SOLID STATE PHYSICS, MINERAL PROCESSING

UDC 539.3

O. Maksymovych1, orcid.org/0000-0002-2892-7735,
T. Solyar2, orcid.org/0000-0003-3826-8881,
A. Sudakov3,4, orcid.org/0000-0003-2881-2855,
I. Nazar5, orcid.org/0000-0003-2592-3592,
M. Polishchuk5, orcid.org/0000-0002-1218-5925

DETERMINATION OF STRESS CONCENTRATION NEAR THE HOLES UNDER DYNAMIC LOADINGS

Purpose. To develop an approach for determining the stress state of plate structural elements with holes under dynamic loads with controlled accuracy.

Methodology. The study was carried out on the basis of the Laplace transform and the method of integral equations.

Originality. A new approach to the regularization of the Prudnikov formula for inverting the Laplace transform as applied to dynamic problems of the theory of elasticity has been developed. For its implementation: convergence of Fourier series based on pre-set stresses at the initial time is improved; the remainder is taken into account in the conversion formula.

Findings. An approach to determining the dynamic stresses at the holes in the plates is proposed, which includes: the Laplace transform in the time coordinate; a numerical method for determining transformants of displacements and stresses based on the method of integral equations; finding originals on the basis of Prudnikov’s formula adapted to dynamic problems of elasticity theory. The problem of determining the Laplace images for displacements is reduced to solving singular integral equations. Integral equations were solved numerically based on the approaches developed in the boundary element method. To find displacements and stresses, the Laplace transform inversion formulas proposed by Prudnikov are adapted to dynamic problems. The study on dynamic stresses at holes of various shapes was carried out.

Literature review. Boundary integral equations method is widely used to solve 2-dimensional dynamic problems of elasticity. Numerical algorithms for solving integral equations, which are based on Stokes singular solutions, are given in the books by C. A. Brebbia, S. Walker [1] and F. J. Sayas [2]. Such algorithms are quite complex, because the discretization of equations is carried out simultaneously by spatial coordinates and time. Therefore, other methods for solving dynamic problems of the elasticity theory are widely used in the literature. In particular, in [3] and Article [4] in the equations, the time derivatives are preliminarily replaced by finite differences with subsequent application to the obtained BIEM equations. Let us note that when using these methods, the solutions are found step by step over time, and therefore errors can accumulate.

Practical value. A method has been developed for calculating the stress concentration at holes of arbitrary shape in lamellar structural elements under dynamic loads. The proposed approach makes it possible to determine stresses with controlled accuracy.

Summary. The studies performed at circular and polygonal holes with rounded tops can be used in strength calculations for dynamically loaded plates. The influence of Poisson’s ratio on the concentration of dynamic stresses is analyzed.

Keywords: concentration of dynamic stresses, method of boundary integral equations, Laplace transforms, inversion formulas

1 – University of Technology and Life Science, Bydgoszcz, the Republic of Poland, e-mail: olesia.maksymovych@utp.edu.pl
2 – Pidstryhach Institute for Applied Problems of Mechanics and Mathematics NASU, Lviv, Ukraine
3 – Dnipro University of Technology, Dnipro, Ukraine
4 – Lviv Polytechnic National University, Lviv, Ukraine
5 – Lutsk National Technical University, Lutsk, Ukraine

https://doi.org/10.33271/nvngu/2021-3/019
modified with respect to the dynamic problems of the theory of elasticity in such a way that when applied, the residual term can be made as arbitrarily small. The convergence of the series that are included in this formula has also been improved. It is shown that with such approach it is possible to control the accuracy of stress concentration calculations near holes of arbitrary shape.

Other methods for solving dynamic problems of the elasticity theory are used in: [10] (based on the integral Fourier transform over time); [11] (the method of Fourier series by spatial coordinates is used); [12] (Galerkin method is used); [13] (the finite element method is used).

The problem statement. Let the plane be weakened by the holes whose boundaries are the contours $L_j, j=1, ..., J$, to the plate boundary the stresses $(\rho_1, \rho_2)$ are applied, where $\rho_{1,2} = \rho_{1,2}(M, \tau, \tau$ is time; $M(x, y)$ is a point on the boundary. Assume that at initial moment the displacements and stresses are absent. To solve the problem we use the Laplace integral time transform and the BIEM.

**Integral representation for the Laplace transform.** For the Laplace transforms from displacements $u_{1,2}$ the integral Somilov representation [1, 5] will be valid

$$\bar{u}_{jk}(M) = \left[ \bar{p}_{jk}U_{\rho_k}(M, M_j) - \bar{u}_{jk}T_{\rho_k}(M, M_j) \right] ds_j, \tag{1}$$

where $M(x, y)$ is an arbitrary point of the plane; $M_0(x_0, y_0)$ is a point on the plane boundary, by which we integrate, are Laplace transforms from the stress vector $(\bar{P}_1, \bar{P}_2)$, $(\rho_1, \rho_2), L = L_1 + ... + L_p$.

$$U_{\rho_k} = \frac{1}{2\Gamma} \left[ \delta_{\rho_k}f_0 - r_f f_1 \right], \quad T_{\rho_k} = \frac{1}{2\pi} \left[ r_{\rho_k}F_0 + \left( \frac{\partial}{\partial n} + r_{\rho_k}F_1 \right) + r_{\rho_k}F_1 \right],$$

where $\delta_{\rho_k}$ is Kronecker’s delta; $(n_1, n_2)$ are the direction cosines of outward normal to the boundary contour in $M_0(x_0, y_0)$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}; \quad n_1 = -x_0/r, \quad n_2 = -y_0/r; \quad \frac{\partial}{\partial n} = \frac{(x_0-x)n_1 + (y_0-y)n_2}{r}.$$

Here and further by the parameter, which is repeated, we will sum up

$$f_0 = K_d(q;r) + \phi(q;r); \quad f_2 = K_d(q;r) - \alpha K_d(q;r) + 2\phi(q;r); \quad \varphi(z) = \frac{K_d(z) - \beta K_s(z)}{z},$$

$$F_1 = -\frac{r^2}{r^2} f_2 + \gamma q_0 K_d(q;r); \quad F_2 = -\frac{r^2}{r^2} f_2 + q_1 K_d(q;r); \quad F_3 = -\frac{8}{r^2} f_2 - 2q_1 K_d(q;r) - \beta' K_t(q;r),$$

where $q_{1,2} = s/C_{1,2}$; $s$ is a parameter of the Laplace time transform; $C_1 = \frac{\lambda + 2G}{\rho}, \quad C_2 = \frac{G}{\rho}, \quad K_d(z), K_t(z)$ are the Macdonald functions; $\rho$ is density; $\lambda, \gamma$ are Lame constants, $\beta = \frac{C_1}{C_2}, \quad \alpha = \beta^2 = \frac{G}{\lambda + 2G}; \quad \gamma_0 = \frac{\lambda + 2G}{2 - 2\nu^2}; \quad \gamma_0 = \frac{\lambda + 2G}{2 - 2\nu^2}$

where $\nu$ is Poisson’s ratio.

The above relations are also valid for the case of the plane stress state (PSS), if the values $G, \lambda, \nu$ are replaced by $G, \lambda, \nu_0$, where $\lambda_0 = 2G/(1 - \nu); \quad \nu_0 = \nu/(1 + \nu)$.

For small values of $z$ we have

$$\varphi(z) = 1 - \frac{\alpha}{2} \left( 1 - \frac{\gamma - \ln \frac{z}{2}}{2} \right); \quad f_2 = \frac{\alpha - 1}{2},$$

where $g = 0.57721566400153$.

Then to find the Laplace transforms from displacements we will start from the potential representation of the solution and as such on the basis of (1) we take

$$\bar{u}_{jk}(M) = \left[ \bar{p}_{jk}U_{\rho_k}(M, M_j)ds_j \right] \frac{f_j}{P(M)}; \quad j=1,2, \tag{2}$$

where $P_k = P_k(M_0)$ are unknown functions, $k = 1, 2$.

We determine the stress vector $(Z_1, Z_2)$, corresponding to displacements (2), at point $M$ on the plane, the normal to which has direction cosine $N_1, N_2$.

Then

$$Z_j(M) = \left[ P_k(M_0)Q_{\rho_k}(M, M_j)ds_j \right] \frac{f_j}{P(M)}; \quad j=1,2. \tag{3}$$

The values $Q_{\rho_k}(M, M_j)$ $(k, j = 1, 2)$ are obtained on the basis of the value $T_{\rho_k}(M, M_j)$ and have the following form

$$Q_{\rho_k} = \frac{1}{2\pi} \left[ R_k N_1 F_1 + \left( \frac{\partial}{\partial N} + R_k N_1 F_1 \right) + R_k N_1 F_1 \right],$$

where

$$R_1 = \frac{x-x_0}{r}; \quad R_2 = \frac{y-y_0}{r}; \quad \frac{\partial}{\partial N} = R_k N_1 F_1.$$

The expression for the sum of stresses is presented as

$$\sigma_x + \sigma_y = -\frac{q_0}{2\pi (1 - \nu)} \left[ \frac{p \partial}{\partial N} + P_k \frac{\partial}{\partial N} - \frac{q_0}{2\pi (1 - \nu)} \right] \left[ P_k N_1 \right] K(q;r)ds.$$

In relation (3) let us direct point $M$ to the plate boundary and assume that $N_1, N_2$ are the direction cosines of the outward normal to the boundary at this point. Then, having used the Sohotski-Plemelj formula, we obtain the integral equations to determine the unknown functions.

$$\frac{1}{2} \int P_k(M) + \int P_k(M_0)Q_{\rho_k}(M, M_j)ds_j = \bar{p}_j(M), \quad j=1,2, \quad M = L. \tag{5}$$

Similarly, we can find the formula for the transform from the sum of stresses

$$\bar{\sigma}_x + \bar{\sigma}_y = \frac{1}{2\pi (1 - \nu)} \left[ \frac{\partial}{\partial N} + \frac{\partial}{\partial N} - \frac{q_0}{2\pi (1 - \nu)} \right] \left[ P_k N_1 \right] K(q;r)ds.$$

**Numerical algorithm of solution of the system of equations.** Integral equations (5) are singular. Besides, the kernels of these equations have logarithmic singularities. Present the direct numerical algorithm of solution of integral equations (5) without their preliminary regularization [14, 15]. Then for simplification we assume that $L$ is one contour. Write down the contour equation in a parametric form $\mathbf{x} = \mathbf{x}(\eta), \mathbf{y} = \mathbf{y}(\eta), A \leq \eta \leq B$.

Having put $x_0 = \alpha(\eta), y_0 = \beta(\eta); \quad A \leq \xi \leq B$ in equation (5), we obtain

$$\frac{1}{2} \int f_j(\eta) + \frac{g}{\Delta} \int f_j(\xi) F_\beta(\eta, \xi) d\xi = g_j(\eta), \quad j=1,2, \quad A \leq \eta \leq B. \tag{6}$$

where

$$F_\beta(\eta, \xi) = s'(\xi)Q_{\rho_k}(M, M_j) \left[ u_\eta(\xi), \xi - \mathbf{u}(\xi), \mathbf{u}_\xi(\xi), \mathbf{u}_\eta(\xi) \right] \left[ u_\eta(\xi), \xi - \mathbf{u}(\xi), \mathbf{u}_\xi(\xi), \mathbf{u}_\eta(\xi) \right]; \quad f_j = P(M_0) \left[ \mathbf{u}(\xi), \mathbf{u}_\xi(\xi), \mathbf{u}_\eta(\xi) \right].$$
\[ g_r (\eta) = \tilde{p}_r (M) \mid_{\eta=0, \ldots, \beta=0} \; ; \]

Let us set the values for parameter \( \eta = \eta_0, \eta = \eta_{12}, \ldots, \eta = \eta_{2N}, \) such that \( \eta_0 = A, \eta_{2N} = B; \eta_{2i+1} > \eta_i. \) Assume that \( \eta_{2i+1} = (\eta_{2i+2} + \eta_{2i})/2, j = 0, \ldots, N - 1. \)

Let us introduce the nodal points \( x_k = \alpha (\eta_k); x_N = \beta (\eta_k); k = 0, \ldots, 2N \) on the contour \( L. \) To solve equation (6) we use the collocation method, demanding that this equation be satisfied at points \( C_k = (x_k, y_k), k = 0, \ldots, 2N. \) Then we obtain a system of equations

\[ \frac{1}{2} f_j (\eta_m) + \int_{A_j} f (\xi, \eta_m) d\xi = g_j (\eta_m), \quad j = 1, 2, m = 0, \ldots, 2N. \]

Note that the integrand functions \( F_j (\eta, \xi) \) in equation (6) have in the vicinity \( \eta = \eta_m \) the Cauchy kernel and logarithmic singularity. That is, in the integral equations the integrals of the form are obtained

\[ I = \int_{A} f (\xi, \eta) d\xi dt, \]

where \( f (\xi) \) is a function discontinuous on the interval \( [A, B]; \) \( F (\xi, \eta) \) is a function which can have singularity on the interval of integration in the vicinity of point \( \xi = c = \eta_m. \) To construct a quadrature formula the integrand function \( f (\xi) \) for \( \eta_m \) \( \leq \eta \leq \eta_m, j = 1, \ldots, N \) is presented by the Lagrange interpolation polynomial

\[ f (\xi) \approx f_{j, \xi} (\xi - \eta_{j, \xi - 1}, \xi) + f_{j, \xi} (\xi - \eta_{j, \xi + 1}, \xi) + f_{j, \xi} (\xi - \eta_{j, \xi}, \xi), \]

where \( 2h_i = \eta_{2i} - \eta_{2i-1}; f_j = f (\eta_j); \)

\[ S_1 (\xi, h) = \frac{2 (\xi - h)}{2h}, \quad S_2 (\xi, h) = \frac{h^2 - 2h (\xi)}{2h}, \quad S_3 (\xi, h) = \frac{2 (\xi + h)}{2h}. \]

Then we obtain a quadrature formula of the form

\[ I = \sum_{j=0}^{2N} A_j f_j, \]

where \( A_0 = I_0 (h); A_N = I_N (h); A_{2i-1} = I_{2i-1} (h); \quad k = 1, \ldots, N; \quad A_{2N} = I_N (h) + I_{2N} (h); \quad n = 1, \ldots, N - 1; \quad I_k = \int_{A_k} f (\xi, \eta) d\xi. \)

Note that those coefficients have the following form

\[ A_{j, \xi} = h_j \int_{A_j} f (\xi - h_j, \eta_j) d\xi, \quad j = 1, \ldots, N; \]

\[ A_{j, \xi} = \frac{1}{2} h_j \left[ \int_{A_1} F (\xi - h_j, \eta_j) d\xi + h_j \int_{A_j} F (\eta_j + h_j, \eta_j) d\xi \right] \times (1 - \xi) (2 - \xi) d\xi, \quad j = 1, \ldots, N - 1. \]

Using the last formula we can also find the coefficients \( A_j \) for \( j = 0 \) and \( j = N, \) when assuming that function \( F (\xi) \) is zero for \( \xi < A \) and \( \xi > B. \)

Here the integrand function \( A_j \) in integrals has a singularity in the vicinity of point \( \xi = 0 \) for \( m = i. \) We can verify that for \( m = 2i - 1 \) in the integrand function the Cauchy kernel is absent and only logarithmic singularities can exist. The first integral, which for \( m = 2i - 1 \) is particular, is written as

\[ A_{j, \xi} = h_j \int_{A_j} F (\eta_j + h_j, \eta_j, 1) + F (\eta_j - h_j, \eta_j, 2), \quad x = 1, \ldots, N. \]

We can also verify that the integrand functions can have only logarithmic singularity in the vicinity of point \( \xi = 0. \) To calculate the integrals we consider such integrals

\[ J = \int_0^a g (\xi) d\xi = a \int_0^a g (au) du, \quad a = \text{const}. \]

Having replaced the variable \( u = z^t, q > 1, \) we obtain

\[ J = a q \int_0^1 g (z) dz. \]

Here the integrand function \( g (z), \) if we choose \( s \geq 3, \) is discontinuous and limited together with the derivative. To calculate this integral it is advisable to use Gauss quadrature formula with nodal points, which do not include point \( z = 0 \) (at this point the initial points have a singularity).

Having applied the quadrature formula (8) to the integrals in equation (6), we obtain the systems of algebraic equations for determination of the unknown functions at nodal points on the boundary

\[ \frac{1}{2} f_j + \sum_{n=0}^{2N} A_{j, m} F (\xi, \eta_m) = g_j (\eta_m), \quad j = 1, 2; \quad m = 0, \ldots, 2N. \]

Here the coefficients \( A_{j, m} = A_{j, m} (F (\xi, \eta_m) - F (\eta_m, \xi)) \) for \( \xi = \eta_m. \)

**Formula of inversion of the Laplace transform.** Consider the problem of finding the function \( f (t) \) based on its integral Laplace transform

\[ F (s) = \int_0^\infty f (t) \exp (-st) dt. \]

We will start from the exact Prudnikov formula, which connects the values of originals and their presentations [7–9]

\[ f (t) = \int_0^\infty \exp (ct/l) \sum_{n=-\infty}^{\infty} F (s_n) \exp (2\pi n t l) - R_i, \]

where \( 0 < t < l, s_n = (c + 2\pi n t l)/l, c \) and \( l \) are constants, choosing which we can improve the convergence of solution, where \( \text{Re} (c) > 0, \)

\[ R_i = \sum_{n=1}^\infty \exp (-n t) f (t + n l). \]

We assume that the known values of original and its derivative for \( t = \text{bare} f (0), f'(0). \) Let us also consider the case when the value of original for large values of the variable \( t, \) denoted by \( f_c = \text{const} \) is known. Then formula (10) can be written in the form [7]

\[ f (t) = \int_0^\infty \exp (c/l) \sum_{n=-\infty}^{\infty} F (s_n) \exp (2\pi n t l) + (1 + \gamma) \left[ f (0) + f'(0) (t + y) \right] + f_c - R_i, \]

where

\[ F_i = F (s_n) - f_c(s_n); \quad f_c = \text{const}; \quad R_i = \sum_{n=1}^\infty \exp (-n t) f (t + n l) - f_c; \quad \gamma = \frac{1}{\text{Re} (c) - 1}. \]

Formula (12) is also exact. The series in (12) is fast convergent since for assumed assumptions the coefficients in it are of the order \( F_i = O(n^{-2}) \) for \( \sigma. \) When we choose parameter \( l \) so that \( t > l f (t) = f_c \) and assume \( c > 3, \) then the value \( R_i \) can be small and it can be neglected. In particular, if \( f \) we choose parameters \( [t - f_c] < \varepsilon \) such that \( t > l \), then the residual term will be of the form

\[ R_i \leq \sum_{n=1}^\infty \exp (-n t) f (t + n l) - f_c \leq \varepsilon \exp (-c)/1 - \exp (-c). \]
When \( c = 3 \) for residual term, the estimate \( |R_2| \leq 0.0524c \) is valid.

Consider now the problem on determination of the hoop stresses which are the basis for strength calculations on the hole boundary by means of (12). The value of the Laplace transform from these stresses will be found by the BIE method, having put above the transform parameter \( s = s_n, n = 0, 1, \ldots \).

Find the value of hoop stresses at initial moment. Then consider the case when at the initial moment the displacements and velocities in the plane are zero. To find the hoop stresses we use Hook’s law, which in the case of plane deformation is of the form [16]

\[
e_n = \frac{1}{E_1}(\sigma_0 - \nu_1 \sigma_n),
\]

where \( \sigma_0, \sigma_n \) are circular and normal stresses on the boundary

\[
E_1 = \frac{E}{1 - \nu_1^2}, \quad \sigma_0 - \nu_1 \sigma_n - \text{h}op \text{ } \text{deformations \text{ on} \text{ } \text{the \text{ boundary \text{ which} \text{ are} \text{ determined \text{ by} \text{ the \text{ formula} \text{ } \sigma_0 = c_{61} + kh_n; \text{ } u_n, u_0 \text{ are} \text{ displacements \text{ in} \text{ the} \text{ hoop} \text{ and} \text{ nodal} \text{ directions \text{ on} \text{ the} \text{ boundary; \text{ k} \text{ is \text{ the \text{ curvature.}}}}}}}}
\]

Then from (13) at the initial moment on the boundary of the hole the hoop stresses will be \( \sigma_0 = \nu_1 \sigma_n \), here the normal stresses \( \sigma_n \) are known. Find the time derivative for \( t = 0 \)

\[
d\sigma_0 = \nu_1 d\sigma_n.
\]

That is the hoop stresses at the initial moment on the boundary are not zero as it is assumed in some publications [4, 17].

We assume that the load applied to the holes boundary is the following: \( p_1 \rightarrow p_1', \quad p_2 \rightarrow p_2' \); for \( t \to \infty \). Then the asymptotic values for hoop stresses \( \sigma_n \), which are necessary for the application of the formula (12), are obtained by solution of the static problem for the plane with holes and boundary stress \( (p_1', p_2') \).

Results of calculations. Consider the plane with a circular hole of radius \( a \), to whose boundary the stress \( \sigma_n = -p(1 + \cos2\theta)H(\theta) \), \( \nu_0 = 0 \) is applied, where \( \theta \) is an angular coordinate, \( H(\theta) \) is the Heaviside function. The calculated hoop stresses on the hole boundary, which are referred to parameter \( p \) depending on the dimensionless time coordinate \( t = C_2\gamma/a \) for \( \theta = 0 \) the values of Poisson’s coefficients \( \nu = 0.1, 0.3, 0.49 \) are given in Fig. 1 (values \( q \) are given at the curves). When solving the problem at 40–80 nodal points on the hole boundary were chosen. When using inversion formula (12) it was set that \( c = 3, f = 15 \) and 60–120 forms of the series were considered.

The plots in Figs. 1, 2 are originally different from the results of calculations obtained for this case in [4], using the method of finite differences by the time coordinate. Therefore, the considered problem was also solved analytically. For this purpose the case is considered when on the boundary of a circular hole the Laplace transforms from the applied stresses read

\[
\sigma_n = \sum_{n=0}^\infty p_1 \cos n\theta; \quad \xi_m = \sum_{n=0}^\infty f_1 \sin n\theta.
\]

The analytical solution of the problem is constructed in [1, 4]. The transforms from the hoop stresses on the hole boundary are determined by the formulas

\[
\sigma_0 = \sum_{n=0}^\infty (A_n f_1 + B_n g_1) \cos n\theta,
\]

where \( A_n, B_n \) are the constants determined from the system of equations

\[
\begin{align*}
A_n f_1 + B_n g_1 &= p_n, \\
A_n f_1 + B_n g_1 &= 2k_n.
\end{align*}
\]

Here

\[
f_1 = s_1 \left( C + c_1^2 \right) K_1(s_1 r); \quad g_1 = -\frac{n^2}{r} \left( 1 - s_1^2 \right) K_1(s_1 r); \\
f_m = 2n^2 \left( 1 - s_1^2 \right) K_1(s_1 r); \quad g_m = \left( n^2 + s_1^2 \right) K_1(s_1 r); \\
f_m = -n^2 \left( 1 - s_1^2 \right) K_1(s_1 r),
\]

where \( r = a; \quad s_1 = s_1/C_1; \quad j = 1, 2; \quad \partial K_1(z) = K'_1(z); \quad \partial^2 K_1(z) = K''_1(z); \quad C = \nu/(1 - 2\nu). \)

The originals of the stresses were found by the given inversion formulas of the Laplace transform. The stresses obtained on the basis of this solution are practically exact since the errors in calculations arise only due to neglecting the residual term \( R_2 \), which in this case is small in consequence of substantially selected parameter \( t \). The stresses calculated by analytical solution are given in Figs. 1, 2 by dots. That is, the used above BIE method guarantees the practically exact results.

Consider a plane with an elliptic hole with semi-axes \( \sigma_n = -pH(\theta) \) and \( p = \text{const} \) whose boundary the normal stresses are applied.

The multipliers for the dynamic stress concentration factors (SCF) are calculated, that is, the relations of SCF under dynamic and static loads, for an elliptic hole with a ratio of semi-axes 0.5 for different values of Poisson’s ratio \( \nu \). Curves 1–5 in Fig. 3 correspond to the results of calculations at the boundary of the hole at point \( A \) (where the radius of curvature is minimal), which are carried out at \( \nu = 0.1, 0.2, 0.3, 0.4, 0.45 \). Curves 1–5’ correspond to the annular stresses assigned to \( p \) at point \( B \) (where the radius of curvature is maximum).

A rectangular hole with semi-axes \( a, b \) and rounded angles of a quadrant of radius \( R \), to whose boundary normal stresses \( \sigma_n = -pH(\tau) \) are applied, is considered. The results of calculations for \( a = b, R = ma, \nu = 0.3 \) are given in Fig. 4. Here near the curves the value of \( m \) is given, which is assumed to be 0.2, 0.5,
Fig. 3. Relative stresses on the boundary of an elliptic hole

Fig. 4. Relative stresses on the boundary of a square hole with nodal apexes

0.8. The solid lines correspond to the hoop stresses, related to $p$, at the center of a rounded apex and the dashed ones -- in the middle of sides $I$. In the same figure the straight lines represent the values of relative stresses under static loading that is based on algorithm [17].

Similar results of calculation are obtained for rectangular holes for relation of the sides $a/b = 1, 2, 4$, when the apexes are rounded by quadrants of radius $R = 0.2A$. The calculated relative maximal hoop stresses on the boundary are given in Fig. 5 by solid lines, near which the relations of sides are presented. The dotted lines represent the relative stresses in the middle of the shorter side.

The calculated dynamic stresses at the values of the time coordinate $t > 6$ go to the set mode and coincide with the corresponding values for the static problem. From Figs. 4, 5 it is seen that the maximum dynamic stresses occur in the region of rounding the vertices of the rectangle. In the middle of the sides of the rectangular and square holes stresses are low.

The performed calculations show a significant influence of the mechanical characteristics of the materials on the concentration of dynamic stresses near the holes (for the considered problems at static loads the stresses do not depend on elastic constants). For materials with low Poisson’s ratios (cast iron, concrete, carbon steels) SCF significantly increases for materials where Poisson’s ratios are close to 0.5. Such materials include, in particular: lead, cold-rolled brass, molybdenum.

Conclusions. The paper presents an approach to solution of the plane boundary dynamic problems of elasticity which is based on the Laplace transform and BIEM. Singular integral equations, whose kernels contain additionally logarithmic singularities, are solved by a direct method (without their regularization). The proposed formula for the inverse of the Laplace transform makes it possible to calculate the stress concentration under dynamic loads with controlled accuracy. In particular, the above dynamic stresses calculated for large values of time coincide with corresponding static stresses, and the stresses for small values of time agree well with corresponding values determined analytically. The series in inversion formula proved to be fast convergent. A residual term $R_\infty$ not considered in dynamic problems of elasticity can be small. From the presented calculations we can see that dynamic SCF increases with the increase in Poisson’s ratio. In the holes of rectangular form with rounded angles, the maximal stress concentration appears near the rectangle apex.

References.
Визначення концентрації напружень біля отворів при динамічних навантаженнях

О. Максимович¹, Т. Соляр², А. Судаков³, И. Назар⁴, М. Поліщук⁵

¹ − Університет технологічно-природничий, м. Бидгощ, Республіка Польща, e-mail: olesia.maksymovych@utp.edu.pl
² − Інститут прикладних проблем механіки та математики імені Я. С. Підстригача НАН України, м. Львів, Україна
³ − Національний технічний університет «Дніпровська політехніка», м. Дніпро, Україна
⁴ − Національний університет «Львівська політехніка», м. Львів, Україна
⁵ − Луцький національний технічний університет, м. Луцьк, Україна

Мета. Розробити методику розрахунку напружених стану пластинчатих елементів конструкцій з отворами при динамічних навантаженнях з контролюваною точністю.

Методика. Дослідження виконано на основі перетворення Лапласа та методу інтегральних рівнянь.

Результати. Запропоновано підхід до визначення динамічних напружень біля отворів у пластинках, що включає: перетворення Лапласа за часовою координатою; числовий метод визначення трансформант переміщень і напружень на основі методу інтегральних рівнянь; знаходження орігіналів на основі адаптованої до динамічних задач теорії пружності формул обернення Пруднікова. Задача визначення зображень Лапласа для переміщень зведена до розв’язування сингулярних інтегральних рівнянь. Розв’язування інтегральних рівнянь проведено чисельно на основі підходів, розроблених в методі граничних елементів. Для знаходження орігіналів переміщень і напружень використані адаптовані до динамічних задач формули обернення Лапласа, що запропоновані Прудніковим. Проведено дослідження динамічних напружень біля отворів різної форми.

Наукова новизна. Розроблено новий підхід до регуляризації формул Пруднікова для обернення перетворення Лапласа багажно до динамічних задач теорії пружності. Для його реалізації: покращена збіжність рядів Фур’є на основі попередньо встановлених напружень у початковий момент часу; ураховано залишковий член у формули обернення.

Практична значимість. Розроблена методика розрахунку концентрації напружень біля отворів довільної форми у пластинчатих елементах конструкцій при динамічних навантаженнях. Запропонований підхід дозволяє визначати напружения з контролюваною точністю. Виконані дослідження біля кругових і многокутних отворів із закругленими вершинами можуть бути використані в розрахунках на міцність динамічно навантажених пластин. Проаналізовано вплив коефіцієнта Пуассона на концентрацію динамічних напружень.

Ключові слова: концентрація динамічних напружень, метод граничних інтегральних рівнянь, перетворення Лапласа, формули обернення

Recommended for publication by D. L. Kolosov, Doctor of Technical Sciences. The manuscript was submitted 24.12.20.