DEVELOPING A MATHEMATICAL MODEL OF LINKAGE PARAMETERS OF AIR FLOW IN A FILTER BOX

**Purpose.** Using the mathematical model, to study the air velocity flow in a filter box of a respirator taking into account its influence on the geometric parameters and protective efficiency.

To define the influence of geometric parameters of a filter box on kinematic characteristics and its protective efficiency.

**Methodology.** To develop the mathematical model, the method of analysis and synthesis was applied. In the course of the analysis, the filter box was divided into a number of zones for estimation of the motion process of dust and gas flows. The synthesis method was used to describe the motion of a two-phase dust-laden flow in the cylindrical coordinate system $r, z, \theta$ in Eulerian variables.

**Findings.** Geometrical parameters of a filter box of a respirator have been optimised including the outlet port diameter; confusor height; outlet port shifting regarding the filter box centre.

**Originality.** A new technique for optimization of the geometrical parameters of a filter box of a respirator based on the mathematical model was presented. Dependencies of distribution of air flow velocities for filter boxes of different geometry have been obtained and dependencies between its aerodynamic and geometrical parameters have been established.

**Practical value.** The results allow defining the parameters of a filter box with minimum resistance to the air flow and improving protective efficiency of the respirator.

**Keywords:** respirator, filter box, mathematical modelling, finite-element method, method of local variations, least square method

**Introduction.** Protective properties of respirators are determined by insulating properties of a half-mask and parameters of a filter box with a filter. The latter ones influence the distribution of air flows in them considerably [1], which results in uneven flow distribution throughout the filter as well as in occurrence of dead zones which do not participate in the filtration process, thus, decreasing a service period of a dust respirator significantly. The air flow is influenced by the ratio of the sizes of the inlet and outlet ports inside the filter box along with their location regarding each other [2]. At the same time particles settle unevenly on some filter segments whereas others do not participate in dust collection [3, 4]. Therefore, design solutions contribute to uneven aerosol particle deposition over the filter, which results in decreasing its protective efficiency. Thus, research on dependence of linkage parameters of the air flow on changes in the filter box geometry is an urgent task.

**Literature review.** Regarding the issue under consideration, research works [5, 6] were reviewed. Most of them deal with choosing filtering material and determining the influence of their structure (the fibre diameter, packaging density and width) on the respirator protection coefficient. The latest research is directed at studying the issues of aerosol particle deposition on fibres of various sizes through computer modelling [7, 8]. They pay much attention to calculations of density of multilayer filter packaging. Studying of changes in filter resistance when polydisperse particles are settling is also of interest.

Work [9] considers issues of even distribution throughout the filter area for extensive corrugation. The purpose of the work is to estimate the filter configuration and, thus, increase its operational life. The authors in [10] established that optimization of a filter box and decrease in the value of pressure drop occur with structural changes in the back of a filter box in the form of a confusor, which reduces the number of dead zones and nonuniformity of the flow velocity distribution. That is why the major geometric parameters which influence the uniformity of the velocity distribution include the outlet port diameter of a filter box, confusor height and outlet port shifting regarding the filter box centre.

**Statement of problem.** Present-day respirators have filter boxes with a limited size due to design features of half-masks. It is considered that the decrease in the filtration rate occurs because of a sufficiently large area of the filtering material placed inside the filter box. However, the influence of the major geometric parameters of the filter box is not considered, whereas according to the...
experimental studies [11] it can degrade filter operation significantly. Thus, a crucial task arises to analyse link-age parameters of the air flow inside the filter box in relation to the geometry changes.

The purpose of the article is to study the air velocity flow in the filter box of a respirator using the mathematical model and taking into account its influence on the geometric parameters and protective efficiency.

Basic theoretical part. To achieve the objective let us apply the mathematical model presented in [12] whose implementation resolves itself to solution of a boundary-value problem for the system of four partial nonlinear differential equations. It describes the motion of a two-phase dust-laden flow in the cylindrical coordinate system of Eulerian variables.

The implementation resolves itself to solution of a boundary-value problem for the system of four partial nonlinear differential equations. It describes the motion of a two-phase dust-laden flow in the cylindrical coordinate system $r, z, \theta$, $\rho$ in Eulerian variables.

$$\frac{\partial V_r}{\partial r} + \frac{V_z}{r} \frac{\partial V_r}{\partial \theta} + \frac{\partial V_r}{\partial z} + \frac{V_r^2}{r} \rho = \frac{1}{\rho} \frac{\partial P}{\partial r}$$

$$\frac{\partial V_z}{\partial r} + \frac{V_r}{r} \frac{\partial V_z}{\partial \theta} + \frac{\partial V_z}{\partial z} + \frac{V_r}{r} \frac{\partial V_z}{\partial z} \rho = \frac{1}{\rho} \frac{\partial P}{\partial z}$$

$$\frac{1}{r} \frac{\partial \rho}{\partial r} + \frac{1}{r} \frac{\partial \rho}{\partial \theta} + \frac{\partial \rho}{\partial z} \rho = 0$$

where $V_r, V_z, \rho$ are tangential, radial and axial velocities in the filter box, $r, z$ are the current radius and research field width, $m$, $P, \rho$ are pressure and density of the flow in the filter box, pascal, kg/m³.

For computer-aided solution of this system of equations, let us write the system of equations (1) as dimensionless form. This will allow obtaining characteristics of process behaviour mode by simple conversion calculation according to the transition coefficients obtained while solving this system.

Let us select the initial parameters: $V_0$ is the filter input velocity; $\rho$ is the density at the filter input; $R_k$ is the radius of the filter box.

Then

$$V_r = \frac{V_r}{V_0}, \quad V_z = \frac{V_z}{V_0}, \quad \rho = \frac{\rho}{\rho_0}; \quad \rho' = \frac{\rho}{\rho_0}; \quad \rho'' = \frac{\rho}{\rho_0};$$

$$H' = \frac{H}{R_k}, \quad R_k' = \frac{R_k}{R_k}; \quad r' = \frac{r}{R_k}, \quad z' = \frac{z}{R_k}.$$  (2)

Conducting rearrangements of the system of equations (1, 2), we will obtain equations of the dust and gas flow motion and continuity in filter zones in dimensionless form

$$\left\{ \begin{array}{l} \frac{1}{r} \frac{\partial V_r}{\partial r} + \frac{\partial V_r}{\partial \theta} + \frac{\partial V_r}{\partial z} + \frac{V_r^2}{r} \rho \frac{\partial V_r}{\partial z} \rho = 0 \\
\frac{V_r}{r} \frac{\partial V_z}{\partial \theta} + \frac{\partial V_z}{\partial z} + \frac{V_r}{r} \frac{\partial V_z}{\partial z} \rho = 0 \\
\frac{1}{r} \frac{\partial \rho}{\partial r} + \frac{\partial \rho}{\partial \theta} + \frac{\partial \rho}{\partial z} \rho = 0 \\
\end{array} \right.$$  \hspace{0.5cm}  (3)

where $k_f$ is the frequency factor of collision of particles with the filter; $R$ is the gas constant; $T$ is the temperature of the dust–laden flow.

To solve the system of equations (3) under specified boundary conditions let us apply the least square method [8, 13]. Let us consider the algebraic and differential form of simultaneous equations

$$f_j(u_1(x, y, z), ..., u_n(x, y, z)) = 0, \quad j = 1, 2, 3, ..., n.$$  (4)

where $x, y, z$ are points of a certain domain $\Omega$.

Let us introduce the functional related to this system

$$I[u_1, u_2, ..., u_n] = \int_{\Omega} f_j^2(u_1(x, y, z), ..., u_n(x, y, z)) \, d\Omega.$$  (5)

It is easy to show that providing the functions $u'_1(x, y, z), ..., u'_n(x, y, z)$ satisfy the system of nonlinear equations for all $x, y, z$ from the domain $\Omega$, the functional (5) becomes zero. On the other hand, it is evident that providing $f(u_1, u_2, ..., u_n) \geq 0$, whereby the equality sign is attained only at the functions $u'_1(x, y, z), ..., u'_n(x, y, z)$ satisfying the system of equations (3), the solution of the system (3) provides the functional (5) with the least value. Therefore, the task of solving the system of nonlinear equations (3) can be replaced by the task of minimizing the functional (5). Such substitution of one task with another allows developing simple and efficient numerical algorithms.

Taking into account the axial symmetry, the integral (5) is decomposed into a double integral of the following form

$$I[V_r, V_z, \rho] = \int_{0}^{R_k} \int_{0}^{\pi} \left[ \frac{1}{r} \frac{\partial V_r}{\partial r} + \frac{\partial V_r}{\partial \theta} + \frac{\partial V_r}{\partial z} + \frac{V_r^2}{r} \rho \frac{\partial V_r}{\partial z} \rho \right] + \left[ \frac{V_r}{r} \frac{\partial V_z}{\partial \theta} + \frac{\partial V_z}{\partial z} + \frac{V_r}{r} \frac{\partial V_z}{\partial z} \rho \right] \frac{dr}{r} \, dz.$$  (6)
Since the motion of the dust-laden flow in the filter box is asymmetrical, only coordinates \( r, z \) will be its significant arguments of the required functions.

The next step of numerical solution of the task involves discretisation of the extremum problem for the functional (5). For discretisation, the finite-element method is used [14–16]. Based on the finite-element method it is necessary to divide the area of study into triangular elements as is shown in Fig. 1.

Within the limits of a finite element each of the the required functions \( V_0(r, z), V_r(r, z), V_z(r, z), \rho(r, z) \) is approximated using the linear expression according to the variables \( r, z \)

\[
\begin{align*}
V_0(r, z) &= a_0 + a_1 r + a_2 z \\
V_r(r, z) &= b_0 + b_1 r + b_2 z \\
V_z(r, z) &= c_0 + c_1 r + c_2 z \\
\rho(r, z) &= d_0 + d_1 r + d_2 z.
\end{align*}
\]  

(7)

In variables \( a_0, a_1, \ldots, d_0, d_1, d_2 \) are expressed in terms of the function values at mesh nodes belonging to the given triangle. Within the introduced approximation, we obtain the partial differential coefficients

\[
\frac{\partial V}{\partial r} = a_1; \quad \frac{\partial V}{\partial z} = a_2; \quad \ldots; \quad \frac{\partial \rho}{\partial r} = d_1; \quad \ldots
\]  

(8)

Let us substitute expression (7) for \( V_0, V_z, V_r, \rho \) and for their partial differential coefficients to the functional (5). Further we evaluate the integral (6) using the simplest investigations [17–19] according to which the integral is substituted with the sum of the product of finite element areas by the value of the subintegral function in the centres of these elements.

Let us satisfy boundary conditions assuming that the required functions at the nodes belonging to the border take up specified values. Then, the task resolves itself to defining the values of the required functions

\( V_0, V_z, V_r, \rho \) at the mesh nodes of the finite elements based on the condition for minimum of the multivariable functional occurring with integral approximation.

Using the method of local variations the solution of minimisation problem is defined as a result of successive approximations – iterations. During one iteration, successive single-fold “improvement” of nodal values occurs in such a way that the minimised function decreases at every step. Let us assume that iterations of the method of local variations have been completed; as a result, values \( V^{(k)}_j, V^{(k)}_r, V^{(k)}_n, \rho^{(k)}_j \) are obtained. The superscript within the brackets shows an iteration number, while the subscript indicates a mesh node number. Then, let the first nodal values \( V^{(0)}_0, V^{(0)}_r, V^{(0)}_r, \rho^{(0)} \) on the \((K + 1)^{\text{th}}\) iteration have been revised and they are equal to

\[
V^{(k+1)}_0, V^{(k+1)}_z, V^{(k+1)}_r, \rho^{(k+1)}_j \quad \text{with} \quad i \leq m.
\]  

(9)

Let us calculate the values \( V^{(k+1)}_0, V^{(k+1)}_r, V^{(k+1)}_r, \rho^{(k+1)}_j \) according to the method of local variations. Let us choose certain \( \delta > 0 \) called a variability step. As value \( V^{(k+1)}_j \) we take one of the variables

\[
V^{(k)}_j \pm \delta, V^{(k)}_j, V^{(k)}_j + \delta.
\]  

(10)

The choice is made based on the condition for minimum of the multivariable function with fixed values of other variables which are equal to

\[
\begin{align*}
V^{(0)}_0 &= \begin{cases} V^{(k+1)}_0, & i \leq m; \\
v^{(k+1)}_0, & i > m + 1; \end{cases} \\
v^{(0)}_r &= \begin{cases} V^{(k+1)}_r, & i \leq m; \\
v^{(k+1)}_r, & i > m + 1; \end{cases} \\
v^{(0)}_z &= \begin{cases} V^{(k+1)}_z, & i \leq m; \\
v^{(k+1)}_z, & i > m + 1; \end{cases} \\
\rho^{(0)}_j &= \begin{cases} \rho^{(k+1)}_j, & i \leq m; \\
\rho^{(k+1)}_j, & i > m + 1; \end{cases}
\end{align*}
\]  

(11)

Having obtained \( V^{(k+1)}_0, V^{(k+1)}_r, V^{(k+1)}_r, \rho^{(k+1)}_j \), we define \( V^{(k+1)}_r \) in a similar way, then \( V^{(k+1)}_r \) and proceed to the node \( m + 2 \).

If, as a result of the iteration

\[
V^{(k+1)}_0 = v^{(k)}, \quad V^{(k+1)}_r = v^{(k)}_r; \quad V^{(k+1)}_r = v^{(k)}_z; \quad \rho^{(k+1)}_j = \rho^{(k)}_j;
\]  

(12)

the velocities and density (12) are equal for almost all the mesh nodes, the value of the variability step will halve, and will be repeated in the process of local variability.

**Results of mathematical modelling.** Research techniques include calculation of finite elements and local variations of linkage parameters of dust-laden air flow throughout the filter box with its present geometric parameters based on the mathematical model using the finite element method.

The initial data for the calculation based on the mathematical model are presented in Table.
The number of iterations and computational accuracy of the results obtained were predetermined. The iterations are terminated if the value of variability step grows smaller than the given permissible error. To make the algorithm operate, it is necessary to define the initial approximation – values \( I_{00}, \ldots, I_{0i} \) at the mesh nodes. Average rates were calculated according to the filter cross-section area and airflow rate in the filter box. The algorithm described converges without regard to choosing the initial approximation; however, to accelerate the iteration of the process it is desirable to choose the approximation which corresponds in nature to the proposed solution. The calculation was considered to be performed if the required computational accuracy was achieved or the predetermined number of iterations was valid.

As a result of modelling, there were obtained distribution diagrams for aerodynamic parameters of the specified calculation zone. The sizes of the study zone were presented by parameters \( z, r \), as is shown in Fig. 1.

When calculated, distribution diagrams for air flow rates within the given zone were obtained. Ratio \( V_{\text{max}}/V_{\text{min}} \), which was minimised in consequence of calculation, was an assessment criterion of uniformity of dust particle settlement on the filter surface. The most uniform velocity distribution occurred in three areas where this ratio was the smallest; in these circumstances no dead zones or non-uniform velocities throughout the studied area were observed.

The system of equations was solved within the Mathcad and allowed obtaining the distribution of air flow rate inside the filter considering the size and position of the outlet port as well as the confusor height.

### Estimated characteristics (Fig. 2)

<table>
<thead>
<tr>
<th>Filter box parameters</th>
<th>Value</th>
<th>Filter parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box radius ( R_b ), mm</td>
<td>45</td>
<td>Initial resistance to the air flow ( P ), pascal</td>
<td>20</td>
</tr>
<tr>
<td>Inlet port radius ( r_b ), mm</td>
<td>15‒30</td>
<td>Filter area ( S ), mm²</td>
<td>460</td>
</tr>
<tr>
<td>Box height ( H ), mm</td>
<td>30</td>
<td>Airflow rate l/min</td>
<td>30</td>
</tr>
<tr>
<td>Confusor height ( h ), mm</td>
<td>5‒20</td>
<td>Outlet port shifting, mm</td>
<td>0‒20</td>
</tr>
</tbody>
</table>

**Table**

Linkage parameters of air flows in the filter box of different geometry were studied. Upon that, relations \( V_{\text{max}}/V_{\text{min}} \) were compared; the results of comparison are given as diagrams in Figs. 3‒5.

**Discussion.** As a result of modelling, interrelation between the geometric parameters of the filter box and filtration rate is obtained. The latter influences the non-uniformity of velocity distribution of a respirator. The

![Fig. 2. The model of structure of the filter box](image-url)

![Fig. 3. Curve of dependence of distribution of relative velocity of the air flow on the value of the outlet port shift with a diameter of 50 mm from the centre of the filter box with a length of the confusor of 15 mm and airflow rate of 30 l/min](image-url)

![Fig. 4. Curve of dependence of distribution of relative velocity of the air flow on the height of the confusor with a 50 mm diameter of the outlet port arranged centrally with the airflow rate of 30 l/min](image-url)

![Fig. 5. Curve of dependence of distribution of relative velocity of the air flow on the diameter of the outlet port with a 90 mm diameter, confusor length of 15 mm and the airflow rate of 30 l/min](image-url)
higher it is, the worse the process of capturing aerosol particles goes. About 80% of all the dust in the filter is captured when using the electrostatic method. An increase in dust and gas flow rate will result in decreasing time of particles being in the electrostatic field created on the surface coating of the filter and, consequently, an increase in slippage of particles through it. The data obtained allow optimising geometrical sizes of filter boxes which will contribute to even air flow velocity along the filter.

The suggested mathematical model features consideration of vortex flows, which can occur because of extreme growth of the airflow rate while performing hard work. In this case, the present-day models for estimating filter operation lead to significant errors.

The analysis of a particular calculated example of a filter box for the respirator RPA with a polypropylene filter shows that the 50 mm diameter is the most acceptable diameter of an outlet port. Further outlet port diameter increase requires production of a respirator inhaling valve of the proper size to protect the filter from moisture ingress when breathing out. Large size respirator inhaling valves feature less resistance and contribute to operation time increment and moist air impingement from underneath the mask to the filter, thus, decreasing their resource.

Confusor height increase is limited by respirator parameters. In particular, it can result in complicated fastening of a respirator to allow the mask to fit the face properly. A large size of a box will change the mass centre of a respirator and, therefore, cause headband load increase. Moreover, it can impair a user’s visibility significantly, which is not allowed. As a result of mathematical modelling, 16–20 mm has proved to be the most appropriate confusor height.

Another element – outlet port shift with relation to the centre of a filter box – is conditioned by increase in service properties of respirators. However, the calculated data shows that outlet port shift with relation to the centre of a filter box results in considerable increase in air flow rate. If this construction of a filter box is required, the outlet port shift relative to the centre of a filter box should not exceed 5 mm.

As a result of performing mathematical modelling, a calculation procedure for designing filter boxes for various work conditions was obtained. Further research will be aimed at decreasing resistance of a filter box and increasing protective effect duration and protection coefficient of dust respirators. Since these characteristics are interrelated, a need arises to define the filtration coefficient

\[
\frac{\Delta P}{\mu v} = \ln \frac{K}{\gamma(a, \beta)}
\]

where \( K \) is the penetration coefficient of the filter, \( v \) is the airflow rate; \( \gamma \) is the filtration coefficient.

Therefore, further studies are to be directed at reducing filter resistance and optimising the airflow rate in it using the mathematical model.

**Conclusions.** Thus, based on the issues considered we can draw the following conclusions:

- the new research technique based on the mathematical model is developed using the least square method, finite-element method and method of local variations; it allows obtaining linkage parameters of dust-laden air throughout the filter box with different geometry;
- geometric parameters of the filter box are optimised including the outlet port diameter, confusor height, and outlet port shift form its centre;
- the new technique for optimising geometric parameters of the filter box based on the mathematical model. Dependencies of distribution of air flow velocities for the filter box with different geometry are obtained and relationship between its aerodynamic and geometrical parameters is established.

**References.**

Розробка математичної моделі кінематичних параметрів потоку воздуха
внутрішньої фільтруючої коробки респіратора
Ю. І. Чеберячко, І. М. Чеберячко, Н. Н. Одновол, Л. С. Коряшкина
Наукова новизна. Представлена нова методика оптимізації геометричних параметрів фільтруючої коробки респіратора на математичній моделі. Отримані залежності розподілу швидкостей повітряного потоку для фільтруючої коробки з різною геометрією, а також встановлені залежності між аеродинамічними й геометричними її параметрами.

Практична значимість. Результати дозволяють визначити параметри фільтруючої коробки з мінімальним опором повітряному потоку та покращити захисну ефективність респіратора.

Ключові слова: респіратор, фільтруюча коробка, математичне моделювання, метод кінцевих елементів, метод локальних варіацій, метод найменших квадратів

Цель. Исследование на математической модели течения скорости воздуха в фильтрующей коробке респиратора с учетом его влияния на геометрические параметры и защитную эффективность.

Определить влияние геометрических параметров фильтрующей коробки респиратора на кинематические характеристики и ее защитную эффективность.

Методика. Для построения математической модели использовался метод анализа и синтеза. При проведении анализа фильтрующая коробка была разбита на несколько зон для оценки процесса движения пылегазовых потоков. Метод синтеза использовался для определения движения двухфазного запыленного потока в цилиндрической системе координат $r, \theta, z$ в Эйлеровых переменных.
Результаты. Оптимизированные геометрические параметры фильтрующей коробки респиратора: диаметр выходного отверстия; высота конфузора; смещение выходного отверстия от ее центра.

Научная новизна. Представлена новая методика оптимизации геометрических параметров фильтрующей коробки респиратора на математической модели. Получены зависимости распределения скоростей воздушного потока для фильтрующей коробки с разной геометрией, а также установлены зависимости между аэродинамическими и геометрическими ее параметрами.

Практическая значимость. Результаты позволяют определить параметры фильтрующей коробки с минимальным сопротивлением воздушному потоку и улучшить защитную эффективность респиратора.

Ключевые слова: респиратор, фильтрующая коробка, математическое моделирование, метод конечных элементов, метод локальных вариаций, метод наименьших квадратов

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