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STRESS STATE OF THE GRINDING TOOL LOADED WITH TANGENTIAL FORCE

Purpose. Determining the interaction mechanism of discrete grains through material that binds them in a tool for abrasive material processing in the case of its loading with a tangentially directed cutting force.

Methodology. Development and analytical solution to a mathematical model formulated on the basis of static equilibrium of an abrasive grain as a system element of tool grains bound with a material with excellent mechanical characteristics.

Findings. A mathematical model has been developed and an algorithm has been formulated to analytically determine the stress-strain state parameters of a tool for abrasive processing of materials, which is loaded with a discrete cutting force, tangential to its working surface. The nature of the dependence of stresses and deformations on mechanical parameters, the quantity of grains and the material binding them in an abrasive tool, loaded with a unit tangential force, has been determined.

Originality. The loading of the extreme grains leads to greater displacements, tangents of the shear angles of the material binding the grains. They decrease with increasing quantity of grain rows in the tool or with increasing quantity of grains to the nearest tool edge.

Practical value. The distribution of interaction forces of grains and stresses in the material binding them has been determined. The found distribution allows one in the process of developing the tool and technology, in which it is involved, to comprehensively assess the influence of tangential load value of the tool working grain on its stress state and the material containing the grains. The determined stress state makes it possible to predict the number of loading cycles until the simultaneous rational wear of the grain and the destruction of that part of the material that contains it. The linear formulation of the problem makes it possible to take into account the mutual influence of the tangential loads of several grains on the stress-strain state of the tool as a whole.

Keywords: *tool for processing, abrasive elements, mechanical interaction, deformations, stress*

Introduction. Grinding process is of great importance in metal fabrication. Wear and self-repairing of the cutting edges of the tool for abrasive processing of materials are the result of their periodic force and thermal interaction with the assembly part in the process of implementing its grinding technology. The gradual destruction of machine parts, caused by their periodic loading, is considered as a gradual accumulation of microscopic damage. The cause of damage is the periodic occurrence of stresses in the parts. If one understands the mechanism of stress distribution, their quantitative dependence on the nature of loads, material properties, it is possible to predict the nature of the part destruction, and in our case, the tool for abrasive processing of materials.

The peculiarity of such a tool is its heterogeneous – composite structure in which abrasive elements – grains are bound into a single structure by another material (a bonding material). The wear of the abrasive grain cutting edges in the tool depends on their loading and mechanical properties. Such wear can be considered as micro-destruction. The tool working surface's macro-destruction depends on the destruction of material that binds the abrasive grains into a single product. The destruction of working grains and the material containing them is accompanied by a positive occurrence of new cutting edges on the tool working surface – the restoration of the tool's cutting properties. An urgent scientific and technical task is to provide conditions for simultaneous destruction (equality of work terms to failure) of cutting grains and the material containing them. Its solution can provide an increase in the process of abrasive machining processing.

Literature review. A significant number of works have been devoted to the issues of determining the interdependence of loads, mechanical properties, and the nature of abrasive tool loading. D. G. Muzychko [1] in his thesis studies the specifics of changing the shape of the grinding wheel cutting surface, taking into account the temperature-force factors. Some aspects of the force interaction of grains and the material containing them are studied in the paper by A. N. Ushakov [2]. The author studies an individual grain. The influence of the grinding wheel material on it is modelled by a system of discrete elastic elements. In [3], the author studies the stress state of the bonding material, in which there is an individual abrasive grain loaded with a cutting force. In [4], the force parameters of the process without centre grinding of roller bearing rings with intermittent grinding wheels are studied. The thesis [5] is devoted to increasing the technology efficiency of grinding the roller bearing rings in the conditions of additional production adjustment. Mathematical modelling methods are used to determine the influence of loads on the tool [6]. In [7], a method for calculating the cutting force on the abrasive grain front surface has been developed. The occurrence of residual stresses in the process of grinding the composite materials is studied in [6]. In [8], it has been revealed that after cryogenic treatment, the static strength of synthetic diamonds increases, caused by an irreversible change in the initial stress-strain state of the crystals, due to the ordering of the crystal lattice defective structure. In [9–11] a flat model of the interaction of elements (cables) in a composite product is presented through a material binding them, which has excellent mechanical properties.

Until now, the problem of formulating, researching and compiling an algorithm for solving a mathematical model of the interaction of tool grains during abrasive processing of materials, as well as determining its stress-strain state, has not been solved. Therefore, the known studies do not allow setting the technological process parameters from the condition of equality of work terms to failure of the working abrasive grains and the material containing them, in particular, in the case of the action of a tangentially directed cutting force.

Purpose. The research purpose is to determine the interaction mechanism of discrete grains through the material binding them in a tool for abrasive material processing in the case of its loading with a tangential cutting force, as well as to develop an algorithm for calculating the stress-strain state of the tool with a complex consideration of its mechanical properties and composite structure.

Methods. The leading research direction is the development and analytical solution to a mathematical model for static equilibrium of an abrasive grain as a system element of orderly located tool grains bound into a single product with a material with excellent mechanical properties.

Basic research material presentation. In the general case, the grains in the bonding material are placed in an arbitrary manner. They are arbitrarily oriented in space. At the same time, the problem of uniform distribution of abrasive grains, their selective selection by the same size and ensuring uniform distribution in the grinding tool remains relevant. It can be assumed that further improvement in the technology of manufacturing grinding tools brings its structure closer to a structure with a regular arrangement of grains in it.

The grain sizes are much smaller than the grinding wheel radius. Accordingly, its radius is assumed to be infinitely large, the working surface is flat, and the tool is prismatic. The grains are considered regularly located along the axes (Δ, i, j) in M rows of N pieces, in K layers. The laying step is taken the same (it can be taken different). The grains can be identified by numbers i, j, Δ. According to this scheme, other values related to specific grains and the bonding material located between the grains can be identified. It can be assumed that a tangential force acts on an arbitrary grain of the working surface of the tool for abrasive material processing. The mathematical model of grain interaction is constructed from the equilibrium condition of an arbitrary grain of length b, loaded with a force T. Let us neglect the self-balanced mutual pressure stresses of grains, which are caused by their rotation around their own mass centres. The pressure force in the direction of the tangential load action (in the direction of the j axis) is considered to be applied in the grain cross-section centre (Fig. 1).

Equilibrium condition for an individual grain is

$$T_{i,j+1,\Delta}-T_{i,j,\Delta}+\begin{pmatrix}(\tau_{1,i,\Delta+1}-\tau_{1,i,\Delta})c+\\+(\tau_{2,i+1,\Delta}-\tau_{2,i,\Delta})b\end{pmatrix}b=0, \qquad (1)$$

where b, c are the grain cross-section sizes normal to the j axis; i, Δ are grain numbers in rows and grain layers in the rows, $(1\le I\le M)$, $(1\le\Delta\le K)$, M is the quantity of grains in the rows; K is the quantity of layers; j is a row number $(1\le j\le N)$; N is the quantity of rows.

$$\tau_{1,i,\Delta}=\frac{G}{h}(u_{i,j,\Delta+1}-u_{i,j,\Delta}); \qquad (2)$$

$$\tau_{2,i,\Delta}=\frac{G}{h}(u_{i+1,j,\Delta}-u_{i,j,\Delta}), \qquad (3)$$

where h is the layer thickness of the bonding material located between the grains; G is bonding material displacement modulus.

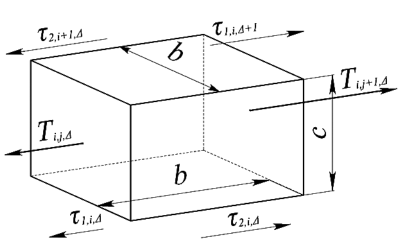

Fig. 1. Arbitrary abrasive grain loaded with a force directed along the j axis parallel to the working plane

By substituting (2 and 3) into (1), a system of homogeneous equations can be obtained. The system order is equal to the product of the quantity of rows M and the quantity of grain layers K.

$$T_{i,j+1,\Delta}-T_{i,j,\Delta}+\frac{G}{h}\begin{pmatrix}(u_{i,j,\Delta+1}-2u_{i,j,\Delta}+u_{i,j,\Delta-1})c+\\+(u_{i+1,j,\Delta}-2u_{i,j,\Delta}+u_{i-1,j,\Delta})b\end{pmatrix}b=0. \qquad (4)$$

According to Hooke's law,

$$T_{i,j,\Delta}=E\frac{b^2}{c+h}(u_{i,j,\Delta+1}-u_{i,j,\Delta}), \qquad (5)$$

where E is reduced elastic modulus of the grain material and the bonding material between the grains located in the same row of one layer.

After calculating (5), equation (4) can be written as follows.

$$\begin{gathered}E\frac{b}{c+h}(u_{i,j+1,\Delta}-2u_{i,j,\Delta}+u_{i,j-1,\Delta})+\\+\frac{G}{h}\begin{pmatrix}(u_{i,j,\Delta+1}-2u_{i,j,\Delta}+u_{i,j,\Delta-1})c+\\+(u_{i+1,j,\Delta}-2u_{i,j,\Delta}+u_{i-1,j,\Delta})b\end{pmatrix}=0.\end{gathered} \qquad (6)$$

It should be noted that shear stresses do not act on all faces of the extreme grains. As a consequence, equation (6) is acceptable for all grains, except the extreme ones, that is under the following condition.

$$0<i<I\quad -1\wedge 0<\Delta<K-1. \qquad (7)$$

The solution to the homogeneous equation (6) in displacements can be found in the following form.

$$u_{i,j,\Delta}=\frac{G(c+h)}{Eh\,b}\begin{bmatrix}\sum_{m=1}^{M-1}e^{\beta_n\Delta\sqrt{-1}+\sqrt{-1}\mu_m i+\chi_{n,m}(b+h)j}+\\ \sum_{n=1}^{K-1}+\sum_{m=1}^{M-1}e^{\sqrt{-1}\mu_m i+\chi_m^M(b+h)j}+\\ +\sum_{n=1}^{K-1}e^{\beta_n\Delta\sqrt{-1}+\chi_n^K(b+h)j}\end{bmatrix}, \qquad (8)$$

where $\mu_m=\frac{\pi m}{M}$; $\beta_n=\frac{\pi n}{K}$; $\chi_{m,n}$, χ_m^M, χ_n^N are array and vectors of the characteristic index values.

The above found expressions for μ_m, β_n values ensure the acceptability of equations (6) for all grains, not only for those determined by the boundaries (7). The solution (8) is substituted into (6). A combination of exponential functions, the multiplier in the arguments of which is an imaginary unit, is represented as trigonometric functions. A combination of exponential functions with real arguments is represented as hyperbolic. After the transformation, the following vectors of characteristic index values can be obtained.

$$\chi_{m,n}(b+h)=\mathrm{ach}\left(1+\frac{G(c+h)}{Ehb}\begin{pmatrix}(1-\cos(\beta_n))c+\\+(1-\cos(\mu_m))b\end{pmatrix}\right); \qquad (9)$$

$$\chi_n^K(b+h)=\mathrm{ach}\left(1+\frac{G(c+h)c}{E_j hb}(1-\cos(\beta_n))\right); \qquad (10)$$

$$\chi_m^M(b+h)=\mathrm{arcch}\left(1+\frac{G(c+h)}{E_j h}(1-\cos(\mu_m))\right). \qquad (11)$$

The required solution of equations (6) in displacements is

$$\begin{gathered}u_{i,j,\Delta}=\sum_{n=1}^{K-1}\sum_{m=1}^{M-1}(A_{m,n}e^{\chi_{n,m}j(b+h)}+B_{m,n}e^{-\chi_{m,n}j(b+h)})f(\mu_m,i)f(\beta_n,\Delta)+\\+\sum_{m=1}^{M-1}(A_m^M e^{\chi_m^M j(b+h)}+B_m^M e^{-\chi_m^M j(b+h)})f(\mu_m,i)+\\+\sum_{n=1}^{K-1}(A_n^K e^{\chi_n^K j(b+h)}+B_n^K e^{-\chi_n^K j(b+h)})f(\beta_n,\Delta),\end{gathered} \qquad (12)$$

where $A_{m,n}, B_{m,n}, A_m^M, B_m^M, A_n^K, B_n^K$ are unknown arrays and coefficient vectors; $f(\rho, \kappa) = \cos(\rho(\kappa - 0.5))$.

The obtained expression for displacements of the abrasive elements (grains) of a tool for abrasive processing and the laws (2, 3, 5) make it possible to determine the shear stresses arising in the bonding material between the abrasive grains and internal forces occurring in them. In particular, the value of the grain interaction forces in the direction parallel to the axis j can be found.

$$T_{i,j,\Delta} = q\left(\begin{array}{l}\sum_{n=1}^{K-1}\sum_{m=1}^{M-1}\left(\begin{array}{l}A_{m,n}e^{\chi_{m,n}j(b+h)}(1-e^{-\chi_{m,n}(b+h)})+\\+B_{m,n}e^{-\chi_{m,n}j(b+h)}\left(1-e^{\chi_{m,n}(b+h)}\right)\end{array}\right)\times\\ \times f(\mu_m,i)f(\beta_n,\Delta)+\\+\sum_{m=1}^{M-1}\left(\begin{array}{l}A_m^M e^{\chi_m^M j(b+h)}(1-e^{-\chi_m^M(b+h)})+\\+B_m^M e^{-\chi_m^M j(b+h)}(1-e^{\chi_m^M(b+h)})\end{array}\right)f(\mu_m,i)+\\+\sum_{n=1}^{K-1}\left(\begin{array}{l}A_n^K e^{\chi_n^K j(b+h)}(1-e^{-\chi_n^K(b+h)})+\\+B_n^K e^{-\chi_n^K j(b+h)}(1-e^{\chi_n^K(b+h)})\end{array}\right)f(\beta_n,\Delta)\end{array}\right), \quad (13)$$

where $q = \dfrac{Eb^2}{c+h}$.

The obtained expressions (12, 13) consist of the sums of products of two functions – trigonometric and exponential ones. The first depends on the location (numbers) of grains in the planes parallel to the working plane. The arguments of the second function are the layer number product (distance from the working surface) and the characteristic index. The latter depends on the quantity of grains, their mechanical properties, and the bonding material.

The dependence on mechanical properties influences the nature of the stress distribution in the tool. For lower characteristic index values, the same changes in the stress-strain state of the grain layers take place with a larger quantity of them. The local disturbance zone, caused by the load on an individual grain, increases. The latter is accompanied by a decrease in the gradient of changes in stresses and displacements.

The characteristic index is proportional to the root of the displacement and elastic moduli ratio. Accordingly, the sizes of the local stress redistribution zones increase inversely proportional to the square root of the ratio of the bonding material displacement modulus and the reduced elastic modulus at the junction of the grain and the surrounding material and transmits normal stresses.

It is possible to determine the distribution of the stress-strain state parameters of the grinding tool with the following characteristics of the grains and the bonding material. The grain has the shape of a cube with a side of 0.1 mm. The thickness of the bonding material between the grains is 0.01 mm. The bonding material displacement modulus is $G = 10^{10}$ Pa. The abrasive grain elastic modulus is $E = 10^{12}$ Pa. There are five grains in each of the seven rows. The rows of grains are placed in five layers. On the plane under the fifth layer, the tool is motionlessly fixed in the direction normal to the working surface. It can be assumed that a tangential force of a unit value is applied to some grain on the tool working surface ($\Delta = 1$) by number (I). The grain is located in a row of grains with a number (J). The nature of loading is written in the form of the following condition,

when $$\Delta = 0 \quad T_{i,j,\Delta} = F(i,j), \quad (14)$$

here $$F(i,j) = \begin{pmatrix} 1 & i = I \wedge j = J \\ 0 & i \neq I \vee j \neq J \end{pmatrix}.$$

The surface opposite to the working surface is formed by grains located in the layer numbered K. This surface is attached to a rigid base, that is, it is motionless. The accepted condition can be written in the form of boundary conditions.

When $$\Delta = K u_{i,j,K} = 0. \quad (15)$$

This condition can be satisfied in the following way. The quantity of layers is conditionally doubled in the tool. The tool grain layer numbered $2K$ is assumed to be loaded with a tangential force directed opposite to the force (13). The latter ensures the asymmetry of the loads of these surfaces and, accordingly, the immobility of the tool surface on which it is held. This can be formulated as follows.

When $$\Delta = 2K \quad T_{i,j,\Delta} = -F(i,j). \quad (16)$$

A conditional increase in layers requires an increase in the terms of sums in expression (8) and in the following. The quantity of terms of the sums, which include the characteristic index vector β_n, increases. Its value, in this case, should be determined by the expression

$$\beta_n = \frac{\pi n}{2K}.$$

External forces do not act on the grinding tool extreme grains in rows by numbers i and layers by numbers Δ, namely when $j = 0$ and when $N = 0$,

$$T_{i,0,\Delta} = T_{i,N,\Delta} = 0. \quad (17)$$

In accordance with conditions (14) and (16), external forces act on grains with coordinates on the axes of their numbers I, J, 1 and I, J, $2K$. The solutions obtained on the basis of the equilibrium condition (1) are inapplicable to them. This inconsistency can be removed. The grinding tool is conditionally divided into two parts with a plane R normal to the axis j. It is drawn through the mass centre of the grain by number J. Parts are numbered as one and two. The numbers are included into the coefficient indices. From the boundary conditions (17) and force values (13), the ratio between the values of unknown arrays and coefficient vectors in expressions (12, 13), describing the grinding tool stress-strain state, can be found.

$$A_{1,m,n} = -B_{1,m,n}\left(\frac{1-e^{\chi_{m,n}(b+h)}}{1-e^{-\chi_{m,n}(b+h)}}\right); \quad A_{1,m}^M = -B_{1,m}^M\left(\frac{1-e^{\chi_m^M(b+h)}}{1-e^{-\chi_m^M(b+h)}}\right);$$

$$A_{1,n}^K = -B_{1,n}^K\left(\frac{1-e^{\chi_n^K(b+h)}}{1-e^{-\chi_n^K(b+h)}}\right);$$

$$A_{2,m,n} = -B_{2,m,n}e^{-2\chi_{m,n}N(b+h)}\frac{1-e^{\chi_{m,n}(b+h)}}{1-e^{-\chi_{m,n}(b+h)}};$$

$$A_{2,m}^M = -B_{2,m}^M e^{-2\chi_m^M N(b+h)}\left(\frac{1-e^{\chi_m^M(b+h)}}{1-e^{-\chi_m^M(b+h)}}\right);$$

$$A_{2,n}^K = -B_{2,n}^K e^{-2\chi_n^K N(b+h)}\left(\frac{1-e^{\chi_n^K(b+h)}}{1-e^{-\chi_n^K(b+h)}}\right).$$

The consistency condition for deformation of two parts of a grinding tool is

$$u_{1,i,J,\Delta} = u_{2,i,J,\Delta}. \quad (18)$$

The above ratios between the values of the unknown arrays, coefficient vectors and the consistency condition of deformation (18) are calculated. The following ratios can be obtained.

$$B_{1,m,n} = \mathrm{B}_{2,\mu,n}\eta_{m,n}; \quad B_{1.m}^M = B_{2.m}^M\eta_m^M; \quad B_{1,n}^K = B_{2,n}^K\eta_n^K,$$

where

$$\eta_{m,n} = \frac{1-\dfrac{1-e^{\chi_{m,n}(b+h)}}{1-e^{-\chi_{m,n}(b+h)}}e^{2\chi_{n,m}(J-N)(b+h)}}{1-\left(\dfrac{1-e^{\chi_{m,n}(b+h)}}{1-e^{-\chi_{m,n}(b+h)}}\right)e^{2\chi_{n,m}J(b+h)}};$$

$$\eta_m^M = \frac{1-\dfrac{1-e^{\chi_m^M(b+h)}}{1-e^{-\chi_m^M(b+h)}}e^{2\chi_m^M(J-N)(b+h)}}{1-\dfrac{1-e^{\chi_m^M(b+h)}}{1-e^{-\chi_m^M(b+h)}}e^{2\chi_m^M J(b+h)}};$$

$$\eta_n^K=\frac{1-\dfrac{1-e^{\chi_n^K(b+h)}}{1-e^{-\chi_n^K(b+h)}}e^{2\chi_n^K(J-N)(b+h)}}{1-\dfrac{1-e^{\chi_m^M(b+h)}}{1-e^{-\chi_m^M(b+h)}}e^{\chi_n^K 2J(b+h)}}.$$

The determined interdependence between arrays and coefficient vectors of conditional parts of the tool is taken into account. The force expressions (13) for the conditional parts of the tool are as follows.

$$T_{1,i,j,\Delta}=q\left(\begin{array}{l}\sum\limits_{n-1}^{2K-1}\sum\limits_{m=1}^{M-1}B_{2,m,n}\eta_{m,n}(e^{-\chi_{m,n}j(b+h)}-e^{\chi_{m,n}j(b+h)})\times\\ \times(1-e^{\chi_{m,n}(b+h)})f(\mu_m,i)f(\beta_n,\Delta)+\\ +\sum\limits_{m=1}^{M-1}B_{2.m}^M\eta_m^M(e^{-\chi_m^M j(b+h)}-e^{\chi_m^M j(b+h)})\times\\ \times(1-e^{\chi_m^M(b+h)})f(\mu_m,i)\qquad+\\ +\sum\limits_{n=1}^{2K-1}B_{2,n}^K\eta_n^K(e^{-\chi_n^K j(b+h)}-e^{\chi_n^K j(b+h)})\times\\ \times(1-e^{\chi_n^K(b+h)})f(\beta_n,\Delta)\end{array}\right);$$

$$T_{2,i,j,\Delta}=q\left(\begin{array}{l}\sum\limits_{n=1}^{2K-1}\sum\limits_{m=1}^{M-1}B_{2,m,n}(e^{-\chi_{m,n}j(b+h)}-e^{\chi_{m,n}(j-2N)(b+h)})\times\qquad+\\ \times(1-e^{\chi_{m,n}(b+h)})f(\mu_m,i)f(\beta_n,\Delta)\\ +\sum\limits_{m=1}^{M-1}B_{2,m}^M(e^{-\chi_m^M j(b+h)}-e^{\chi_m^M(j-2N)(b+h)})\times\qquad+\\ \times(1-e^{\chi_m^M(b+h)})f(\mu_m,i)\\ +\sum\limits_{n=1}^{2K-1}B_{2,n}^K(e^{-\chi_n^K j(b+h)}-e^{\chi_n^K(j-2N)(b+h)})\times\\ \times(1-e^{\chi_n^K(b+h)})f(\beta_n,\Delta)\end{array}\right).$$

In the section $j=J$, conditions (14 and 16) are satisfied. We equate the difference in grain loading forces in the section $j=J$ of the function specified by the Fourier series on the axes of limited lengths in coordinate systems – grain numbers in the interval $1\le\Delta\le 2K$. The following ratios are obtained.

$$B_{2,m,\Delta}\left(\begin{array}{l}\eta_{m,\Delta}(e^{-\chi_{m,\Delta}J(b+h)}-e^{\chi_{m,\Delta}J(b+h)})-\\ -(e^{-\chi_{m,\Delta}J(b+h)}-e^{\chi_{m,\Delta}(J-2N)(b+h)})\end{array}\right)(1-e^{\chi_{m,\Delta(b+h)}})=$$

$$=\frac{2(c+h)}{MKEb^2}f(\mu_m,I)(f(\beta_n,1)-f(\beta_n,2K));$$

$$B_{2.m}^M\left(\begin{array}{l}\eta_m^M(e^{-\chi_m^M J(b+h)}-e^{\chi_m^M J(b+h)})-\\ -(e^{-\chi_m^M J(b+h)}-e^{\chi_m^M(J-2N)(b+h)})\end{array}\right)(1-e^{\chi_m^M(b+h)})=$$

$$=\frac{(c+h)}{MKEb^2}f(\mu_m,I);$$

$$B_{2,\Delta}^K\left(\begin{array}{l}\eta_\Delta^K(e^{-\chi_\Delta^K J(b+h)}-e^{\chi_\Delta^K J(b+h)})-\\ -(e^{-\chi_\Delta^K J(b+h)}-e^{\chi_\Delta^K(J-2N)(b+h)})\end{array}\right)(1-e^{\chi_m^M(b+h)})=$$

$$=\frac{(c+h)}{MKEb^2}(f(\beta_n,1)-f(\beta_n,2K)).$$

From the last ratios, the values of arrays and coefficient vectors can be determined.

$$B_{2,m,n}=\frac{2(c+h)f(\mu_m,I)(f(\beta_n,1)-f(\beta_n,2K))}{MKEb^2(1-e^{\chi_{m,n(b+h)}})\left(\begin{array}{l}\eta_{m,n}(e^{-\chi_{m,n}J(b+h)}-e^{\chi_{m,n}J(b+h)})-\\ -e^{-\chi_{m,n}J(b+h)}+e^{\chi_{m,n}(J-2N)(b+h)}\end{array}\right)};$$

$$B_{2.m}^M=\frac{(c+h)f(\mu_m,I)}{MKEb^2(1-e^{\chi_m^M(b+h)})\left(\begin{array}{l}\eta_m^M(e^{-\chi_m^M J(b+h)}-e^{\chi_m^M J(b+h)})-\\ -e^{-\chi_m^M J(b+h)}+e^{\chi_m^M(J-2N)(b+h)}\end{array}\right)};$$

$$B_{2,n}^K=\frac{(c+h)(f(\beta_n,1)-f(\beta_n,2K))}{MKEb^2(1-e^{\chi_n^K(b+h)})\left(\begin{array}{l}\eta_n^K(e^{-\chi_n^K J(b+h)}-e^{\chi_n^K J(b+h)})-\\ -e^{-\chi_n^K J(b+h)}+e^{\chi_n^K(J-2N)(b+h)}\end{array}\right)}.$$

We use the found values of arrays and coefficient vectors to study cases of applying a tangential force to the middle and extreme cables of the middle row of their location. The displacements of grains, the tangential forces acting on them, the shear stress distribution in the grinding tool with five grain layers in the tool are determined. Figs. 2–4 show the forces acting on the grinding tool grains, their displacements and the shear angle tangents of the bonding material in the grinding tool with a real

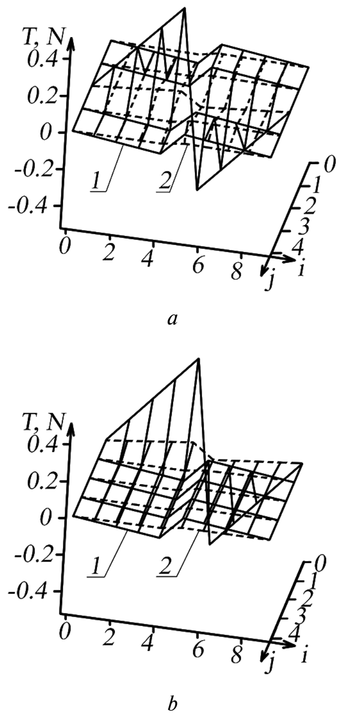

Fig. 2. Distribution of forces between the abrasive grains in the working layer (Curve 1) and the layer under it (Curve 2) when loading the middle (a) and extreme (b) grains in the middle row

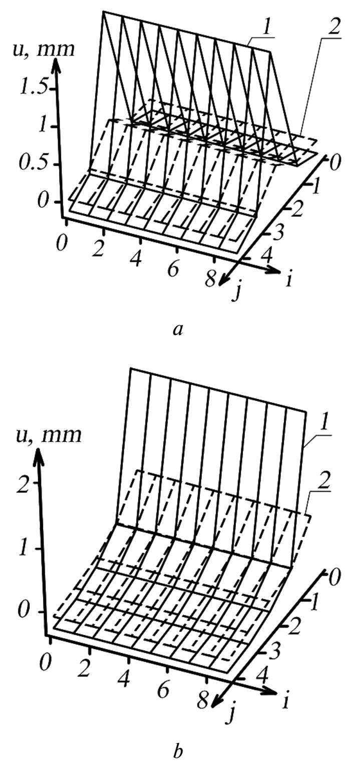

Fig. 3. Displacement of abrasive grains in the working layer (Curve 1) and the layer under it (Curve 2) when loading the middle (a) and extreme (b) grains in the middle row

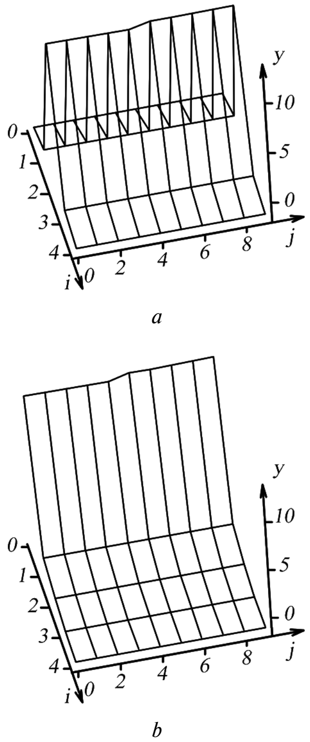

Fig. 4. Tangents of mutual shear angles of abrasive grains in the working layer when loading the middle:
a – and extreme; b – grains in the middle row

quantity of layers in section *J*. In this case, in the indicated section, the stress-strain state parameters in the second part of the tool are specified conditionally shifted by the grain pitch value.

The given graphs show that the extreme grain interaction forces are realized in the upper (working) layer – Curve (1), the pressure forces of the layer under it are much smaller – Curve (2). Tensile and compressive stresses are realized in the bonding material. This distribution of forces is caused by the application of a tangential force. It is also influenced by the nature of grain displacements (Fig. 3).

A characteristic peculiarity of grain displacements is that in the tool for abrasive processing of materials, under the action of force parallel to working surface, grains with numbers corresponding to the number of the loaded grain move mainly. This leads to a significant displacement of the grains with the indicated numbers relative to the adjacent grains of the working layer and grains with the same layer numbers under the working layer (Figs. 4, 5).

The nature of distributing tangents of the mutual shear angles of abrasive grains with numbers corresponding to the numbers of the loaded grain in the working layer and the layer under it, when loading the middle and extreme grains in the middle row, coincides. The maximum tangents of the angles in the latter case are greater by 10 %.

The found dependences of stresses in the constituent elements of a composite tool for abrasive processing of materials make it possible to determine the conditions for their operating time to failure, based on the known characteristics of these materials. It is possible to ensure the simultaneous loss of the tool working surface cutting ability and its restoration by selecting the parameters for mechanical processing, as well as the bonding material of the tool for such processing. In this way, the efficiency of the technological process as a whole can be improved.

Conclusions and prospects for further research development in this direction. The well-known studies on the abrasive tool stress-strain state do not take into account its composite structure. Based on the equilibrium condition of an individual abrasive grain, a mathematical model of its equilibrium and an algorithm for determining the stress-strain state in the case of a tangential load arbitrarily located on the working surface of a discrete-grain tool have been developed. The following has been determined. The loading of the extreme grains leads to greater displacements, tangents of shear angles of the material binding the grains. The most dangerous stresses are the shear and tensile stresses of the material binding the grains. They decrease with an increasing quantity of grain rows in the tool or with increasing distance from the row of grain location to the nearest tool edge. The determined nature of the distribution of forces of grains and stresses interacting in the material that bonds them allows comprehensively assessing the influence of the tangential load value of the tool's working grain on its stress state and the material that bonds the grains. In addition, it makes it possible to predict the probable quantity of loads to the simultaneous rational wear of the grain and the destruction of the material part that contains it. The linear formulation of the problem makes it possible to determine the stress state for an arbitrary scheme of loading the grains by tangential forces as the sum of separately determined stresses for each individual case. A further research direction should be the development of an algorithm for determining the stress-strain state of a tool for abrasive processing due to thermal changes in the size of its working grains.

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Напружений стан інструменту шліфування навантаженого дотичною силою

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Мета. Уточнення механізму взаємодії дискретних зерен через матеріал, що їх з'єднує в інструменті абразивної обробки матеріалів, у разі його навантаження дотично спрямованою силою різання.

Методика. Побудова та аналітичний розв'язок математичної моделі, сформульованої на основі статичної рівноваги абразивного зерна як елемента системи зерен інструменту, з'єднаних поміж собою матеріалом із відмінними механічними характеристиками.

Результати. Розроблена математична модель і сформульовано алгоритм аналітичного визначення показників напружено-деформованого стану інструменту абразивної обробки матеріалів навантаженого дискретною дотичною до його робочої поверхні силою різання. Встановлено характер залежності напружень і деформацій від механічних параметрів, кількості зерен і матеріалу, що їх з'єднує в абразивному інструменті, навантаженому одиничною дотичною силою.

Наукова новизна. Навантаження крайніх зерен призводить до більших переміщень, тангенсів кутів зсуву матеріалу, що з'єднує зерна. Вони зменшуються зі зростанням кількості рядів зерен в інструменті або зі зростанням кількості зерен до найближчого краю інструменту.

Практична значимість. Встановлений розподіл сил взаємодії зерен і напружень у матеріалі, що їх з'єднує. Відомий розподіл дозволяє у процесі розробки інструменту й технології, в якій він задіяний, комплексно оцінювати вплив величини дотичного навантаження робочого зерна інструменту на його напружений стан, матеріалу, що утримує зерно. Відомий напружений стан дозволяє прогнозувати кількість циклів навантажень до одночасного раціонального зносу зерна та руйнування тієї частини матеріалу, що його утримує. Лінійна постановка задачі дозволяє враховувати взаємний вплив дотичних навантажень декількох зерен на напружено-деформований стан інструменту в цілому.

Ключові слова: *інструмент для обробки, абразивні елементи, механічна взаємодія, деформації, напруження*

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