CONTROL LAWS OF ELECTRIC DRIVES AS A RESULT OF AN IN-DEPTH KINEMATIC ANALYSIS OF THE DELTA ROBOT

Purpose. To provide a simple and clear approach to kinematic analysis and motion computations useful to those who may wish to program and employ nice delta robots.

Methodology. A circle and sphere intersection model is used to describe positioning of the elements, which allows obtaining the analytical solution for the forward and inverse kinematics problem. For the verification of the proposed solution, the results were processed configuring the mechanical model of the kinematic system using SimMechanics Blocks in MATLAB/Simulink environment, which allows simulating various geometric configurations and reactions to mechanical stress and develop effective control strategies.

Findings. A mathematical expression describing the movement of the end-effector of the delta robot taking into account the mutual positioning of the elements of the kinematic system is obtained. A synthesis algorithm that is convenient for scaling and replication in automatic mode of operation is proposed.

Originality. For the first time, the solution was obtained that takes into account the mutual positioning of the elements and the parameters of the linear dimensions of the mechanism with the involvement of IT technologies and real equipment. The distinctive feature of the proposed solution is the adaptation to the control system of the electromechanical system.

Practical value. A parallel robot consisting of three arms connected to universal joints at the base is most effective when it is necessary to perform a quick displacement along a complex or simple trajectory while simultaneously changing the coordinates x, y and z. This fact makes the task actual to develop an algorithm for obtaining mathematical expressions for the simultaneous control of the electric motors of the delta robot. The obtained mathematical expressions for the inverse and forward kinematics problem are the first step in developing a control system that ensures the coordination and consistency of required displacements of all executive bodies in accordance with a specified program, which is understood as the set of requirements to ensure the implementation of the technological process.

Keywords: delta-robot, inverse kinematics, forward kinematics, electromechanical system
and joins, the constraints. Before proceeding to any analysis or design, the number of degrees of freedom is always determined. The paper [1] suggests a new matrix method, that always gives the correct results compared to conventional approaches like Grubler and Kutzbach formulas, but it is more complex and requires high computational efforts. As a proof, the method was applied to the mechanisms like the four-bar planar linkage, augmented 4-bar linkage, university of Maryland manipulator, Cartesian parallel manipulator and delta robot and orthoglide robot scheme. The results obtained using different methods for these mechanisms were compared and effectiveness of the suggested matrix approach was shown. The number of degrees of freedom for delta robot to be considered in this paper is calculated to be 3.

The design process of the parallel robot consists of the following stages: kinematics analysis, dynamic optimization and path tracking control. This paper deals with kinematics. Kinematic analysis is the first of the steps in the design of most industrial robots. Kinematic analysis allows the designer to obtain information on the position of each component within the mechanical system. This information is an inevitable part for subsequent dynamic analysis along with control paths.

There are many methods devoted to this problem such as an analytical solution using geometric method [2], where a problem is simplified to defining intersection point of two circles and then transforming the coordinated systems to get the final solution. In paper [3] the main part is devoted to proper determination of kinematic parameters leading to a desired workspace, but the kinematics analysis is also briefly given using similar approach requiring solution of multiple coupled nonlinear algebraic equations. There are also iterative methods based on neural network algorithms as described in paper [4]. This work proposes a methodology using the ability of adaptive neuro-fuzzy inference system to solve an inverse kinematics problem. The network applies known combination of least squares as well as back-propagation gradient descent search algorithm for training the neurons the input-output map of the inverse kinematics. This solution is quite complex, but still acceptable as an alternate approach to solving the inverse kinematics problem. An approach based on probability theory with special focus on the Levenberg-Marquardt algorithm is presented in [5]. The parameter estimation of the dynamic system is based on Functional Mock-up Interface and the underlying optimization problem the Ceres Solver is utilized. The results seem promising. There are also different numerical algorithms like real coded genetic algorithms, polynomial methods and so on. Recently, research on haptic devices for application in rehabilitation and virtual reality has become popular; for example, paper [6] where an interesting approach to kinematics of delta robot is presented very briefly. And the current paper addresses this point and investigates the kinematics problem in a numerical study using similar approach with the purpose of obtaining simple mathematical expressions that are suitable for implementation with conventional control systems. The efficient dynamic position control of the actuators could be performed in each direction using methodologies described in paper [7].

**Unsolved aspects of the problem.** It is necessary to take into account the fact that the obtained equations describing the motion of the elements of the kinematic system of the delta robot are to be implemented based on low-performance digital devices. A distinctive feature of the proposed solution is the dependence between the numerical values characterizing the instantaneous position of the kinematic system and their graphical interpretation, providing also the possibility of solving the inverse kinematics problem when the equations of the kinematic system of the delta robot are obtained from predefined coordinates in space.

**Objective of the article.** It is required to develop a simple and clear method for kinematic analysis for both forward and inverse kinematics problems using geometric constructions describing accurately the mutual positioning of the robot chains for efficient control in order to enhance the dynamic behavior.

**Presentation of the main research and explanation of scientific results.** Now let us proceed with the examination of the inverse and direct kinematics with the help of geometric constructions. These are pretty easy to understand following this paper. In order to build a parallel delta robot, two main problems have to be solved. The first problem is determining the corresponding angles of each of three arms, in case the desired position of the end effector is known to set each motor in proper position. Such a process is called an inverse kinematics. And the second problem, knowing the angles, the end effector position has to be determined for example, to make some corrections of its current position. It is called forward kinematics. Theoretical part of parallel delta robot kinematics comes further.

**Inverse kinematics.** The kinematic model of a parallel delta robot in three-dimensional Cartesian coordinate system with origin at point $T_0(0, 0, 0)$ and axis lines $X, Y$ and $Z$, oriented by arrows, is shown in Fig. 1.

Top and bottom planes of the robot are presented as equilateral deltas. Top plane is stationary. Control motors are mounted on it at points $T_1$, $T_2$ and $T_3$. Bottom plane is movable. Point $B_3$ is the position of the end effector. All physical dimensions in Fig. 1 are determined by design of the robot.

Let us consider a part of the robot model containing arm $T_1H_1$ and projections of $H_1B_3$ along $X$ and $Z$ axes of the coordinate system in Fig. 2. The target angle $\alpha_1$ is
The next step is to designate the left-hand side and the right-hand side arguments of the system of equations (2) as \( r_1^2 = r_1^2 \), \( r_2^2 = r_2^2 \), \( x_1 = (t/2)\tan 30^\circ \), \( x_2 = x_0 + b\tan 30^\circ \) and open all brackets. Then the system of equations looks like

\[
\begin{align*}
X_H^2 - 2X_H x_1 + x_1^2 + Z_H^2 &= r_1^2 \\
X_H^2 - 2X_H x_2 + x_2^2 + Z_H^2 &= 2Z_H^2 z_0 + z_0^2 = r_2^2.
\end{align*}
\] (3)

Subtract from the first equation of the system (3) the second equation, make some math manipulations and then rewrite the system as

\[
\begin{align*}
X_H^2 - 2X_H x_1 + x_1^2 + Z_H^2 &= r_1^2 \\
X_H^2 - 2X_H x_2 + x_2^2 + Z_H^2 &= r_1^2.
\end{align*}
\] (4)

Make the following designations in the first equation of (4), \( a_1 = 2x_1 - 2x_1 q = r_1^2 - r_2^2 + x_1^2 + z_0^2 \) and \( b_1 = 2z_0 \). Afterwards, resolve this expression relatively \( Z_H^2 \)

\[
Z_H^2 = \frac{q - a_1 X_H}{b_1}.
\] (5)

Substitute (5) into the second equation of the system (4) in order to get an expression resolved relatively \( X_H^1 \)

\[
X_H^1 + 1 = \frac{a_1^2}{b_1^2} + X_H^1 - 2x_1 - 2q a_1/b_1^2 + \left( x_1^2 + q^2/b_1^2 - r_1^2 \right) = 0.
\]

We have got a quadratic equation, let us reduce it to a common view. For the sake of simplicity, designate the equation coefficients as follows \( a = 1 + 4q^2/b_1^2 \), \( b = -2\left(x_1 + qa_1/b_1^2\right) \), \( c = x_1^2 + q^2/b_1^2 - r_1^2 \). The final view of quadratic equation

\[
aX_H^2 + bX_H + c = 0,
\]

where \( X_H \) represents an unknown, and \( a \), \( b \) and \( c \) are the coefficients of the equation. To find roots of this equation, firstly, the discriminant of the quadratic equation that is often represented using as upper case \( D \) must be found

\[
D = b^2 - 4ac.
\]

A quadratic equation with real coefficients can have either one or two distinct real roots, or two distinct complex roots. In this case the discriminant determines the number and nature of the roots. There are three cases:

If \( D < 0 \), the equation possesses no solutions for our problem, which means that the desired position of end effector is beyond the region of admissible displacements (i.e. workspace).

Otherwise if \( D = 0 \), then there is exactly one real root. It can be found as \( X_H = -b/2a \).
Otherwise $D \geq 0$, then the equation possesses two roots, which can be calculated as $-b + \sqrt{D}/2a$ and $-b - \sqrt{D}/2a$. The choice for our problem is the root with the most positive value.

The next step is to substitute the calculated value of $X_u$ into (5). Thereby $X_u$ and $Z_u$ are both defined. This fact allows calculating the target angle

$$\alpha_i = \arctan \left( \frac{Z_u - X_{Tu}}{X_u - X_{Tu}} \right).$$

Such algebraic simplicity comes from a good choice of the reference frame. To pursue even further this advantage and find the two remaining angles we use the symmetry of deltas and just simply rotate the coordinate system by dint of the next rotation matrix

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \end{bmatrix},$$

which rotates points in the $XY$-Cartesian plane counterclockwise through an angle $\theta = 120^\circ$ to get $\alpha_i$ and $\theta = 240^\circ$ to get $\alpha_j$ round $Z$-axis of the Cartesian coordinate system. To perform a rotation using the given matrix, the position of the point must be represented by a column vector, containing its coordinates $x_0$ and $y_0$. The rotated vector is obtained by using the matrix multiplication

$$\begin{bmatrix} x'_0 \\ y'_0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}.$$  

Thus, the coordinates $x'_0$ and $y'_0$ of the end-effector position $B_0$ after rotation

$$x'_0 = x_0 \cos\theta + y_0 \sin\theta;$$  
$$y'_0 = -x_0 \sin\theta + y_0 \cos\theta.$$  

Substituting these coordinates into the system of equations (2) instead of $x_0$ and $y_0$, the remaining angles $\alpha_2$ and $\alpha_3$ can be calculated easily using the same technique.

**Forward kinematics.** Now let us examine the forward kinematics via geometric constructions. The solution of this problem is based on similar, but not quite identical geometry. As mentioned earlier $H_1$, $H_2$, $H_3$, $B_1$, $B_2$, $B_3$ are the so-called universal joints. It means that arms $H_1 B_1$, $H_2 B_2$ and $H_3 B_3$ can rotate freely around points $B_1$, $B_2$ and $B_3$, respectively, forming spheres with radiuses $r_i$. Thereby, the most obvious and simple way to determine the position of the end effector knowing angles $\alpha_1$, $\alpha_2$ and $\alpha_3$ beforehand is to compose a system of equations for these three spheres, one of the solutions of which will be the point $B_0$ containing the coordinates of the end effector. Firstly, as angles $\alpha_1$, $\alpha_2$ and $\alpha_3$ are known, the coordinates of universal joints $H_1$, $H_2$ and $H_3$ have to be determined

$$H_1\left(OT_1 + r_i \cos\alpha_1, 0, r_i \sin\alpha_1\right);$$

$$H_2\left(-\left[\begin{array}{c} OT_2 + r_i \cos\alpha_2 \\ OT_2 + r_i \cos\alpha_2 \cos 30^\circ \\ OT_2 + r_i \cos\alpha_2 \sin 30^\circ \end{array}\right], r_i \sin\alpha_2\right);$$

$$H_3\left(-\left[\begin{array}{c} OT_3 + r_i \cos\alpha_3 \\ OT_3 + r_i \cos\alpha_3 \cos 30^\circ \\ OT_3 + r_i \cos\alpha_3 \sin 30^\circ \end{array}\right], r_i \sin\alpha_3\right).$$

In order to obtain point $B_0$ from as a root of the system of equations for the spheres, its centers must be shifted as shown in Figs. 3 and 4.

According to Fig. 4 coordinates of shifted centers of the spheres are

$$H'_1\left(OT_1 + r_i \cos\alpha_1 \tan 30^\circ, 0, r_i \sin\alpha_1\right);$$

$$H'_2\left(-\left[\begin{array}{c} OT_2 + r_i \cos\alpha_2 \tan 30^\circ \\ OT_2 + r_i \cos\alpha_2 \sin 30^\circ \end{array}\right], r_i \sin\alpha_2\right);$$

$$H'_3\left(-\left[\begin{array}{c} OT_3 + r_i \cos\alpha_3 \tan 30^\circ \\ OT_3 + r_i \cos\alpha_3 \sin 30^\circ \end{array}\right], r_i \sin\alpha_3\right).$$

For the sake of simplicity, we designate the coordinates of these points as $H'_1(x_1, y_1, z_1)$, $H'_2(x_2, y_2, z_2)$, $H'_3(x_3, y_3, z_3)$. Then the general view of the system of equation for the spheres appears like

$$\begin{align*}
(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 &= r_1^2, \\
(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 &= r_2^2, \\
(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 &= r_3^2.
\end{align*}$$  

(6)

Mathematical manipulations with this system come further with the purpose of obtaining simple expressions for roots calculation that could be used in programming and simulation without any issues. Firstly, let us open all brackets in each equation of the system (6) and group up similar variables

$$\begin{align*}
x^2 + y^2 + z^2 - 2xx_1 - 2zz_1 &= r_1^2 - x_1^2 - z_1^2; \\
x^2 + y^2 + z^2 - 2xx_2 - 2zz_2 &= r_2^2 - x_2^2 - z_2^2; \\
x^2 + y^2 + z^2 - 2xx_3 - 2yy_2 - 2zz_3 &= r_3^2 - x_3^2 - y_3^2 - z_3^2.
\end{align*}$$  

(7)  

(8)
\[ x^2 + y^2 + z^2 - 2xx_3 - 2yy_3 - 2zz_3 = r_0^2 - x_3^2 - y_3^2 - z_3^2. \]  
(9)

Subtract (7‒8)

\[ 2x (x_3 - x_1) + 2y y_3 + 2z (z_3 - z_1) = -x_1^2 - z_1^2 + x_3^2 + y_3^2 + z_3^2. \]

Subtract (7‒9)

\[ 2x (x_1 - x_3) + 2y y_1 + 2z (z_1 - z_3) = -x_3^2 - z_3^2 + x_1^2 + y_1^2 + z_1^2. \]

Subtract (8‒9)

\[ 2x (x_3 - x_2) + 2y (y_3 - y_2) + 2z (z_3 - z_2) = -x_2^2 - z_2^2 + x_3^2 + y_3^2 + z_3^2. \]

Let us introduce the following designation

\[ Q_i = x_i^2 + y_i^2 + z_i^2. \]

Taking into account the above manipulations and just introduced designation the set of equations looks like

\[ x (x_2 - x_1) + y y_2 + z (z_2 - z_1) = (Q_2 - Q_1) / 2; \]  
(10)

\[ x (x_1 - x_3) + y y_3 + z (z_1 - z_3) = (Q_1 - Q_3) / 2; \]  
(11)

\[ x (x_3 - x_2) + y (y_3 - y_2) + z (z_3 - z_2) = (Q_3 - Q_2) / 2. \]  
(12)

Continue further mathematical manipulations with equations (10‒12). Subtract (10‒11)

\[ x \left( \frac{x_2 - x_1}{y_2} - \frac{x_3 - x_1}{y_3} \right) + z \left( \frac{z_2 - z_1}{y_2} - \frac{z_3 - z_1}{y_3} \right) = \frac{Q_2 - Q_1}{2y_2} - \frac{Q_3 - Q_2}{2y_3}. \]

Represent this expression with respect to \( x \) as

\[ x = a_1 z + b_1, \]  
(13)

where

\[ a_1 = \left( \frac{z_3 - z_1}{y_3} - \frac{z_2 - z_1}{y_2} \right) / d_1; \]

\[ b_1 = \left( \frac{Q_3 - Q_1}{2y_3} - \frac{Q_1 - Q_2}{2y_2} \right) / d_1; \]

\[ d_1 = \frac{x_2 - x_1}{y_2} - \frac{x_3 - x_1}{y_3}. \]

Now subtract (11‒12)

\[ y \left( \frac{y_3}{x_3 - x_1} - \frac{y_1}{x_3 - x_2} \right) + z \left( \frac{z_3 - z_1}{x_3 - x_1} - \frac{z_2 - z_1}{x_3 - x_2} \right) \]

\[ = \frac{Q_3 - Q_1}{2(x_3 - x_1)} - \frac{Q_1 - Q_2}{2(x_3 - x_2)}. \]

Represent this expression with respect to \( y \) as

\[ y = a_2 z + b_2, \]  
(14)

where

\[ a_2 = \left( \frac{z_3 - z_2}{x_3 - x_1} - \frac{z_1 - z_2}{x_3 - x_2} \right) / d_2; \]

\[ b_2 = \left( \frac{Q_3 - Q_1}{2(x_3 - x_1)} - \frac{Q_1 - Q_2}{2(x_3 - x_2)} \right) / d_2; \]

\[ d_2 = \frac{y_3}{x_3 - x_1} - \frac{y_1}{x_3 - x_2}. \]

Finally, substitute (13) and (14) into (7). After opening all brackets and grouping similar variables the expression looks as follows

\[ \left[ a_1^2 + a_2^2 + 1 \right] z^2 + 2 \left[ a_1 (b_1 - x_1) + a_2 b_2 - z_1 \right] z + \left[ (b_1 - x_1)^2 + b_2^2 + z_1^2 - r_0^2 \right] = 0. \]

So, we get quadratic equation which can be represented in common view as

\[ az^2 + bz + c = 0, \]

where \( z \) represents an unknown, and \( a, b \) and \( c \) are the coefficients of the equation. To find roots of this equation, firstly, the discriminant of the quadratic equation must be found just like it was done in the section describing inverse kinematics problem. If \( D \geq 0 \), then the equation possesses two roots, which can be calculated as

\[ -b + \sqrt{D} / 2a \quad \text{and} \quad -b - \sqrt{D} / 2a. \]

The choice for our problem is the root with the most positive value of \( z \).

The next step is to substitute the calculated value into (13) and (14). Thereby the two remaining coordinates \( x \) and \( y \) are both defined and the problem of forward kinematics is solved.

**Conclusions.** The problem of inverse and direct kinematics with the help of geometric constructions is exam-
ined. The mathematical expressions describing the movement of the end-effector of the delta robot taking into account the mutual positioning of the elements of the kinematic system are obtained. It is claimed that the simplest analytical solution to the inverse and direct kinematics problems for high speed delta robots has been exposed for the first time herein. The proposed solution can be adapted to the control system of the electromechanical system. The future work will be devoted to the development of the algorithms for the simultaneous control of the electric motors of the delta robot to perform quick displacements along complex or simple trajectories.

References.

Мета. Розробити простий і чіткий підхід до кінематичного аналізу та обчислень руху, корисних для техніки, хто бажає програмувати й використовувати дельта-роботи.

Методика. Для опису позиціонування елементів використовується модель перетину кола та сфери, що дозволяє отримати аналітичне рішення для задачі прямої та косої кінематики. Для перевірки адекватності рішення результати були оброблені при налаштуванні механічної моделі кінематичної системи з використанням блоків бібліотеки SimMechanics у середовищі розробки MATLAB/Simulink, що дає змогу імітувати різні геометричні конфігурації й реакції на механічні впливи, а також розробляти ефективні стратегії керування.

Результати. Отримано математичний вираз, що описує рух робочого органа дельта-робота з урахуванням взаємного позиціонування елементів кінематичної системи. Запропоновано алгоритм інтерпретації, зручний для масштабування й тиражування в автоматичному режимі роботи.

Наукова новизна. Уперше було отримано рішення, що враховує взаємне розташування елементів і параметри лінійних розмірів механізму із урахуванням IT-технологій і реального обладнання. Відмінністю ризи запропонованого рішення є адаптація до системи керування електромеханічною системою.

Практична значимість. Паралелльний робот, що складається з трьох важелів, приєднаних за допомогою карданних шарнірів до основи, є найбільш ефективним, коли необхідно швидко здійснювати переміщення по складній або простій траєкторії, однакові замінюючи координати x, y та z. Цей факт робить актуальним завдання розробки алгоритму отримання математичних виразів для одночасного управління електродвигунами дельта-робота. Отримані математичні вирази для задачі прямої та косої кінематики є первістю кроком у створенні системи керування, що забезпечує координацію та згладженість необхідних переміщень усіх виконавчих органів відповідно до певної програми, яка розуміється як суккупність вимог щодо забезпечення реалізації технологічного процесу.

Ключові слова: дельта-робот, кінематика, пряма кінематика, електромеханічна система

Цель. Разработать простой и четкий подход к задаче кинематического анализа и вычислений движения, полезный для тех, кто желает программировать и использовать дельта-роботы.

Методика. Для описания позиционирования элементов используется модель пересечения окружности и сферы, которая позволяет получать аналитическое решение для задачи прямої и обратной кинематики. Для проверки предложенного решения результаты были обработаны при настройке механической модели кинематической системы с использованием блоков библиотеки SimMechanics в среде разработки MATLAB/Simulink, которая дает возможность имитировать различные геометрические конфигурации и реакции на механические воздействия, а также разрабатывать эффективные стратегии управления.

Результаты. Получено математическое выражение, описывающее движение рабочего органа дельта-робота с учетом взаимного позиционирования элементов кинематической системы. Предложен алгоритм интерпретации, удобный для масштабирования и тиражирования в автоматическом режиме работы.

Научная новизна. Впервые было получено решение, учитывающее взаимное расположение элементов и параметры линейных размеров механизма с привлечением IT-технологий и реального оборудования. Отличительной чертой предложенного ре-
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ELECTROMECHEANICS SYSTEM MODELLING
OF HYDROTTRANSPORT AT AN ENRICHMENT PLANT

Purpose. To construct and analyze electromechanical system model for hydrotransport used at factories, that will be able to take into account changes related to equipment wear, and also produce the accumulation of technical changes caused by the exploitation of the pipeline system.

Methodology. The dynamic model of the electromechanical system of hydrotransport is developed on the basis of data about physical parameters of hydrotransport systems, received empirically. It is based on the methods for identification of dynamic systems in the form of differential equations for elements of the inside-factory hydraulic transport technological object.

Findings. A model of the electromechanical system of hydrotransport is developed. Verifications of homogeneity by the Fisher’s and Bartlett’s criteria showed the homogeneity of the estimates of the variance of reproducibility. For the Fisher’s criterion rating was 2.59; for the Bartlett’s criterion verification showed that the coefficient is significant at the level less than 0.02, and this is indicating the reliability of the calculation of the correlation matrix.

Originality. For the first time a model for systems of the hydraulic transport, based on the Jeffcott’s multi-mass rotor models, was applied. While modelling, wear of equipment in process is taken into account.

Practical value. Efficiency of usage of Jeffcott’s multi-mass rotor models based model, has been proven. It allows describing the behavior of an object in specific technological regimes reliably and improves efficiency of the processes.

Keywords: dynamic modeling, Jeffcott’s rotor, multi-mass model, hydrotransport