MODELING OF MANIPULATOR GRIP REACHES WITH REGARD TO GENERALIZED COORDINATE CONSTRAINTS

Purpose. Maintaining operational characteristics when using the robot in extreme conditions and ensuring the reach of the grasp of the manipulator, despite the limitations of the generalized coordinates.

Methodology. The kinematic analysis of the manipulator is carried out on the basis of coordinate transformation by the Denavit – Hartenberg method. Polynomial laws of change in generalized coordinates are considered taking into account restrictions in the actuators of kinematic pairs.

Findings. Algorithms for the kinematic analysis of the manipulator have been developed taking into account the constraints of the generalized coordinates, which in real time make it possible to adjust the control actions on the actuator drives for the successful implementation of software technological operations.

Originality. A technique for solving problems of manipulator kinematics is proposed, taking into account the limitations of many permissible values, drive power and friction coefficients for all generalized coordinates. This allows, in contrast to the known techniques, determining in real time the actual coordinates, positioning accuracy and reach of the gripper, implemented considering the arisen limitations of the generalized coordinates.

Practical value. The research results can be used at the stage of design, implementation, modernization and operation of manipulators. At the design stage, simulation of the manipulator movement, with considering the constraints of the generalized coordinates, makes it possible to optimize the parameters of the kinematic scheme of the manipulator. For autonomous manipulative mobile robots operating in extreme conditions, the developed software makes it possible to carry out kinematic analysis, adjust the target function of the adaptive control system, synthesize control actions on the actuator drives, and implement software technological operations, despite the arisen limitations of generalized coordinates.

Keywords: robot manipulator, kinematic diagram, gripper pole, coordinate transformation, workspace, reach limits

Introduction. When using manipulative mobile robots (MR) in a previously unknown environment, the timely identification of faults or limitations, and the corresponding correction of the programmed movements of the manipulator, increases the survivability of the structure and ensures the success of the mission. Indeed, when overcoming the consequences of man-made and natural disasters, conducting reconnaissance and demining, partial damage to the MR structure is possible, namely, limiting the mobility of kinematic pairs due to an increase in friction and a decrease in the power of the actuator drives. This causes a change in the set of values of the generalized coordinates and reach of the gripper, a narrowing of the size of the working space, a decrease in the number of permissible manipulator configurations, and a decrease in positioning accuracy. Failure to comply with the listed restrictions in the conditions of autonomous operation and remote control may lead to loss of functionality of the MK.

The success of the implementation of software technological operations depends on the geometric parameters of the kinematic diagram of the manipulator, the laws of change in generalized coordinates, the power of the drives, the design and characteristics of the kinematic pairs. At the stage of designing a universal anthropomorphic manipulator (UAM), it is advisable to carry out mathematical modeling of the movement of the pole of the grip of the point P, taking into account the listed restrictions. In the case of autonomous operation of the MR, the analysis of the simulation results will make it possible to carry out kinematic analysis in real time, correct the target function of the adaptive control system, synthesize control actions on the actuator drives and implement software technological operations, despite the arisen limitations of the generalized coordinates.

Literature review. Currently, robotic systems are widely used when working in hazardous and harmful conditions for humans. In the article [1], the authors consider the use of robotic autonomous systems for earthmoving and construction work in the construction of fortifications and military structures. The article [2] provides an overview of the latest achievements in the field of military robotics, in article [3] the author analyzes the effectiveness of the use of groups of ground-based robotic systems in the armed forces.

At present, the process of decommissioning nuclear facilities is underway in Europe. In the article [4], the authors propose an algorithm for optimizing the path for the protection of personnel and robots participating in radiation work in a limited and unknown environment. The article [5] discusses the technical support system for the decommissioning of nuclear power facilities. Evaluation of the effectiveness of decontamination is carried out using an electro-pneumatic robot for sampling. The robotic system presented in [6] allows monitoring the storage of spent nuclear fuel and remotely detecting chlorine deposits on stainless steel containers. The article [7] presents the design of an autonomous underwater vehicle (AUV) for surveillance and reconnaissance (SR), built within the framework of the SABUVIS project. The results of research on the positioning and navigation of an underwater robot in a complex and changing environment are presented in [8]. The article [9] proposes a new distributed targeting strategy for several underwater robots in order to reduce the average target search time and increase the probability of target detection and hitting. The system presented in [10] allows the use of a collaborative robot and an industrial robot interacting in a hazardous industrial environment.

In Ukraine, along with the “classical” formulation of problems for extreme robotics, there is its own specificity. A serious challenge for Ukraine is the demining of territories in Donetsk and Luhansk regions and the elimination of the consequences of explosions at ammunition depots in Novobohdanivka, Lozova, Svatove, Balaklia, Kalyntivka and Ichne. The use of MR for this purpose would save the life and health of servicemen and civilians.
Consequently, the development of a UAM as part of a robotic complex for eliminating the consequences of natural and man-made emergencies is an urgent scientific and practical task. The use of UAM will allow for visual reconnaissance, monitoring of the level of radiation and the concentration of harmful and hazardous substances within the working area, search and primary diagnosis of suspicious objects, evacuation and disposal of explosive devices and explosive objects.

The relevance of the work lies in the need to create a software method for the kinematic analysis of the manipulator to determine the reach of the gripper pole, taking into account the limits in the kinematic pairs, the power of the drives and the coefficients of friction. Mathematical modeling of the manipulator movement allows one to perform real-time software kinematic analysis, determine the permissible manipulator movement allows one to perform real-time software kinematic analysis, determine the optimal trajectory, correct the target function of the adaptive control system, synthesize control actions on the actuator drives and implement software technological operations, despite the arisen limitations of the generalized coordinates.

This will preserve the functionality and operational characteristics when using the MR in extreme conditions of a previously unknown environment, taking into account the limitations of the generalized coordinates.

**Purpose.** Maintaining operational characteristics when using the robot in extreme conditions and ensuring the reach of the grip of the manipulator, despite the limitations of the generalized coordinates.

**Methods.** The movement of each link of the manipulator relative to the platform can be represented as a sequence of rotations and parallel transfers by the Denavit – Hartenberg method [11]. The kinematic analysis of the manipulator based on the coordinate transformation method is carried out using the Mathcad software package [12]. An analytical solution to the first kinematics problem for UAM is obtained by the coordinate transformation method [11]. The kinematic analysis of the manipulator based on the coordinate transformation method is carried out using the Mathcad software package [12]. An analytical solution to the first kinematics problem for UAM is obtained by the coordinate transformation method [11].

**Results.** The object of the research is a robotic complex for work in extreme conditions, consisting of a controlled platform and an UAM. The design of the UAM provides for the mobility of the complex (its cross-country ability, maneuverability and speed), and the UAM, equipped with attachments, performs technological operations. The range of attachments for the manipulator (bucket, scraper, probe, and so on) significantly expands the scope of its application. The manipulator is controlled remotely, the control system provides three independent communication channels with the operator. The results of the manipulator performance of technological operations are monitored using a computer vision system.

**Coordinate systems.** For the kinematic analysis of the manipulator, let us introduce the right-hand coordinate systems (Fig. 1).

$O_X Y Z_M$ is an inertial coordinate system. $M_X Y_M Z_M$ is a movable base coordinate system. The coordinate axes coincide with the main central axes of inertia of the platform, and the origin is the center of mass of the platform — point $M$. The $M_X$ axis is located in the plane of movement of the platform and is directed towards the movement. The $M_Z$ axis is perpendicular to the platform movement plane and is directed upward to the manipulator docking disk. The $M_Y$ axis is located in the plane of the platform movement and complements the coordinate system to the right.

$O_X Y Z$ is a connected mobile coordinate system. The origin is aligned with point $O_1$ — the center of mass of the docking disk. The coordinate axes coincide with the main central axes of inertia of the docking disk. In the initial position of the docking disk of the manipulator, the axes of the $O_X Y Z$ coordinate system are parallel to the axes of the $M_X Y_Z$ coordinate system.

$O_X Y Z$ for $(i = 2, 3, 4)$ is a connected moving coordinate systems. The origin is connected to point $O_i$ — the center of the kinematic pair. The $O_X$ coordinate axes coincide with the longitudinal axes of the rod links of the manipulator arm. In the initial position of the manipulator links, the axes of the coordinate systems $O_X Y Z_i$ are parallel to the axes of the $M_X Y_Z_i$ coordinate system.

**Analytical research.** The movement of each link of the manipulator relative to the platform can be represented as a sequence of turns and parallel transfers. For an example of the construction shown in Fig. 1, let us assume that the change in the generalized coordinate $q_i$ occurs around the $O_i Z_i$ axis, the change in the generalized coordinates $q_i$ (for $i = 2, 3, 4$) — around the $O_i Y_i$ axis.

For each kinematic pair, let us compose the matrix transformation of the coordinates of Denavit–Hartenberg [11–13]

$$A_i^1 = \begin{bmatrix} C(q_i) & 0 & S(q_i) & c_{i3} \\ 0 & 1 & 0 & 0 \\ -S(q_i) & 0 & C(q_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_i^2 = \begin{bmatrix} C(q_i) & 0 & S(q_i) & c_{i2} \\ 0 & 1 & 0 & 0 \\ -S(q_i) & 0 & C(q_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_i^3 = \begin{bmatrix} C(q_i) & 0 & S(q_i) & c_{i1} \\ 0 & 1 & 0 & 0 \\ -S(q_i) & 0 & C(q_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_i^4 = \begin{bmatrix} C(q_i) & 0 & S(q_i) & c_{i0} \\ 0 & 0 & 1 & c_{i0} \\ -S(q_i) & 0 & C(q_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where $c_{i3}$, $c_{i2}$, $c_{i1}$, $c_{i0}$ are the geometrical dimensions of the manipulator structure.
For each generalized coordinate, let us define the law of change \( q_i(t) \). For example, the law of uniformly accelerated motion

\[
q_i(t) = a_i \cdot t^2 / 2 + b_i \cdot t + q_{i0},
\]

where \( a_i \) and \( b_i \) are the parameters of the drive in the \( p \)th degree of mobility; \( q_{i0} = q_i(0) \) is the initial value of the generalized coordinate \( q_i(t) \). The design features of the kinematic pair impose restrictions on the range of values of the corresponding generalized coordinate \( q_i(t) = [q_{i\min}, q_{i\max}] \). Fig. 2 shows a fragment of a program in Mathcad that simulates the law of change in a generalized coordinate \( q_i(t) \) taking into account restrictions.

The drive of the selected degree of mobility generates an increase in the generalized coordinate according to the law

\[
Q_i(t) = \frac{0.3 \cdot t^2}{2} - 0.03t + \frac{\pi}{6}.
\]

The actual implementation \( q_i(t) \) takes into account the design features of the kinematic pair and limitations

\[q_i(t) \in \left[ \frac{\pi}{180} \frac{2\pi}{3} \right] .\]

Graphs of changes in the calculated \( Q_i(t) \) and actual \( q_i(t) \) values of the generalized coordinate obtained in Mathcad are shown in Fig. 3.

The authors have developed software that allows modeling in Mathcad the movements of a universal anthropomorphic manipulator, taking into account the constraints of generalized coordinates. This allows building an actual model of the manipulator working area, estimating the reach and calculating the positioning accuracy of the gripper. The results obtained are used to select the optimal geometric dimensions of the structure from the set of admissible values. Fig. 4 shows a fragment of a program in Mathcad for the analytical solution of the first kinematics problem.

Table 1 shows the results of mathematical modeling in Mathcad of the reach of the manipulator grip depending on the geometric parameters of the kinematic scheme. The software developed in Mathcad makes it possible to simulate the movements of a universal anthropomorphic manipulator for various laws of change in generalized coordinates, taking into account the drive power, friction coefficients, design features and characteristics of kinematic pairs (Table 2). Table 3 shows the results of mathematical modeling of the reach of the manipulator, taking into account the constraints of the generalized coordinates.

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**Table 1**

<table>
<thead>
<tr>
<th>( c_{30} )</th>
<th>( c_{21} )</th>
<th>( c_{12} )</th>
<th>( c_{43} )</th>
<th>( q_i(t) ) range</th>
<th>( q_i(t) ) values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>( x_{\min}(t) = x(0.1) = 1.481 )</td>
<td>( x_{\max}(t) = x(3.5) = -0.175 )</td>
</tr>
<tr>
<td>0.05</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>( y_{\min}(t) = y(2.8) = -0.198 )</td>
<td>( y_{\max}(t) = y(0.8) = -0.875 )</td>
</tr>
<tr>
<td>0.05</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>( z_{\min}(t) = z(3.7) = -0.037 )</td>
<td>( z_{\max}(t) = z(1.6) = -1.180 )</td>
</tr>
</tbody>
</table>

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**Table 3**

<table>
<thead>
<tr>
<th>( c_{30} )</th>
<th>( c_{21} )</th>
<th>( c_{12} )</th>
<th>( c_{43} )</th>
<th>( q_i(t) ) range</th>
<th>( q_i(t) ) values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>( x_{\min}(t) = x(0.1) = 1.276 )</td>
<td>( x_{\max}(t) = x(4.1) = -1.015 )</td>
</tr>
<tr>
<td>0.05</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>( y_{\min}(t) = y(2.7) = -0.201 )</td>
<td>( y_{\max}(t) = y(0.1) = -0.734 )</td>
</tr>
<tr>
<td>0.05</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>( z_{\min}(t) = z(3.5) = -0.037 )</td>
<td>( z_{\max}(t) = z(1.5) = -1.014 )</td>
</tr>
</tbody>
</table>
Results of mathematical modeling in Mathcad of the reach of the manipulator grip, taking into account the drive power and friction coefficients

<table>
<thead>
<tr>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>max</th>
</tr>
</thead>
</table>
| $a_1 = 0.3$; $b_1 = -0.03$ | $a_1 = 0.3$; $b_1 = -0.03$ | $a_1 = 0.3$; $b_1 = -0.03$ | $a_1 = 0.3$; $b_1 = -0.03$ | $x_{max}(t) = x(0.12) = 1.483$
| $y_{max}(t) = y(4.95) = -0.175$
| $z_{max}(t) = z(3.61) = 0.032$

<table>
<thead>
<tr>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>max</th>
</tr>
</thead>
</table>
| $a_1 = 0.3$; $b_1 = -0.01$ | $a_1 = 0.3$; $b_1 = -0.01$ | $a_1 = 0.3$; $b_1 = -0.01$ | $a_1 = 0.3$; $b_1 = -0.01$ | $x_{max}(t) = x(0.05) = 1.472$
| $y_{max}(t) = y(4.95) = -0.175$
| $z_{max}(t) = z(3.54) = 0.032$

<table>
<thead>
<tr>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>max</th>
</tr>
</thead>
</table>
| $a_1 = 0.6$; $b_1 = -0.01$ | $a_1 = 0.6$; $b_1 = -0.01$ | $a_1 = 0.6$; $b_1 = -0.01$ | $a_1 = 0.6$; $b_1 = -0.01$ | $x_{max}(t) = x(0.05) = 1.480$
| $y_{max}(t) = y(4.95) = -0.175$
| $z_{max}(t) = z(4.95) = -0.059$

<table>
<thead>
<tr>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>$q(t)$</th>
<th>max</th>
</tr>
</thead>
</table>
| $a_1 = 0$; $b_1 = 0.03$ | $a_1 = 0.3$; $b_1 = 0.03$ | $a_1 = 0.3$; $b_1 = 0.03$ | $a_1 = 0.3$; $b_1 = 0.03$ | $x_{max}(t) = x(0.01) = 1.475$
| $y_{max}(t) = y(2.70) = -0.317$
| $z_{max}(t) = z(3.34) = -0.266$

Results of mathematical modeling in Mathcad of the reach of the manipulator grip, taking into account the constraints of generalized coordinates

<table>
<thead>
<tr>
<th>$q(t) = \frac{0.3-t^2}{2} - 0.03t + \frac{\pi}{6}$</th>
<th>$q(t) = \frac{0.3-t^2}{2} - 0.03t + \frac{\pi}{6}$</th>
<th>$q(t) = \frac{0.3-t^2}{2} - 0.03t + \frac{\pi}{6}$</th>
<th>$q(t) = \frac{0.3-t^2}{2} - 0.03t + \frac{\pi}{6}$</th>
<th>max</th>
<th>min</th>
</tr>
</thead>
</table>
| $q(t)$ ∈ $[0; \frac{2\pi}{4}]$ | $x_{max}(t) = x(0.10) = 1.481$
| $y_{max}(t) = y(2.81) = -0.198$
| $z_{max}(t) = z(4.90) = 0.014$
| $x_{max}(t) = x(3.7) = -0.316$
| $y_{max}(t) = y(0.8) = -0.875$
| $z_{max}(t) = z(1.5) = -1.187$
| $q(t) ∈ [0; \frac{2\pi}{3}]$ | $x_{max}(t) = x(0.10) = 1.481$
| $y_{max}(t) = y(2.81) = -0.198$
| $z_{max}(t) = z(3.80) = 0.037$
| $x_{max}(t) = x(3.7) = -0.175$
| $y_{max}(t) = y(0.9) = -0.876$
| $z_{max}(t) = z(1.6) = -1.180$
| $q(t) ∈ [0; \pi]$ | $x_{max}(t) = x(0.10) = 1.481$
| $y_{max}(t) = y(4.40) = 0.500$
| $z_{max}(t) = z(4.30) = 0.050$
| $x_{max}(t) = x(4.4) = -0.500$
| $y_{max}(t) = y(4.10) = -0.875$
| $z_{max}(t) = z(1.5) = -1.187$

Fig. 5 shows a fragment of the program in Mathcad, which simulates the movements of the UAM gripper, taking into account the listed restrictions. Fig. 6 shows the projections of the reach of the manipulator grip, built in Mathcad, based on the results of mathematical modeling, taking into account the listed restrictions. In this case, the reach of the manipulator grip is projected onto the coordinate planes of the associated movable coordinate system $O_XY_1Z_2$.

The developed software makes it possible to carry out mathematical modeling of UAM movements, taking into account the restrictions, to calculate the reach of the gripper and to build the working area of the manipulator.

**Conclusions.** The authors completed the first stage of designing the UAM as part of a robotic complex for eliminating the consequences of natural and man-made emergencies. The kinematic scheme of the manipulator is selected. Software for mathematical modeling of manipulator movements has been developed. A software method for the kinematic analysis of the manipulator has been created to determine the reach of the gripper pole of the manipulator in the presence of the constraints of the generalized coordinates. Mathematical modeling allows optimizing the geometric parameters of the kinematic scheme of the manipulator, the laws of change in generalized coordinates, the design and characteristics of the kinematic pairs.
Моделювання меж досяжності схвatu маніпулятора з урахуванням обмежень узагальнених координат

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Мета. Збереження експлуатаційних характеристик при використанні робота в екстремальних умовах і забезпечення меж досяжності схвatu маніпулятора, незважаючи на обмеження узагальнених координат.

Методика. Кінематичний аналіз маніпулятора проводиться на підставі перетворення координат методом Денавіта-Хартенберга. Розглядаються поліноміальні зміни узагальнених координат з урахуванням обмежень у виконавчих приводах кінематичних пар.

Результати. Розроблені алгоритми кінематичного аналізу маніпулятора з урахуванням обмежень узагальнених координат, що в режимі реального часу дозволяють скорегувати керуючі впливи на виконавчі приводи для успішної реалізації програмних технологічних операцій.

Наукова новизна. Запропонована методика рішення завдань кінематики маніпулятора з урахуванням обмежень множин допустимих значень, потужності приводів і коефіцієнтів тертя за всіма узагальненими координатами. Це дозволяє, на відміну від відомих методик, визначати в режимі реального часу фактичні координати, точність позиціонування й межі досяжності схвatu, реалізовані з урахуванням виниклих обмежень узагальнених координат.

Практична значимість. Результати досліджень можуть бути використані на етапі проектування, впровадження, модернізації та експлуатації маніпуляторів. На етапі проектування моделювання руху маніпулятора з урахуванням обмежень узагальнених координат дозволяє оптимізувати параметри кінематичної схеми маніпулятора. Для автономних маніпуляційних мобільних роботів, що діють в екстремальних умовах, розроблене програмне забезпечення дозволяє провести кінематичний аналіз, скорегувати цільову функцію адаптивної системи керування, синтезувати керуючі впливи на виконавчі приводи і реалізувати програмні технологічні операції, незважаючи на виникнення обмежень узагальнених координат.

Ключові слова: маніпулятор робота, кінематична схема, полюс схвatu, перетворення координат, робочий простір, межі досяжності.

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References.