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# ANALYSIS OF THE ERROR OBTAINING WHILE DETERMINING CHARACTERISTICS OF ROTOR SYSTEM WITHIN THE RUN-DOWN

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# АНАЛІЗ ОТРИМАННЯ ПОХИБОК ПРИ ВИЗНАЧЕННІ ХАРАКТЕРИСТИК РОТОРНОЇ СИСТЕМИ НА ВИБІГУ

**Purpose.** The objective of the paper is to determine reasons of origin and conditions of reducing errors while obtaining combined geometrical (axial moment of inertia, mass centre location), mass (rotor weight), frictional (relative radii of bearings) characteristics of rotor system within the run-down. This is required to determine constant (gravitational moment), linear (being proportional to rotational velocity), and quadratic (ventilator moment) components of complete moment of rotor rotation resistance with the following balance problem solving with the help of peak method.

**Methodology.** Theoretical research of rotating rotors relies upon basic theoretical provisions (dynamics) of machines and mechanisms, pre-developed algorithm of 1.2 ... of N-planar balancing of rigid rotors using peak method as well as analysis of calculation error obtaining.

**Findings.** A statement concerning the necessity of equality of initial errors while measuring experimental values (i.e. time, turning angle, rotational velocity) of run-down has been developed. The statement results in minimization of an error while determining required characteristics of a rotor. The developed algorithm of serial computations identifies definitely geometrical, mass, and frictional characteristics of balanced rotor system in the case of negative discriminant of total moment of rotational resistance. In the context of another, arbitrary relation of rotor system parameters (not negative discriminant), both methodology and order of rotor characteristics determination remain similar. It is planned to use the algorithm as one of the potential alternatives to obtain values of the rotor characteristics with the following substantiated selection of sets of characteristics using the least square method and to come to the balancing problem.

**Originality.** The basic reason or a source to form the error while determining geometrical, mass, and frictional characteristics of rotor systems within the run-down has been identified – difference in measuring errors of certain run-down parameters: time and rotational velocity of rotor turning angle. Theoretically, in terms of any measuring errors being similar in value within the run-down (for example, its time) we obtain true value of target characteristics in the process of each experiment.

**Practical value** is in potential use of permanent stoppages of rotor systems to monitor basic total rotor characteristics – wear and tear of operating devices, block bearings, changes in a value of a process moment, redistribution of bulk weight of a rotor, and the following moment balancing problem solving with the help of a peak method. The proposed solution technique involving the least square method will make it possible to select reasonably the most adequate set of characteristics from the solution set for zero (two possible solutions), non-zero (generalized case), and positive discriminant of a rotor rotational resistance moment.

**Keywords:** *rotor system, permanent stoppages, run-down, axial inertia moment, block bearings, frictional characteristics, rotor weight, mass centre location*

**Introduction.** The importance of dynamic balancing in the context of modern machine-building and operation of high-tech equipment results in following traditional problems:

- balancing time cutting owing to the decreased number of test runs or special test run – permanent machine stoppage coincidence;
- accuracy of balancing in progress at the expense of identification of origin and accumulation of measuring errors of experimental values;
- no specific vibratory equipment (i.e. sensors, phase meters) without strict recommendations concerning their use and application;
- identification of other reasons (except balance weights) of origin of increased rotor rotational resistance within its bearings.

Simultaneous solution of the abovementioned problems is possible only in the context of a new approach development. The approach should be based upon the analysis of rotor system run-down process to determine its geometrical, mass, and frictional characteristics with minimum possible error. Solution of the formulated problem is to identify the most accurate values characteristics of a rotor on the basis of the simplest experimental data (i.e. rotation period, rotor rotational angle) determined with their errors.

**Analysis of the recent research and unsolved aspects of the problem.** Modern balancing of rotor systems means elimination of harmful oscillations of rotor supports. Traditionally, it is put into effect by means of contact vibration detectors, various phase indicators [1, 2], other dedicated equipment as well as the developed and tested methods (phases [2], phases and amplitudes [3]). Entering of correction balance weight into the rotor simply on the basis of indirect measurement of oscillation amplitudes (and phases) can be calculated by means of integral method but not only for the reaction of a single support. In this context problems of mechanics in terms of planes perpendicular to rotor axis are solved. Study of dynamic balance in turning direction will be another approach of using indirect measurements while balancing rigid rotors. That will allow obtaining simultaneous effect of all the distributed balance weights to all the rotor supports in terms of changeable basic parameter of run-down process – rotation velocity ω, i.e. within the whole time interval (within each point) of run-down. Each of the listed (both traditional and during the run-down) approaches to solve the balancing problem applies the data obtained experimentally. Thus, the issue of their further processing in terms of minimum accumulation of error of the obtained theoretical results is of great importance.

It goes without saying that reject of the abovementioned expensive devices and instruments, use of standard rotor stops, simplification of the experimental measuring activities, and systematization of the processing of the obtained data in relation to the importance and widespread occurrence of balancing problem is **topical set of both scientific and practical tasks**. First of them is the task preceding the run-down balancing, i.e. determination of total rotor characteristics. These cover:

- $J_P$ is axial moment of rotor inertia;
- $\mu$, $m$, $M$ are coefficients of quadratic trinomial characterizing the approximate total moment of rotational resistance $M_{SUP} = \mu\omega^2 + m\omega + M$;
- $(r_S f)_A$ and $(r_S f)_B$ are friction characteristics of bearing supports $A$ and $B$ of a rotor (equivalent friction radii);
- $G_P$ is the rotor weight; $x_P$ is axial arrangement of its mass centers.

Availability of $J_P$ within the mentioned total characteristics indicates close interrelation of the specified preliminary balancing problem and the proper balancing problem as their major searching option is the very moment of inertia. Similar solution of two shared problems will make it possible to monitor basic units of rotor system that is rotor wear ($J_P$, $G_P$), condition of bearings ($(r_S f)_A$, $(r_S f)_B$), change in geometry ($J_P$, $x_P$), air resistance to rotation $\mu^1$ with following transition to the determination of addend-balance weight $d_k = m_k \cdot r_k$ from rotor unbalance $\mu\omega^2 = (\mu^1 + m_k \cdot r_k)\omega^2$. The objective of the research is to determine the efficient causes and conditions of the reduction of errors while obtaining total characteristics $J_P$, μ, $m$, $M$ of rotor system in terms of run-down.

**Presentation of the main research.** A process of balancing trial runs (run-downs) involves determination of the simplest and most accessible values – time of process $t$ and rotor turning angle φ with the computer-aided data recording. Algorithm of approximate or graphic differentiation as well as representation of the functions in the form of spline functions can be used to determine the rotating velocity ω and rotating deceleration ε. The paper considers dynamically balanced rotor and analyzes errors while determining $J_P$, μ, $m$, $M$ characteristics, i.e. the problem being preliminary to the one of run-down balancing.

**General statement of the problem.** Equation of decelerated run-down rotation is as follows

$$-J_P \cdot d\omega/dt = M_{SUP} = \sum \alpha_j \omega^j \approx m\omega^2 + m\omega + M, \quad j = 0, 1, 2, \ldots, \infty \qquad (1)$$

in terms of i $\varphi_{t=0} = 0$ и $\omega_{t=0} = \omega_0$ initial conditions. In this case we are restricted by the first three terms of a series according to the basic parameter of the process that is rotation velocity ω. Effect of total radial effort values effecting on bearings or its components (in the context of every support) is considered further only as $M = (r_S f)_{A,B} R_{A,B}$. Depending on the combinations of μ, $m$, $M$ coefficients and value of $\Delta = 4\mu M - m^2$ discriminant (the corresponding $\mu\omega^2 + m\omega + M = 0$ square equation), the first integral (1) will be as follows

$$-J_p \cdot \int_{\omega_0}^{\omega} \frac{d\omega}{\mu\omega^2 + m\omega + M} = J_p \cdot \left[W(\omega_0) - W(\omega)\right] = t(\omega),$$

where antiderivative $W$ is determined according to one of the four expressions

$$W(\omega)=\begin{cases}\dfrac{\omega}{M}, & \text{when } \mu=0, m=0, \Delta=4\mu M-m^2\\ \dfrac{-2}{m+2\mu\omega}, & \text{when } \Delta=0, \mu+m\neq 0\\ \dfrac{2}{\sqrt{\Delta}}\operatorname{arctg}\left(\dfrac{m+2\mu\omega}{\sqrt{\Delta}}\right) & \text{if } \Delta>0\\ \dfrac{2}{\sqrt{-\Delta}}\operatorname{Arth}\left(\dfrac{m+2\mu\omega}{\sqrt{-\Delta}}\right) & \text{if } \Delta<0\end{cases}. \quad (2)$$

Fuller use of square equation properties makes it possible to combine the last two particular cases by means of the introducing

$$-J_p\frac{d\omega}{dt}=\mu(\omega-\omega_1)(\omega-\omega_2),$$

where $\omega_{1,2}=\dfrac{-m\pm\sqrt{m^2-4\mu M}}{2\mu}$ are the roots of the equation; then the first integral of a differential equation will have following final form

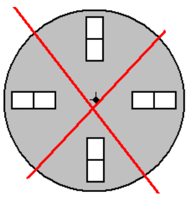
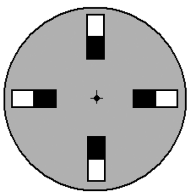
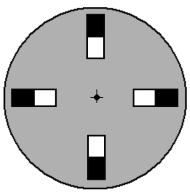
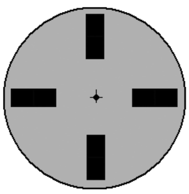
$$t(\omega)=\frac{J_p}{\mu(\omega_1-\omega_2)}\ln\frac{(\omega_0-\omega_1)(\omega-\omega_2)}{(\omega_0-\omega_2)(\omega-\omega_1)}.$$

Divide the determination of rotor system characteristics into two stages: stage is identification of the inertia moment $J_P$ by introducing specific disturbances into a rotor system – $J_\Delta$ and $J_\Delta^I$ additives (Table 1); stage two identifies $\mu$, $m$, $M$ coefficients according to experimental dependences $\varphi(t)$ or $\omega(t)$ with the application of least square method (LSM) or selection of three points-conditions to develop the system relative to three unknown $\mu$, $m$, $M$.

It is planned to select the required variant of four from (2) or (3) of the final solution, for instance, according to minimum difference of sum of squares of theoretical and experimental dependence deviations. In this context, inertia moment $J_P$ acts as a scale coefficient; that is why it should be predefined. Time $t_{RD}$ and angel $\varphi_{RD}$ of a full run-down (rotation velocity $\omega$ changes from $\omega_0$ to 0) being measured by the simplest techniques are the most convenient characteristics of the rotor system run-down. They are total characteristics which changes are being monitored continuously within the whole $t_{RD}$ interval.

*Table 1*

Runs (run-downs) required to determine rotor characteristics

| Run number | Scheme of extra disc | Total | | Full run-down period $t_{RD}$ |
|---|---|---|---|---|
| | | inertia moment | gravity force | |
| 0 |  | $J_P$ | $G_P$ | $t_{RD}^0=\dfrac{2J_P}{\sqrt{m^2-4\mu M}}\cdot\operatorname{Arth}\left[\dfrac{\omega_0\sqrt{m^2-4\mu M}}{2M+m\omega_0}\right]$ |
| 1 |  | $J_P+J_\Delta$ | $G_P+G_\Delta+G_D$ | $t_{RD}^1=\dfrac{2(J_P+J_\Delta)}{\sqrt{m^2-4\mu M'}}\cdot\operatorname{Arth}\left[\dfrac{\omega_0\sqrt{m^2-4\mu M'}}{2M'+m\omega_0}\right]$ |
| 2 |  | $J_P+J_\Delta^1$ | | $t_{RD}^{11}=\dfrac{2(J_P+J_\Delta^1)}{\sqrt{m^2-4\mu M'}}\cdot\operatorname{Arth}\left[\dfrac{\omega_0\sqrt{m^2-4\mu M'}}{2M'+m\omega_0}\right]$ |
| 3 |  | $J_P+J_\Delta^{11}$ | $G_P+G_\Delta+2G_D$ | $t_{RD}^{111}=\dfrac{2(J_P+J_\Delta^{11})}{\sqrt{m^2-4\mu M''}}\cdot\operatorname{Arth}\left[\dfrac{\omega_0\sqrt{m^2-4\mu M''}}{2M''+m\omega_0}\right]$ |

*Note*: the Table shows change in full run-down period for $\Delta<0$

**Determination of axial moment of rotor inertia $J_P$.** To simplify the calculations and stress the role of each disturbance being introduced it is necessary to consider four runs with the additional disc shown in Table 1. If we take the time ratio of runs 1 and 2 in terms of the foregone $J_\Delta$ and $J_\Delta^1$ values, we will get following final formula for axial moment of the rotor inertia

$$J_P = \frac{J_\Delta \cdot t_{RD}^{11} - J_\Delta^1 \cdot t_{RD}^1}{t_{RD}^1 - t_{RD}^{11}}, \qquad (4)$$

involving implementation of stage 1concerning determination of the rotor system characteristics.

Analysis for obtaining value error $J_P$ on the basis of the specified errors of the measurements of full run period $t_{RD}^1$ and $t_{RD}^{11}$ is performed numerically (Table 2) for the case of $\Delta_t = 0.05$ and time $t_{RD}^1 \cdot (1 + k^1 \cdot \Delta_t)$, $t_{RD}^{11} \cdot (1 + k^{11} \cdot \Delta_t)$ specified with error in terms of similar sets $k^1 = k^{11} = (-1; -1/2; 0; 1/2; 1;)$. The calculation assumes: $J_P = 700$; $\omega_0 = 500$; $J_\Delta^1 = 16$; $J_\Delta^{11} = 14$; $\mu = 1.25$; $m = 11$; $M = 20$.

Calculations (Table 2) show that to minimize errors while $J_P$ identifying it is required to measure run-down time with similar errors (principal diagonal main diagonal of the last-mentioned Table). Otherwise the error in the process involving determination of axial inertia moment in terms of run-down may be rather significant.

**Determination of μ, *m*, *M* coefficients.** Traditionally similar problems are solved by means of LSM. The method means solution of an optimizing problem which requires the task (unknown in general case) of initial approximation; it may have several extrema-solutions. Thus, in this case it is desirable to develop algorithm giving the unique variant of $\mu$, $m$, $M$ determination based on the selection of three (according to the number of unknown) conditions forming closed system. Table 3 shows analytical expressions corresponding to the full solution (1) with graphic dependences for $\Delta < 0$ case. The variant of algorithm giving the required unique solution is proposed.

Divide any time interval of 0...3 runs into 4 equal parts $\delta = t_{RD}/4$. Introduce possible runs corresponding to different $t_{RD} = \delta(1\ 2\ 3)$ and initial rundown velocities $\Omega_k = \omega(k \cdot \delta)$, $k = 1, 2, 3$ (Fig. 1).

Develop following equation system

*Table 2*

Error of the value of axial rotor inertia moment in the context of run-down

| $\frac{J_p^1 - J_p}{J_p} \cdot 100\,\%$ | | $k^1$: −1 | −1/2 | 0 | 1/2 | 1 |
|---|---|---|---|---|---|---|
| $k^{11}$ | −1 | 0 | −15 | −27 | −36 | −43 |
| | −1/2 | 23 | 0 | −15 | −26 | −35 |
| | 0 | 57 | 22 | 0 | −15 | −27 |
| | 1/2 | 118 | 57 | 22 | 0 | −15 |
| | 1 | 256 | 118 | 57 | 22 | 0 |

$$\begin{cases} \dfrac{\Omega_1\sqrt{-\Delta}}{m\Omega_1 + 2M} = \mathrm{th}\left(1 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right) \\[2ex] \dfrac{\Omega_2\sqrt{-\Delta}}{m\Omega_2 + 2M} = \mathrm{th}\left(2 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right) = \dfrac{2\,\mathrm{th}\left(1 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right)}{1 + \mathrm{th}^2\left(1 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right)} \\[2ex] \dfrac{\Omega_3\sqrt{-\Delta}}{m\Omega_3 + 2M} = \mathrm{th}\left(3 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right) = \\[2ex] = \dfrac{3 + \mathrm{th}^2\left(1 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right)}{1 + 3\,\mathrm{th}^2\left(1 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right)} \cdot \mathrm{th}\left(1 \cdot \delta \cdot \dfrac{\sqrt{-\Delta}}{2J}\right) \end{cases} . \qquad (5)$$

Taking th(1 · δ...) away obtain intermediate system

$$\begin{cases} \dfrac{\Omega_2}{\Omega_1\left(m\Omega_2 + 2M\right)} = \dfrac{2\left(m\Omega_1 + 2M\right)}{\left(m\Omega_1 + 2M\right)^2 + \Omega_1^2\left(m^2 - 4\mu M\right)}, \\[2ex] \dfrac{\Omega_3\left(m\Omega_1 + 2M\right)}{\Omega_1\left(m\Omega_3 + 2M\right)} = \dfrac{3\left(m\Omega_1 + 2M\right)^2 + \Omega_1^2\left(m^2 - 4\mu M\right)}{\left(m\Omega_1 + 2M\right)^2 + 3\Omega_1^2\left(m^2 - 4\mu M\right)} \end{cases}$$

and then final solution

$$\frac{M}{m} = \frac{\Omega_1\left(\Omega_3 - \Omega_2\right)}{\Omega_2 + \dfrac{\Omega_2\Omega_3}{\Omega_1} + \Omega_1 - 3\Omega_3}; \quad \frac{\mu}{m} = \frac{\Omega_2 - 2\Omega_1}{\Omega_1^2\Omega_2} \cdot \frac{M}{m} - \frac{1}{\Omega_2};$$

$$m = \frac{2J_P}{\delta\sqrt{1 - 4\dfrac{\mu}{m}\dfrac{M}{m}}}\,\mathrm{Arth}\left(\frac{\Omega_1\sqrt{1 - 4\dfrac{\mu}{m}\dfrac{M}{m}}}{\Omega_1 + 2\dfrac{M}{m}}\right).$$

Error accumulation in the process of $\mu$, $m$, $M$ identification is followed in terms of previous numerical example when determination accuracy of three $\Omega_{1,2,3}$ values with possible $\Delta_\omega = \pm 0.05$ and 0.15 spread takes place (Table 4). Average total error

$$\delta_\Sigma = \frac{1}{3} \cdot \left(\frac{\left|\mu_P - \mu\right|}{\mu} + \frac{\left|m_P - m\right|}{m} + \frac{\left|M_P - M\right|}{M}\right) \cdot 100\ \%.$$

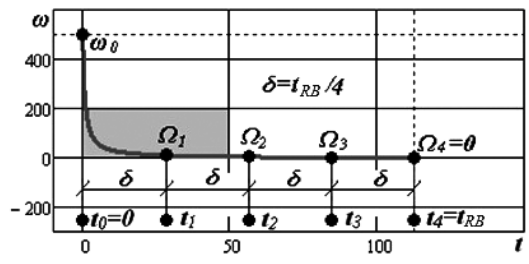
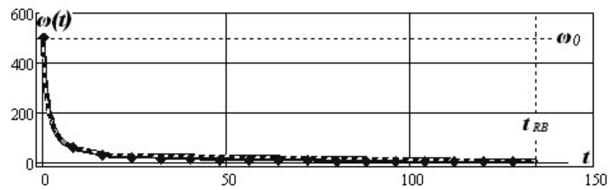
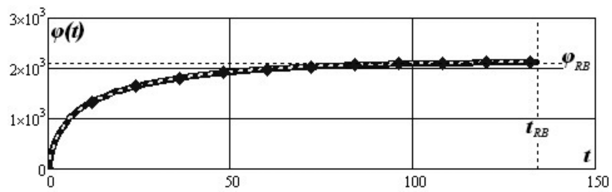
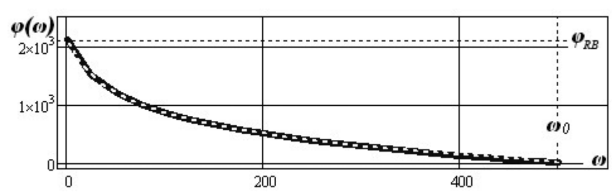



*Fig. 1. Graphic representation to obtain conditions of rundown variants*

*Table 3*

Solution of differential equation (1) if $\Delta = 4\mu M - m^2 < 0$

| Parameters | Analytical expressions and symbol names |
|---|---|
| Original integral | $\int \frac{Jd\omega}{\mu\omega^2+m\omega+M} = \frac{2}{\sqrt{-\Delta}}\text{Arth}(\alpha(\omega)), \quad \alpha(\omega)=\frac{m+2\mu\omega}{\sqrt{-\Delta}}$ |
| Solution of differential equation 1 | $t(\omega)=t_0+\frac{2J}{\sqrt{-\Delta}}\left[\text{Arth}(\alpha(\omega_0))-\text{Arth}(\alpha(\omega))\right]=t_0+\frac{2J}{\sqrt{-\Delta}}\cdot\text{Arth}\left[\frac{(\omega_0-\omega)\sqrt{-\Delta}}{2M+m(\omega_0+\omega)+2\mu\omega_0\omega}\right]$ |
| Period of complete rundown | $t_{RD}=t(0)\Big\vert_{t_0=0}=\frac{2J}{\sqrt{-\Delta}}\cdot\text{Arth}\left[\frac{\omega_0\sqrt{-\Delta}}{2M+m\omega_0}\right]$ |
| Change in rotation velocity | $\omega(t)=\frac{\sqrt{-\Delta}}{2\mu}\text{th}\left[\text{Arth}(\alpha(\omega_0))+\frac{(t-t_0)\sqrt{-\Delta}}{2J}\right]-\frac{m}{2\mu}$, control: $\omega(t_{RD})=0$ |
| Solution of differential equation 2 | $\varphi(t)=\varphi_0+\frac{J}{\mu}\ln\left[\cos\beta-\frac{m+2\mu\omega_0}{\sqrt{-\Delta}}\cdot\sin\beta\right]-\frac{m}{2\mu}(t-t_0), \quad \beta=\frac{(t-t_0)\sqrt{-\Delta}}{2J}$ |
| Rotation angle from rotation velocity | $\varphi(\omega)=\frac{J}{2\mu}\left[\ln\frac{\mu\omega_0^2+m\omega_0+M}{\mu\omega^2+m\omega+M}+\frac{2m}{\sqrt{-\Delta}}\left(\text{Arth}[\alpha(\omega_0)]-\text{Arth}[\alpha(\omega)]\right)\right]$ |
| Complete rundown angle | $\varphi(t_{RD})\Big\vert_{t_0=0}=\varphi_{RD}=\varphi(0)=\frac{J}{2\mu}\left[\ln\left(\frac{\mu\omega_0^2+m\omega_0+M}{M}\right)-\frac{2m}{\sqrt{-\Delta}}\text{Arth}\left(\frac{\omega_0\sqrt{-\Delta}}{m\omega_0+2M}\right)\right]$ |
| Graphs of the dependences obtained | |
| The calculations assume: | $J$ = 700 kg · m²; $M$ = 20 N · m; $\mu$ = 1.25 kg · m²; $m$ = 11 kg · m²/s; $\Delta$ = –21 (kg · m²/s)²; $t_0$ = 0 s; $\varphi_0$ = 0 rad; $\omega_0$ = 500 rad/s |
| Complete rundown period | $t_{RD}$ = 134.402 s |
| Complete rundown angle | $\varphi_{RD}$ = 2117 rad = 336.993 rad |
| Rotation velocity from time is hyperbolic dependence |   |
| Angle of rotor rotation from time is logarithmic dependence, extremum is in ($t_{RD}$ ; $\varphi_{RD}$ ) point |   |
| Angle of rotor rotation from rotation velocity is logarithmic dependence, extremum is in (0 ; $\varphi_{RD}$ ) point |   |

Variation (or velocity determined with error) of $\Omega_{1,2,3}$ velocities will be performed with the same coefficients $\Omega_j \cdot (1 + k_j \cdot \Delta_\omega)$ like we have done it before, where $k_j$ = = (–1; –1/2; 0; 1/2; 1) are sets of numbers corresponding to the predefined measuring errors. Value $k = 0$ corresponds to zero error, i. e. accurate value of the rotation velocity. Table 4 shows that minimum values of errors will occur in case of coincidence of the initial errors of

$\Omega_{1,2,3}$ velocities (grey cells in Tables 2 and 4). The performed calculations if $k_1 = 1$ (Table 4) makes it possible to conclude that average error $\delta_\Sigma$ is proportional to $\Delta_\omega$ value spread.

Taking into account the complexity of performing the measurement of experimental values with obligatory equality of errors, ways to clarify the calculations of rotor system characteristics should be searched in:

- connection of various methods to search for a solution (LSM and equations (Table 3));
- simplification of complex transcendental functions with the help of initial series terms;
- application of simplified equation of decelerated rotation (1) with the use of double-numerical differentiation;
- determination of characteristic points-conditions of the dependences (Table 3) with the application of the points of maximum curvature of graphs (grey square in Fig. 1).

Errors of the determination of other characteristics of rotor systems will be related directly to $\delta_\Sigma$ value.

**Determination of frictional characteristics $(r_S f)_A$ and $(r_S f)_B$.** Consider 0, 2, and 3 rundowns (Table 1). Equations (1) for each of them are united in following system

$$\begin{cases} -J_P\varepsilon = \mu\omega^2 + m\omega + M = \mu\omega^2 + m\omega + \\ +\left[\left(r_S f\right)_A \cdot R_A + \left(r_S f\right)_B \cdot R_B\right] \\ -\left(J_P + \dfrac{J_\Delta}{J_\Delta^1}\right)\varepsilon = \mu\omega^2 + m\omega + M_\Delta = \mu\omega^2 + m\omega + \\ +\left[\left(r_S f\right)_A\left(R_A + G_D\dfrac{L-x_D}{L}\right) + \left(r_S f\right)_B\left(R_B + G_D\dfrac{x_D}{L}\right)\right] \\ -\left(J_P + J_\Delta^{11}\right)\varepsilon = \mu\omega^2 + m\omega + M_\Delta^{11} = \mu\omega^2 + m\omega + \\ +\left[\left(r_S f\right)_A\left(R_A + G_D^{11}\dfrac{L-x_D}{L}\right) + \left(r_S f\right)_B\left(R_B + G_D^{11}\dfrac{x_D}{L}\right)\right] \end{cases}.$$

Deduction of $M$ (equation 1) from $M_\Delta$ of two other equation results in

$$\begin{cases} \left(r_S f\right)_A G_D\dfrac{L-x_D}{L} + \left(r_S f\right)_B G_D\dfrac{x_D}{L} = M_\Delta - M \\ \left(r_S f\right)_A G_D^{11}\dfrac{L-x_D}{L} + \left(r_S f\right)_B G_D^{11}\dfrac{x_D}{L} = M_\Delta^{11} - M \end{cases},$$

a new system which solution is introduced comfortably in terms of matrix form

$$\begin{bmatrix} \left(r_S f\right)_A \\ \left(r_S f\right)_A \end{bmatrix} = L \cdot \begin{bmatrix} G_D\left(L-x_D\right) & G_D x_D \\ G_D^{11}L - x_D & G_D^{11}x_D \end{bmatrix}^{-1} \cdot \begin{bmatrix} M_\Delta - M \\ M_\Delta^{11} - M \end{bmatrix}. \quad (6)$$

**Determination of $G_P$-$x_P$ relation.** Considering single-span rotor (with $L$ length), $G_P$ weight and in-line arrangement centre of mass (gravity) $x_P$) being identified statically enter the following for reactions of bearing supports of the rotor

$$R_A = G_P\frac{L-x_P}{L} \quad \text{and} \quad R_B = G_P\frac{x_P}{L}.$$

Table 4

Averaged errors of *μ, m, M* values determination

| $\delta_\Sigma$ | $k_2$ | | | | | $k_2$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | –1 | –1/2 | 0 | 1/2 | 1 | –1 | –1/2 | 0 | 1/2 | 1 |
| $k_3$ | $k_1 = -1$, $\Delta_\omega = 0.05$ | | | | | $k_1 = -1/2$, $\Delta_\omega = 0.05$ | | | | |
| –1 | 3 | 15 | 29 | 43 | 56 | 1 | 4 | 19 | 33 | 46 |
| –1/2 | 7 | 11 | 25 | 38 | 51 | 16 | 2 | 15 | 28 | 41 |
| 0 | 11 | 7 | 21 | 34 | 47 | 20 | 5 | 10 | 24 | 37 |
| 1/2 | 14 | 6 | 17 | 30 | 42 | 23 | 9 | 6 | 20 | 33 |
| 1 | 18 | 8 | 13 | 26 | 39 | 27 | 12 | 5 | 16 | 29 |
| $k_3$ | $k_1 = 0$, $\Delta_\omega = 0.05$ | | | | | $k_1 = 1/2$, $\Delta_\omega = 0.05$ | | | | |
| –1 | 23 | 7 | 9 | 23 | 37 | 35 | 18 | 5 | 13 | 27 |
| –1/2 | 27 | 11 | 4 | 18 | 32 | 39 | 22 | 6 | 9 | 22 |
| 0 | 31 | 15 | 0 | 14 | 27 | 43 | 26 | 11 | 5 | 18 |
| 1/2 | 35 | 19 | 4 | 10 | 23 | 47 | 30 | 15 | 2 | 14 |
| 1 | 38 | 23 | 8 | 6 | 19 | 50 | 34 | 18 | 4 | 10 |
| $k_3$ | $k_1 = 1$, $\Delta_\omega = 0.05$ | | | | | $k_1 = 1$, $\Delta_\omega = 0.15$ | | | | |
| –1 | 46 | 29 | 13 | 8 | 18 | 177 | 98 | 36 | 26 | 60 |
| –1/2 | 51 | 33 | 17 | 6 | 14 | 191 | 114 | 53 | 19 | 43 |
| 0 | 55 | 37 | 21 | 6 | 10 | 203 | 126 | 66 | 16 | 30 |
| 1/2 | 58 | 41 | 25 | 10 | 7 | 212 | 136 | 77 | 30 | 19 |
| 1 | 62 | 45 | 29 | 14 | 3 | 220 | 145 | 87 | 40 | 9 |

Use of predefined $M$, $(r_S f)_A$ and $(r_S f)_B$ values will help obtain following expression for run 0 (Table 2)

$$G_P = \frac{M \cdot L}{\left(r_S f\right)_A L + \left[\left(r_S f\right)_B - \left(r_S f\right)_A\right]x_P}, \quad (7)$$

to detect integral (re-)distribution of rotor mass during the period of its operation in terms of any standard shutdown. In case of similar rotor supports and their equivalent wear, i. e. formally for $(r_S f)_A = (r_S f)_B$ it follows from the latter expression that

$$M = (r_S f)_{A,B}\, G_P,$$

which confirms the known thesis concerning formation of gravitation moment value as separate addend of total moment of rotational resistance in bearings.

**Conclusions and recommendations for further research.** Causes of the error formation while determining characteristics of rotor system during run-down have been determined.

It has been shown that the equality of initial errors of the measurement of experimental values (time, turning angle, and rotation velocity) of a run-down results in minimization of the errors of rotor characteristics determination.

The developed algorithm of the sequential calculations (3–7) determines geometrical, weight, and friction characteristics of the balanced rotor system for the case of negative discriminant of total moment of rotational resistance.

The solved problem is the preliminary one for the following rotor balancing; it also makes it possible to transform to the balancing by the method of amplitudes.

The proposed solution is not the unique one as it does not eliminate the use of LSM with the optimization task for multimodal function.

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**Мета.** Визначити причини виникнення та умови зменшення похибок отримання сумарних геометричних (осьовий момент інерції, розташування центру мас), масових (вага ротора), фрикційних (приведені радіуси підшипників) характеристик роторної системи на вибігу. Це необхідно для знаходження постійної (гравітаційний момент), лінійної (пропорційний швидкості обертання) та квадратичної (вентиляторний момент) складових повного моменту опору обертанню ротора з подальшим вирішенням завдання балансування методом амплітуд.

**Методика.** Теоретичні дослідження обертових роторів базуються на основних положеннях теорії (динаміки) машин і механізмів, розробленому раніше алгоритмі 1,2, ..., *N*-площинного балансування жорстких роторів методом амплітуд і аналізі отримання похибок розрахунків.

**Результати.** Отримано твердження щодо необхідності рівності початкових похибок вимірювання експериментальних величин (часу, кута повороту, швидкості обертання) вибігання, що призводить до мінімізації похибки знаходження шуканих характеристик ротора. Побудований алгоритм послідовних обчислень однозначно визначає геометричні, масові й фрикційні характеристики врівноваженої роторної системи в разі негативного дискримінанту сумарного моменту опору обертанню. При іншому, довільному співвідношенні параметрів роторної системи (не негативний дискримінант), методика й послідовність визначення характеристик ротора залишаються аналогічними. Планується використовувати алгоритм як один із потенційних варіантів отримання значень характеристик ротора, із подальшим обґрунтованим вибором набору характеристик за методом найменших квадратів і переходом до задачі балансування.

**Наукова новизна.** Виявлено основну причину або джерело формування похибки визначення геометричних, масових і фрикційних характеристик роторних систем на вибігу – різниця в похибках вимірювання окремих параметрів вибігу: часу й швидкості обертання кута повороту ротора. Теоретично за будь-яких, але однакових за величиною, похибок виміру на вибігу (наприклад, його часу) кожного досліду отримуємо точне значення шуканої характеристики.

**Практична значимість.** Полягає в можливому використанні штатних зупинок роторних систем для моніторингу основних сумарних характеристик роторів – зносу робочих органів, опорних підшипників, зміни величини технологічного моменту, перерозподілу маси ротора, проведення подальшої задачі моментного балансування за методом амплітуд. Пропонований варіант рішення з використанням методу найменших квадратів дозволить, за своїми критеріями, обґрунтовано вибрати найбільш відповідний набір характеристик із комплекту рішень для нульового (два можливих рішення), не нульового (узагальнений випадок) та позитивного дискримінанту моменту опору обертання ротора.



**Ключові слова:** *роторна система, штатні зупинки, вибіг, осьової момент інерції, опорні підшипники, фрикційні характеристики, вага ротора, розташування центру мас*



**Цель.** Определить причины возникновения и условия уменьшения погрешностей получения суммарных геометрических (осевой момент инерции, расположение центра масс), массовых (вес ротора), фрикционных (приведенные радиусы подшипников) характеристик роторной системы на выбеге. Это необходимо для нахождения постоянного (гравитационный момент), линейного (пропорционален скорости вращения) и квадратичного (вентиляторный момент) слагаемых полного момента сопротивления вращению ротора с последующим решением задачи балансировки методом амплитуд.

**Методика.** Теоретические исследования вращающихся роторов базируются на основных положениях теории (динамики) машин и механизмов, разработанном ранее алгоритме 1, 2, ..., *N*-плоскостной балансировки жестких роторов методом амплитуд и анализе получения погрешностей расчетов.

**Результаты.** Получено утверждение о необходимости равенства начальных погрешностей измерения экспериментальных величин (времени, угла поворота, скорости вращения) выбега, которое приводит к минимизации погрешности нахождения искомых характеристик ротора. Построенный алгоритм последовательных вычислений однозначно определяет геометрические, массовые и фрикционные характеристики уравновешенной роторной системы в случае отрицательного дискриминанта суммарного момента сопротивления вращению. При другом, произвольном соотношении параметров роторной системы (не отрицательный дискриминант), методика и последовательность определения характеристик ротора остаются аналогичными. Планируется использовать алго-

ритм как один из потенциальных вариантов получения значений характеристик ротора, с последующим обоснованным выбором набора характеристик по методу наименьших квадратов и переходом к задаче балансировки.

**Научная новизна.** Выявлена основная причина или источник формирования погрешности определения геометрических, массовых и фрикционных характеристик роторных систем на выбеге – разница в погрешностях измерения отдельных параметров выбега: времени и скорости вращения угла поворота ротора. Теоретически при любых, но одинаковых по величине, погрешностях измерения на выбеге (например, его времени) каждого опыта получаем точное значение искомой характеристики.

**Практическая значимость.** Заключается в возможном использовании штатных остановок роторных систем для мониторинга основных суммарных характеристик роторов – износа рабочих органов, опорных подшипников, изменения величины технологического момента, перераспределения погонной массы ротора, проведения последующей задачи моментной балансировки по методу амплитуд. Предлагаемый вариант решения с использованием метода наименьших квадратов позволит, по своим критериям, обосновано выбрать наиболее подходящий набор характеристик из комплекта решений для нулевого (два возможных решения), не нулевого (обобщенный случай) и положительного дискриминанта момента сопротивления вращения ротора.

**Ключевые слова:** *роторная система, штатные остановки, выбег, осевой момент инерции, опорные подшипники, фрикционные характеристики, вес ротора, расположение центра масс*

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# DESIGN OF TIERED OPERATING UNIT FOR DEEP DIFFERENTIATED TILLAGE

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# КОНСТРУКЦІЯ ЯРУСНОГО РОБОЧОГО ОРГАНА ДЛЯ ГЛИБОКОЇ ДИФЕРЕНЦІЙОВАНОЇ РОЗРОБКИ ҐРУНТУ

**Purpose.** Determining the parameters of tillage operating unit design to provide energy-efficient tillage by means of operating units of excavation machinery.

**Methodology.** The theoretical studies were based on the general provisions of agricultural mechanics, the elements of continuum theory, Coulomb-Mohr theory of strength. Analytical and graphical analysis of mathematical models was implemented by means of their visual reproduction in space and time on a PC using applied and developed software.

**Findings.** The basic principles of adapted passive operating unit creation for vertically differentiated deep tillage and recultivation after opencast mining (e.g., the case of amber extraction) are described.

**Originality.** The mathematical models for the construction of energy-efficient, environmentally focused tools for loosening the soil and differentiated tillage based on the regularity research of the tool for loosening the soil chunk by means of two plane buckle have been obtained.

**Practical value.** Methods for the design and engineering calculation of tiered operating units for differentiated deep soil tillage have been developed.

**Keywords:** *operating unit, soil, loosening, tier*