839 > 1. \] (29)
Thus, it is advisable to use investment deposits from the income of PJSC “PivdHZK” to increase its profits.

According to (24), the regression equation (14) will take the form
\[ y = -585 + 1.839 \cdot x. \] (25)

The analysis of the correlation field in Fig. 3 demonstrates that almost all values are above the specified limit, which indicates the possibility of applying internal investment of the PJSC “PivdHZK” to increase its profits.

The last column of the table shows the values of parameter \( b \). The values of this parameter are greater than one, i.e. they satisfy condition (17).

Let us consider the use of the conducted research findings on internal investment of PJSC “PivdHZK” to increase its profits in 2020.

Based on the given results, parameter \( b = 2.304 \). Then equation (16) will take the following form
\[ P_0 - P = 1.304 \cdot \delta y, \] (30)
Considering the data of Table, one has, UAH
\[ P = y - x = 16,836 - 7306 = 9530. \] (31)
Considering (31), equation (30) can be reduced to the following form
\[ P_0 = 9530 + 1.304 \delta y. \] (32)

Fig. 3 plots function (32).

Analysis of the graph in Fig. 3 shows that growth of internal investment of PJSC “PivdHZK” results in its increased profits. Visual linear positive growth is indicative of the suggested analytical analysis viability.
Thus, achieving the highest profit of the enterprise is one of the most important problems of market relations. One of the possible ways to solve this problem is to use the method of economic and mathematical modeling of the impact of investment processes on enterprise profits. Based on the analysis of the proposed economic and mathematical model, it is possible to investigate the conditions under which you can get the most profit.

**Conclusions.** In market conditions, Ukraine’s investment activities require application of modern economic management methods based on digital economy. Research into investment impacts on enterprise profits is one of important issues whose solution is impossible without application of economic and mathematical modeling that involves building regression models. One of the methods for increasing profits consists in use of internal investment. The conducted research lays the foundation for determining expediency of implementing the investment process at an industrial enterprise. Analytically, the mechanism should contain two stages reflecting structural synthesis of activities of an enterprise. The synthesis underlies economic and mathematical modeling of investment impacts on profits of the enterprise. The analysis of activities of the PJSC “PivdHZK” proves viability of using economic and mathematical modeling when making efficient managerial decisions on investment.

**References.**

**Моделювання впливу інвестицій на прибуток промислових підприємств**

Н. В. Лохман1, Т. М. Берідзе2, З. П. Бараник3,

1 – Донецький національний університет економіки та торгівлі імені Туган-Барановського, м. Кривий Ріг, Україна
2 – Криворізький національний університет, м. Кривий Ріг, Україна, e-mail: beridze2016@cloud.com
3 – Кіївський національний економічний університет імені Вадима Гетьмана, м. Київ, Україна

Мета. Побудова економіко-математичної моделі впливу інвестиційних вкладів із власних джерел на прибуток промислового підприємства.

Методика. Побудовано економіко-математичну модель функціонування підприємства, що дозволяє розробити методику аналізу впливу величини інвестиційних вкладів із власних джерел на прибуток підприємства шляхом урахування особливостей його функціонування. У процесі дослідження використані методологічні засади економічної кібернетики, а саме представлення підприємства як багатопільосників з невідомою структурою. Заострення системного підходу дозволило всебічно дослідити процес впливу величини інвестиційних вкладів із власних інвестиційних вкладів на прибуток промислового підприємства.

Результати. Математичне моделювання функціонування промислового підприємства, як складного об’єкту надало можливість чисельно з’ясувати вплив величини інвестиційних вкладів із власних джерел на прибуток промислового підприємства. Такий підхід дав можливість за допомогою регресійних моделей отримати аналітичну залежність прибутку підприємства від величини інвестиційних вкладів із власних джерел. Тeorетичні дослідження зазначили можливість побудови економіко-математичної моделі впливу величини інвестиційних вкладів із власних джерел на прибуток промислового підприємства. Доцільність поділення процесу побудови моделі функціонування промислового підприємства на два етапи: перший етап — структурний синтез; другий етап — ідентифікація параметрів моделі.

Наукова новизна. Уперше задля аналізу впливу інвестиційних вкладів із власних джерел на прибуток промислового підприємства використана економіко-математична модель функціонування промислового підприємства на два етапи: перший етап — структурний синтез; другий етап — ідентифікація параметрів моделі.

Ключові слова: економіко-математичне моделювання, "чорна скриня", прибуток, інвестиційні вклади, регресійна модель, промислове підприємство

*The manuscript was submitted 29.08.21.*