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MODELING OF THE EARTH'S SURFACE SUBSIDENCE DURING ITS UNDERMINING BY STOPING IN COAL MINES

Purpose. To substantiate methods for modeling the earth's surface subsidence in the process of its undermining by stoping in the Western Donbas mines. The methods make it possible to develop the automated system determining areas and volume of the earth's surface flooding to minimize hydroecologic risks while closing down mines in the Western Donbas.

Methodology. The work substantiates methods to model processes of the earth's surface subsidence helping identify areas of the flooded surface as well as its volume. Therefore, the two types of models (the interpolation model and polynomial one) to construct surfaces with a regular network have been considered. The models make it possible to get adequate idea of surfaces; moreover, they are the basis to calculate volumes, represented in the form of total unit prisms in terms of network node.

Findings. The developed mathematical and algorithmic apparatus has made it possible to develop effective models of the earth's surface and an aquifer as well as to calculate the zone volumes between the reference earth's surface and the consolidated one as well as between the aquifer to identify the flooded areas.

Originality. Methods to model processes of the earth's surface subsidence for hydroecologic risk minimization in the process of mine closing down are substantiated.

Practical value. The work develops hardware and software to provide efficient modeling of the earth's surface subsidence while mining coal seams and to reduce errors while calculating the immersed surface zone volumes.

Keywords: *automation, subsidence, surface modeling, immersion, hydroecologic risks*

Introduction. Due to the current political and economic situation and in accordance with the Strategy of coal industry development up to 2030, it is planned to close down a substantial number of coal mines. In this context, deficient capacities of coal mining are scheduled to be provided while increasing output of those mining enterprises having higher economic performance. Reduction of the lifetime of mine complexes as well as their closure is a result of the intensification of mine field development. However, the earth's surface deformation is the key problem of underground extraction of mine fields since the deformation depends upon the total thickness of coal seams being mined. In terms of certain Western Donbas mines, it may achieve seven meters. After the mines are closed down and water pumping from mine workings is terminated, underground water level rises up to the surface; thus, some areas of the mine field will turn out to be immersed. Mire formation will result in severe negative ecological implications [1].

Literature review. Owing to the progress in the development of computer facilities as well as achievements in the advance of the required software, the updated computer-assisted solutions become more and more popular among the top mining companies and research institutes in many world countries [2]. In Ukraine, scientists and researchers of Dnipro University of Technology are the first ones who started implementing the solutions while modeling rock mass around stopes and development workings in the process of coal deposit extraction [3]. The processes result in rock shift through formation over the worked-out area. Values of maximum vertical and horizontal displacements of the earth's surface are the basic parameters of the shift through [4]. In the process of coal extraction, geometrization of mining and surveying characteristics of a mine field is quite a complex task. The matter is that in our case it is required to take into consideration the number of seams being mined, their thickness, spatial coordinates of pillars and prospecting wells in addition to planar coordinates of land plots within the mine field surface [5]. The abovementioned predetermines the necessity to set adequate database.

Unsolved aspects of the problem. DTEK Pavlohradvuhillia PJSC made efforts to identify separate flooded areas with further recultivation. However, the activities were of the isolated nature due to the lack of methods helping develop the automated system to identify the flooded areas at the earth's surface as well as corresponding volumes between subsidence zones and the aquifer surface.

Accuracy of methods to calculate the zones is another important problem connected with their dimensions. Hence, even minor calculation errors may factor into significant economic loss in the context of considerable volumes of inundable surfaces.

Purpose is to substantiate methods of the earth's surface subsidence in terms of its undermining by means of coal extraction operations in the Western Donbas mines. The methods help develop the automated system to identify zones and volumes of the earth's surface to minimize hydroecologic risks while closing down the Western Donbas mines.

Methods. In the context of the paper**,** development of the earth's surface and aquifer to identify the zones and volumes of the mine field terrain inundation is the key modeling mission.

To select a method to construct a complex surface, the following basic criteria are applied:

- the model adequacy for a real-life object;

- possibility for effective formation of the model as well as the complex surface construction;

- opportunity of 3D visual representation of the model.

National and foreign GIS packages are available. They make it possible to model the earth's surface while mining. Usually, the packages use network types: irregular network (i. e. triangulated) and regular one (i. e. uniformly rectangular) [6].

Irregular triangulated network relies upon a system of non-intersecting triangles where the initial geodetic points are corners. In this case, a terrain is represented with the help of the faceted surface; Delaunay triangulation is applied to model the terrain [7] (Fig. 1).

The following theorems are the basis for the majority of algorithms to construct Delaunay triangulation:

1. Obtaining random triangulation on the same points while reconstructing pairs of points failing to meet the requirements of Delaunay condition. A sequence for Delaunay triangulation construction is as follows: construction of the initial triangulation with its following improvement.

2. Total of all minimum angles should be maximum one.

3. Total of radii of circles, circumscribed about the triangles, should be minimum one.

Delaunay triangulation may be obtained while optimizing totals of minimum angles or radii which, being the combined characteristics, occur within the theorems.

Check of Delaunay condition for the specified pairs of triangles is one of the most important operations performed to construct the triangulation.

Type of structural data, applied for triangulation, is essential in the process of the earth's surface modeling. Selection of the structure type is of particular importance in the context of complex mining models with large databases influencing the model implementation quickness during the triangulation completion.

In terms of triangulation, structures are shaped with the help of such basic objects as edges (segments), triangles, and nodes (points and corners).

Non-necessity of the initial data transformation is one of differential advantages of the triangular models. Such a model excludes the introduced errors which may originate in the models if alternative methods are applied.

However, when complex earth's surfaces, undermined by subsurface mining and having steep slopes and altitude differences are constructed, Delaunay triangulation method cannot provide largely adequate representation of realistic conditions.

It can be demonstrated in terms of a separate site of a mine field shown in Fig. 2 in a plan view and in a 3D graphics. In this context, the surface should follow accurately a bench edge line (Fig. 2, *a*). Modeling of the site using the abovementioned construction rules of Delaunay triangulation (Fig. 2, *b*) dem-
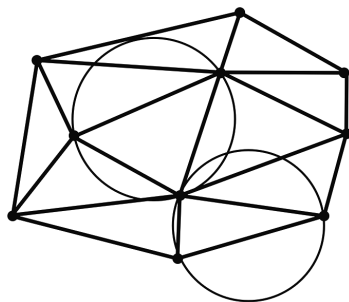
onstrates angularity origination. It means that the surface, being modeled, is impaired.

The problem can be solved if the edge lines are added by the intermediate points. It should be taken into consideration that such point concentration is not always helpful to avert construction errors.

Moreover, the actions will prolong the operating period of algorithms. In addition, it levels up the method advantage in the form of nonavailability of initial data transformation.

Triangulation-based models are implemented as part of GIS packages by ESRI Company [8], software system SURPAC Gemcom [9].

Irregular system allocation of control points uses piecewise-polynomial interpolation where both orthogonal and non-orthogonal polynoms are applied as well as Fourier series, the weighted moving average methods and other ones. λ function, being inversely proportional to the distance from the considered point to a control one to some *r* power, is applied as a weight function in terms of the weighted moving average. In practice, $r = 2$ value is the most applicable one, i. e. spatial interpolation procedure is used called as inverse distance squared method.

Moreover, local-stochastic methods of spatial interpolation are applied known as kriging interpolation or just kriging. The method relies upon the consideration of regularities of statistical structure of spatial distribution of the analyzed variable owing to which it is more preferable to compare with local deterministic methods including piecewise-polynomial interpolation method as well as the weighted moving average one. The ability to substantiate a radius around the considered point which should be involved in terms of interpolation; a type of the weighted moving average; as well as the potential to evaluate the accuracy of spatial interpolation are among the advantages. Kriging is based upon the construction of variograms. In the context of a surface, varying drastically, the variograms will be unstable; moreover, their use may result in significant errors.

Plenty of methods are available to describe surfaces analytically. They are taken as the basis for spatial interpolation of
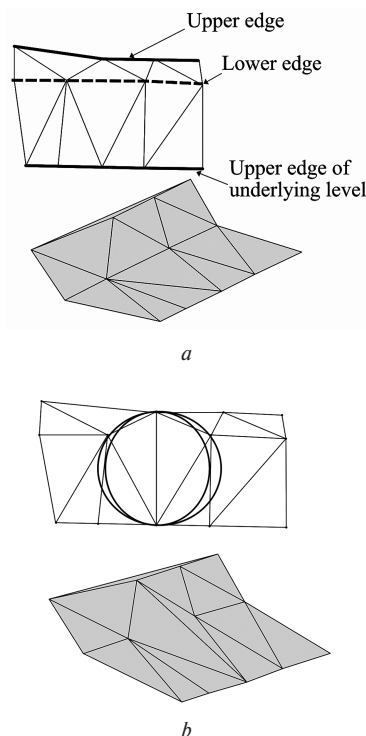

a

b

Fig. 2. Example of a surface construction:

a — is relevant representation; b — is with the use of Delaunay triangulation

Fig. 1. Triangulation meeting the requirements of Delaunay condition

the control (reference) points. In this context, spatial interpolation results, obtained with the use of different methods, vary. Sometimes, the variation is rather essential. A network step-size control is a distinct problem.

If regular altitudes are applied then the terrain models turn out to be the simplest and convenient ones for various calculations. In this case, an area in the X and Y coordinates should be of a rectangular form. A network of vertical (if $x_1, x_2, ..., x_n$) and horizontal (if $y_1, y_2, ..., y_m$) lines is specified within it. A function is constructed within each rectangular (more often squared) box $[x_i, x_{i+1}] \times [y_j, y_{j+1}]$ according to z altitudes specified within the box corners. Actually, it is the network dimensioned in accordance with the accuracy requirements of the definite current task.

Spatial interpolation of the control points is based upon numerous methods of analytical description of surfaces.

A network step-size control for the constructed model is a distinct problem as well.

The use of interpolation model and polynomial model is the most relevant solution for our problem.

Construction of the models is based on a regular network. They make it possible to evaluate to a high precision and calculate dimensions of the constructed surfaces basing upon the use of numerical techniques.

Spatial interpolation of point data relies upon the selection of analytical model of a surface. In general, the surface is a function of two variables $Z = f(X, Y)$ specified within the certain spatial area with the help of point data. The number of the points and their allocation may be absolutely random. The interpolation task is to construct functions for the selected periphery using the data. Namely, it is required to set calculation algorithm for $f(X, Y)$ function in any point having X and Y coordinates. Since it is impossible to describe a surface within the whole territory using one function, local interpolation methods are applied for spatial interpolation of surfaces with regular allocation of reference points. A value of a variable within the considered point (node) is determined by means of measurement results within the points in certain neighbourhood of the point rather than the whole set of available data. In this context, polynomial interpolation is applied.

Computer application to evaluate the surfaces is to use numerical techniques, i.e. representation of the volume as the total of volumes of unit prisms. In terms of such interpretation, volumetric palette method, used for manual calculations, is the most relevant for automation. Manually, the volumetric palette method is applied to identify volumes when a surface is represented in terms of numerical labels along the rectangular network or when a plain terrain is represented in contours. It is unacceptable to calculate volumes while using numerical labels on the lines of the upper and lower bench edges.

Moreover, there are methods of correct geometric figures. They may be applied if only any solid is represented in the form of such standard geometric bodies as a prism, cone, pyramid, conoidal frustum, top, integral prismatoid, antiprism, and pile. It is a problem to select adequate geometric figure. Particularly, the problems arise if it is necessary to calculate volumes of the worked out cross-over when the mine section is of an irregular shape. Generally, it is required to solve a problem concerning the evaluation of mine section volumes in term of lines of upper and lower bench edges with different areas and spatial allocation relative to each other.

Despite the disadvantages of the considered methods, the ideas to replace a mine section by a geometric figure and represented its geometry as the total of unit prisms are the basis for the development of innovative universal methods making it possible to improve significantly the accuracy of determination of volumes of the mined sections having any configurations, i.e. in terms of any spatial allocations of surface lines.

Results. Control points with X, Y, and Z coordinates are the source information to construct surfaces. In terms of the task, X and Y are wellhead coordinates; and Z is a reference mark of the wellhead and an aquifer. Generally, Z is a random mark used to construct the surface.

A rectangle, described relative to the ultimate coordinates of the reference points (i.e. X_{min}, X_{max}, Y_{min}, Y_{max}) is the study area [10].

The rectangle is covered by a uniform network with the specified step [11].

Interpolation model. While constructing the interpolation model, Z factor in the ij^{th} node of the uniform network is calculated with the help of inverse distance method using the formula

$$Z_{ij} = \sum_{k=1}^{N} Z_k d_k^{-\alpha} \Big/ \sum_{k=1}^{N} d_k^{-\alpha},$$

where N is the number of points neighbouring the ij^{th} node; Z_k is the mark value at k^{th} point; d_k is the distance from the nearest k^{th} well to a node of ij network; and α is the power interpolation exponent.

In this context, search area of the nearest wells is performed in the plan (i.e. in terms of X and Z coordinates) being limited by a circle with a centre in the interpolated node which radius is equal to the specified distance. To avoid influence of the effect, connected with the concentration of points in certain directions and their absence in other ones, the circle around the uniform network node is divided into n sectors. It is expedient to assume value 8 as n. One point, being the closest to the node, is within each sector (Fig. 3).

Interpolation network is the basis to construct column and wire-frame models represented by the research.

A column model is constructed based upon the obtained values within the interpolation network nodes. First, squares, contacting each other, are formed. Their centres are in the interpolation points; and their sides are equal to the interpolation network step. As a result, a surface is shaped in the form of rectangular prism whose height is comparable with the calculated value of Z index being (Fig. 4).
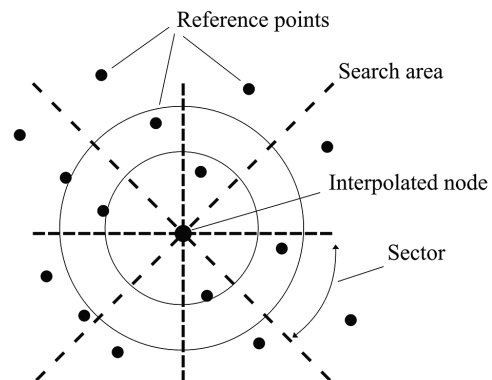

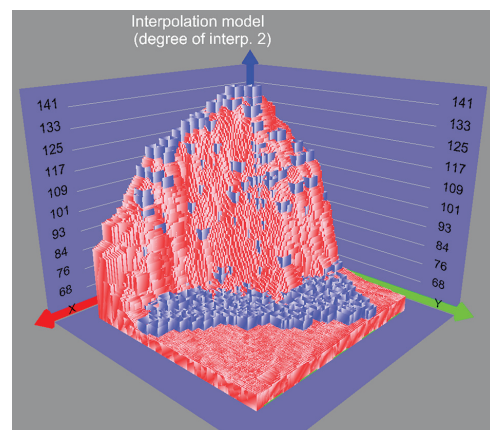*Fig. 3. Search for the nearest points in the plan*
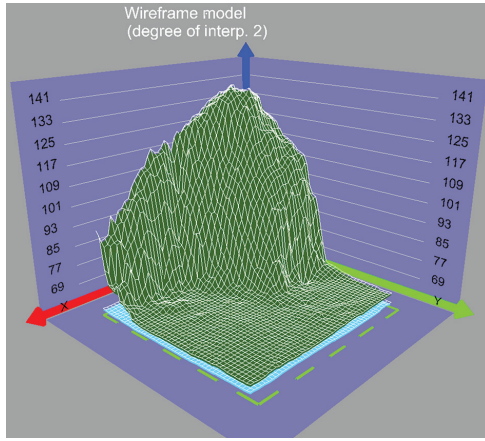

Fig. 4. Column model of the surface

Fig. 5. Wire-frame model of the surface

The wire-frame model construction needs connection of the lines by means of interpolation points. The link is performed in two orthogonally related directions (Fig. 5).

Polynomial model. Construction of a 3D polynomial model $z = f(x, y)$ involves an algorithm by A. S. Zelenskiy [12].

As for the 5th degree polynom, the dependence may be represented as follows

$$z = A_0 + A_1 x + A_2 y + A_3 x^2 + A_4 xy + A_5 y^2 + A_6 x^3 + A_7 x^2 y + A_8 xy_2 +$$
$$+ A_9 y^3 + A_{10} x^4 + A_{11} x^3 y + A_{12} x^2 y^2 + A_{13} xy^3 + A_{14} y^4 + A_{15} x^5 +$$
$$+ A_{16} x^4 y + A_{17} x^3 y^2 + A_{18} x^2 y^3 + A_{19} xy^4 + A_{20} y^5,$$

where A_0, A_1, ..., A_{20} are the polynom coefficients to be identified.

Depending upon a polynom degree, the number of coefficients in terms of the unknowns (linear equations) is identified using the formula

$$K = \frac{(\alpha + 1)(\alpha + 2)}{2},$$

where $K = 21$ is for the 5th degree polynom.

According to A_i parameters, the system, consisting of 21 linear equations, is obtained

$$\sum_{s=0}^{20} a_{\ell S} A_S = b_\ell, \quad \ell = 0, 1, \dots, 20,$$

i. e.

$$\begin{cases} a_{00} A_0 + a_{01} A_1 + \dots + a_{0\,20} A_{20} = b_0 \\ a_{10} A_0 + a_{11} A_1 + \dots + a_{1\,20} A_{20} = b_1 \\ \overline{a_{20\,0} A_0 + a_{20\,1} A_1 + \dots + a_{20\,20} A_{20} = b_{20}} \end{cases},$$

where $a_{\ell S}$ are coefficients of the linear equations; b_ℓ are free terms of the equation system; ℓ is a line number (of an equation); and S is a column number.

Simple dependences have been derived to determine $a_{\ell S}$ coefficients in terms of the A_0, A_1, ..., A_{20} unknowns, and free terms b_ℓ

$$a_{\ell S} = \sum_{i=1}^{n} x_i^{k_\ell + k_s} y_i^{p_\ell + p_s}; \quad b_\ell = \sum_{i=1}^{n} z_i x_i^{k_\ell} y_i^{p_\ell},$$

where k_ℓ and p_ℓ are exponents depending upon a line number ℓ; k_S and p_S are exponents depending upon a column number S

$$k_\ell = c_1(c_1 + 1)/2 - (\ell + 1); \quad k_S = c_2(c_2 + 1)/2 - (S + 1);$$
$$p_\ell = \ell - c_1(c_1 - 1)/2; \quad p_S = S - c_2(c_2 - 1)/2;$$
$$c_1 = \text{int}\left(\frac{1 + \sqrt{8(\ell + 1) - 7}}{2}\right); \quad c_2 = \text{int}\left(\frac{1 + \sqrt{8(S + 1) - 7}}{2}\right).$$

Fig. 6 represents spatial implementation of a 3rd degree polynom.

Construction of wire-frame and polynomial models involves displaying of several models to one scene; for instance, displaying of the earth's surface and aquifer.

Fig. 7 shows displaying of two wire-frame models together with the peripheries in terms of two selected seams.

The earth's surface subsidence takes place within the specified peripheries.

To implement the methods, *Model* software has been developed in the Visual C++ 2015 programming language with the use of OpenGL library. The software includes two basic modules [13]:

1. Database operation − data storing, editing, and deleting.
2. Models − data visualization with the help of digital models.

Database module involves:
- objects (coal mines in this context);
- prospecting wells;
- surfaces, including the earth's surface and aquifer;
- peripheries of production units for each seam;
- the seam points including occurrence depth and the seam thickness in addition to X and Z coordinates.

In turn, the data consist of:
- the well name;
- X coordinates, m;
- Y coordinates, m;
- $Z1$ coordinates, m (wellhead mark);
- $Z2$ coordinates, m (the well depth);
- H, m (the coal seam thickness).

The data are complemented by the information concerning the undermining peripheries being available within each seam:
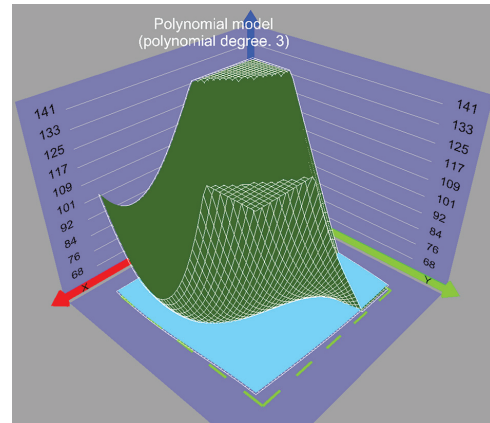- name of the periphery point;

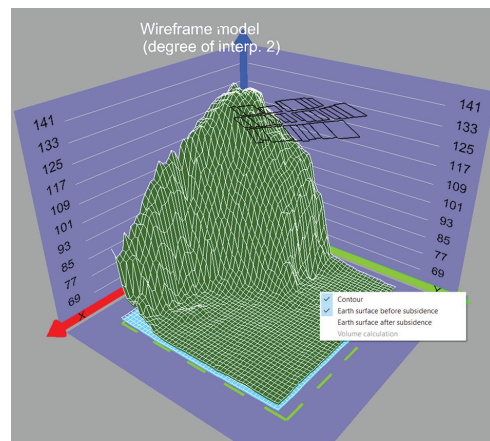*Fig. 6. Polynomial model of a surface*

Fig. 7. Wire-frame models of the earth's surface and aquifer inclusive of the peripheries

- name of the coal seam where the periphery is allocated;
- *X* coordinate of the periphery point, m;
- *Y* coordinate of the periphery point, m.

It is planned that the database will consist of the following tables:

1) objects – a table, involving data on the mines. The data are stored by the program. Owing to the fact, the program can display and process data on several mines;

2) surfaces – the table stores data on coal seams in terms of each mine;

3) points (general data) – since there is much information, concerning each well, it would be better to share the data according to their content. The general data may involve *X* coordinate, *Y* coordinate, and *Z*1 coordinates of the earth's surface;

4) points (data on the seams) – all other information, i.e. occurrence depth of the minerals for each seam;

5) periphery points – since each seam contains a set of undermining peripheries, a separate table is required to store such data as a point name, a seam name, *X* coordinate, *Y* coordinate.

Hence, when formulation of data, to be stored in the data base, is over, it is time to build up tables with the help of a constructor.

To do that, the data were complemented and organized in the following tables:

1. Object_mines – Surfaces – where Object_id field is combined with the help of connection one to many. The abovementioned makes it possible to assign surfaces to each Object (i. e. a mine).

2. Surfaces – Contour – where Point_id field is also combined with the help of connection "1 to many". The abovementioned makes it possible to combine the periphery points with one object.

3. Object_mines – Points_main – Object_id field helps combine a set of general information of well with the specific mine.

4. Points_main – Points_on_surface – Point_number field helps assign to each well from Points_main table representing the general data concerning wells and occurrence depth of each seam from Points_on_surface.

Fig. 8 demonstrates the developed database.

Database operation helps correct information within each of the tables. ADO universal technique has been developed for that purpose.

Models module includes:
- initial data;
- interpolation model;
- wire-frame model;
- polynomial model.

Surface volume determination relative to the specified surface and with the use of interpolation or polynomial methods is performed as the total of unit prisms by means of the following formula

$$V = \Delta x^2 \cdot \sum_{i=1}^{k} h_i,$$

where *k* is the number of points of the specified network; Δx is the network step; h_i is the height of the ith unit prism being identified.

In terms of our case, areas and volumes between the initial earth's surface and the compacted one are calculated as well as between the aquifer and the compacted surface to determine flowage lands. Fig. 9 shows the calculation results concerning the areas and volumes.

Here you can find the initial data concerning the interpolation network as well as the calculation results. The following should be mentioned: the smaller the network step is, the more accurate computations are concerning the volumes between the specified surfaces. 30 m network step is optimal for the case.

It has been obtained while varying the network step in terms of the developed program. In terms of further decrease in the step, the volumes remain constant supporting maximum accuracy of the calculation results.

Hence, undermining of the four coal seams while closing down the mines results in the following volumes and areas of the surface variation:

1. The area between the initial surface and the compacted one is 10 659 600 m² (10.66 km²).

2. The volume between the initial surface and the compacted one is 25 778 655 m³ (0.257 km³).

3. The area between the aquifer and the compacted surface is 1 474 200 m² (1.47 km²).

4. The volume between the aquifer and the compacted surface is 1 339 708.41 m³ (0.0013 km³).

The data demonstrate that processes of the earth's surface subsidence will occupy almost 10.66 km² area; 1.47 km² of them will be flooded.

Table demonstrates the basic calculation results.

The Table explains that the area and volume between the initial surface and the compacted surface vary depending upon the coal seam dimensions. At the same time, the volume between the aquifer and the compacted surface remains almost invariable since a section of the changed surface with the aquifer is not available after one seam extraction.

Conclusions. The authors are the first who have substantiated modeling methods of the earth's surface subsidence while undermining it by means of stoping in the Western Donbas mines. The authors propose optimum models to construct surfaces with regular networks (the interpolation model and polynomial one). The models make it possible to get an adequate insight into surfaces; moreover, they are the basis to calculate the volumes represented in the form of the total of unit prisms in terms of each network node. Spatial interpolation of the point data relies upon the selection of a surface analytical
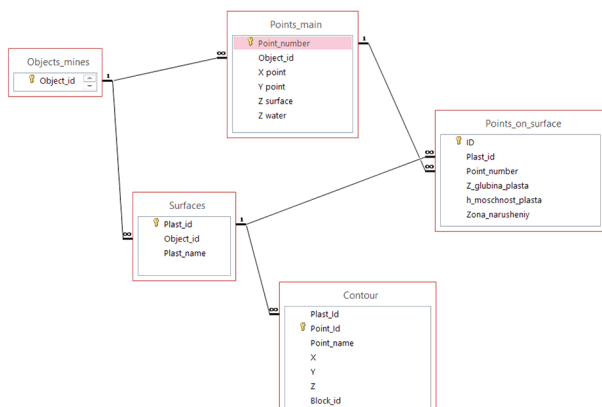

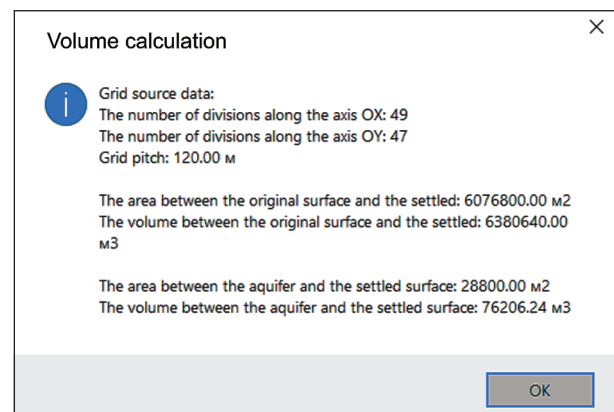*Fig. 8. Scheme of the developed database*

Fig. 9. Calculation result concerning the areas and volumes

Calculation results

Index	C4 seam	C5 seam	C6 seam	C8 seam	All the seams taken together
The area between the initial surface and the compacted surface, km^2	6.07	7.17	9.17	2.08	10.66
The volume between the initial surface and the compacted surface, km^3	0.064	0.0075	0.0096	0.0022	0.257
The area between the aquifer and the compacted surface, km^2	0.032	0.032	0.032	0.032	1.47
The volume between the aquifer and the compacted surface, km^3	0.000073	0.000073	0.000056	0.000056	0.0013

model. Since it is impossible to describe the whole surface by means of one function, local interpolation methods are applied for spatial interpolation of surfaces with regular allocation of reference points. In this context, a value of a variable within the considered point (node) is determined by means of measurement results within the points in certain neighbourhood of the point rather than the whole set of the available data.

Software to model surfaces was developed by the authors in Visual C^{++} 2015 program language for operating system Windows with the use of tools of open graphic library OpenGL. The mathematical and algorithmic apparatus, developed by the authors, has made it possible to develop efficient models of the earth's surface and aquifer as well as to calculate the volumes between the initial earth's surface and the compacted surface, and between the aquifer and the compacted surface to identify flowage lands.

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Моделювання опускання земної поверхні при її підробці очисними роботами на вугільних шахтах

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Мета. Обґрунтування методики моделювання опускання земної поверхні при її підробці очисними роботами на шахтах Західного Донбасу, що дозволяє розробити автоматизовану систему визначення зон і обсягів затоплення поверхні землі для мінімізації гідроекологічних ризиків при закритті шахт Західного Донбасу.

Методика. У даній роботі обґрунтовується методика моделювання процесів осідання земної поверхні, що дозволяє визначати площі й обсяги затопленої поверхні. Для цього розглянуті два типи моделей побудови поверхонь із регулярною сіткою: інтерполяційна й поліноміальна. Дані моделі дозволяють отримати адекватне уявлення поверхонь, а також є базисом для підрахунку обсягів, представлених у вигляді суми елементарних призм по кожному вузлу сітки.

Результати. Розроблений математичний і алгоритмічний апарат дозволив створити ефективні моделі земної поверхні та водоносного горизонту, а також підрахувати обсяги зон між вихідною земною поверхнею та осілою, а також між водоносним горизонтом і осілою поверхнею для визначення площ і обсягів зон ділянок затоплення.

Наукова новизна. Полягає в обґрунтуванні методики моделювання процесів осідання земної поверхні для мінімізації гідроекологічних ризиків при ліквідації вугільних шахт.

Практична значимість. Полягає в розробці апаратного та програмного комплексу для забезпечення ефективного моделювання осідання земної поверхні при відпрацюванні вугільних пластів, а також для зменшення похибок при підрахунку обсягу зон затопленої поверхні.

Ключові слова: *автоматизація, осідання, моделювання поверхні, затоплення, гідроекологічні ризики*

Моделирование опускания земной поверхности при ее подработке очистными работами на угольных шахтах

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Цель. Обоснование методики моделирования опускания земной поверхности при ее подработке очистными работами на шахтах Западного Донбасса, позволяющей разработать автоматизированную систему определения зон и объемов затопления поверхности земли для минимизации гидроэкологических рисков при закрытии шахт Западного Донбасса.

Методика. В данной работе обосновывается методика моделирования процессов оседания земной поверхности, позволяющая определять площади и объемы затопленной поверхности. Для этого рассмотрены два типа моделей построения поверхностей с регулярной сетью: интерполяционная и полиномиальная. Данные модели позволяют получить адекватное представление поверхностей, а также являются базисом для подсчета объемов, представленных в виде суммы элементарных призм по каждому узлу сети.

Результаты. Разработанный математический и алгоритмический аппарат позволил создать эффективные модели земной поверхности и водоносного горизонта, а также подсчитать объемы зон между исходной и осевшей земной поверхностью, а также между водоносным горизонтом и осевшей поверхностью для определения площадей и объемов зон участков затоплений.

Научная новизна. Заключается в обосновании методики моделирования процессов оседания земной поверхности для минимизации гидроэкологических рисков при ликвидации угольных шахт.

Практическая значимость. Заключается в разработке аппаратного и программного комплекса для обеспечения эффективного моделирования оседания земной поверхности при отработке угольных пластов, а также для уменьшения погрешностей при подсчете объема зон затопленной поверхности.

Ключевые слова: *автоматизация, оседание поверхности, моделирование поверхности, затопление, гидроэкологические риски*