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SEARCH OF VARIANTS OF ASSEMBLIES OF STRUCTURAL GROUPS IN PLANAR LINKAGES

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ПОШУК ВАРІАНТІВ СКЛАДАНЬ СТРУКТУРНИХ ГРУП У ПЛОСКИХ СТРИЖНЕВИХ МЕХАНІЗМАХ

Purpose. To determine the effect of the presence of prismatic kinematic pairs on possible versions of assemblies of the structural group of the third class in modern planar mechanisms.

Methodology. In the work an analytical study is carried out of possible assemblies of a four-link third class structural group, in which there are prismatic kinematic pairs in two links. The study was carried out using Mathcad mathematical packages.

Findings. Possible variants of the mutual arrangement of the links of a third class structural group at fixed positions of its external kinematic pairs are found. A polynomial of degree six is obtained and all its coefficients are defined. It means that the polynomial is of degree six both for the structural group with two prismatic pairs and for the group with revolute pairs.

It is shown that, for the variant of the group under consideration, the search for assemblies reduces itself to finding the real roots of a sixth-degree polynomial.

Dependences of the polynomial coefficients on the geometric parameters of the structural group are obtained. The finding of the roots of the polynomial is performed using the standard procedure of the Mathcad, which does not require specifying the initial approximations of unknowns.

Originality. It consists in determining the number of possible assemblies of a four-links third class structural group, in which two links have prismatic kinematic pairs. It has been determined for the studied structural group that maximum assembling number is four.

Practical value. The results of the research can be used in designing the mechanisms of new machines when choosing a specific variant of assembly of a third class structural group appropriate in the best way of set problem. The search for possible assemblies of third-class structural groups can be carried out numerically with the help of modern computer technologies, which makes it possible to determine both the number of possible solutions and the solutions themselves without specifying initial approximations.

Keywords: *mechanism assembly, structural group, geometrical analysis, search for polynomial roots, Mathcad program*

Introduction. Geometrical analysis is the first and the most important problem of kinematic mechanism analysis. As a result, link positions are determined. The problem of determining link position of the group is not uniquely defined even for the simplest mechanisms with the lowest kinematic pairs containing only one structural group of the second class. Non-uniqueness is manifested in various positions of the group links under given coordinates of its external kinematic pairs. When the class of the group and the number of its links are increased the amount of the group assemblies is also increased. If in the context of 2nd degree groups two possible assemblies are easily determined with the help of square equation, in the context of higher class groups (e.g. 3rd and higher) calculated dependences are of more complicated non-linear nature; moreover, cannot be solved analytically.

Analysis of the recent research. As far as back in the mid of last century V. A. Zinoviev proposed analytical method to determine positions of links of high-class mechanisms. The method is based on the idea that links of leverage are replaced by adequate vectors to develop a system of nonlinear equations of vector contours closeness. Angles determining position of the mechanism each link are unknown within the equation system. The method principle is in approximate numerical solution for the developed equation system with prescribed accuracy.

E. E. Peisakh was engaged in studies of positions of links in four-link Assur groups with turning pairs. He has shown that for the groups a problem amounts to determine real roots of sextic polynom. Moreover, he has also identified that maximum possible number for assembling variants of four-link group with turning pairs is six in terms of fixed positions of external joints.

One may state that numerical techniques are the only applicable to determine assembling variants. As for the equation systems or polynom, various techniques can be applied [1].

Already in this century works have been published that describe a method for finding of assemblies variants, based on the partition of a group of the 3rd class into simple groups of the 2nd class. Then for each simple group equations are making up (again nonlinear) the solution of which is found using the iteration method. All these works differ only in the mechanisms of the 3rd class, which demonstrate the application of the proposed method. In [2] was considered P. L. Chebyshev's rowing mechanism, in [3] – the mechanism of window control, in [4] – the mechanism of the press and in [5] – the mechanism of the 3rd class of arbitrary kind.

The authors propose to reviewers' attention numerical methods to estimate equation roots required to determine assembling variants for high-class mechanisms. They believe that numerical solution for nonlinear equations have following key disadvantages:

- solution convergence needs relatively accurate determination of initial approximations of variables;
- nonlinear equations always have several roots which amount is unknown.

The facts are involved to draw a conclusion on the necessity to develop an ingenious method for assembling variants determination.

Not disputing right of the technique to exist we cannot fall into line with their argumentation.

First, the necessity of determination as for accurate initial approximation of variables. One of the authors of [6] paper shows that current computer mathematical programs (Mathcad, Maple, Wolfram Mathematica [7, 8] etc.) are very potential to solve nonlinear equations numerically. In terms of three-link group of 3rd class with turning pairs (Fig. 1), paper [6] describes algorithm to solve problems of assembling variants using Mathcad program.

First kinematic pair C connecting link 2 and basic link 3 is unclosed; easily seen trajectories are made for breaking points; to do that visualization tools of Mathcad are applied. Circular curve is trajectory of a point belonging to link 2; coupler curve four-link chain is trajectory of point 2 (Fig. 2). Intersection points of the trajectories mean possible assemblies of the group.

Then tracking mode *X–Y* Trace can be used to deduce initial approximations for each of the assemblies. Exact values are determined with the help of operational unit Given – Find. Fig. 3 demonstrates all the six assemblies of the group deduced using the technique.

Hence, statements concerning complexity of procedure to determine initial angle approximation and uncertainty of possible solutions number can be considered as untenable.

Paper [3] analyzes specific mechanism to open window involving 3rd class group. The group contains two prismatic kinematic pairs of 5th class. The technique has helped the authors identify several assemblies of the specific group; however, it cannot reply to a question concerning the number of possible assemblies for such a group.

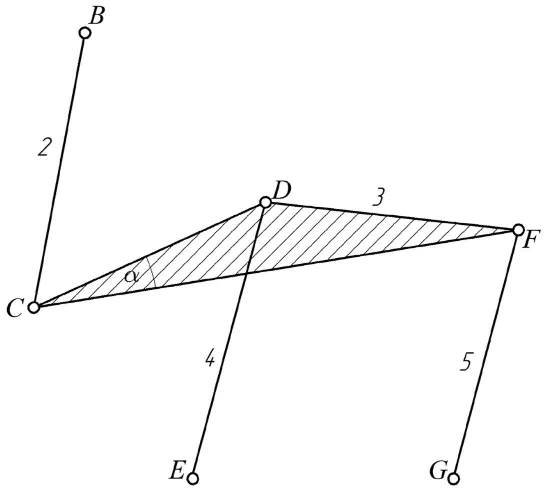

Fig. 1. Structural group of 3rd class including:

basic link – 3; links – 2, 4, 5; turning pairs B, C, D, E, F and G

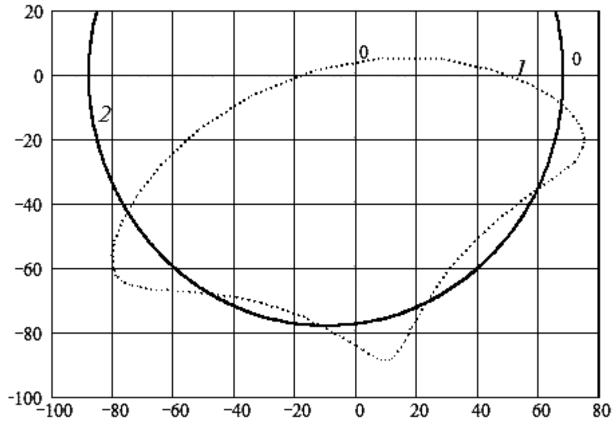

Fig. 2. Visualization of possible assemblies of 3rd class group:

1 is trajectory of point C (link 3); 2 is trajectory of point C (link 2)

Formulation of the research objective and purpose. Objective of the research is to analyze possible variants for 3rd class group assemblies where two links involve prismatic pairs of 5th class.

To succeed, following tasks have been formulated:

- analyze a mechanism similar to that described in [3] (Fig. 4) where one link (1) is a slider, and another one (4) is rotating guide;
- the developed nonlinear equations should be solved numerically with the help of Mathcad program procedure.

Presentation of the main research. Place reference point rectangular coordinate system in point *A*.

Let geometric parameters of group are specified: $x_A = 0$; $y_A = 0$; $x_F = 0.15$ m; $y_F = 0.7$ m; $l_{AD} = 0.6$ m; $l_{BD} =$ $= 0.6$ m.

It is required to determine every possible variant of relative position of the group links in terms of fixed positions of its external kinematic pairs *A*, *C* and *F*.

Replace links with proper vectors; specify them as it is shown in Fig. 5: $l_1 = l_{AB}$ is horizontal vector determin-

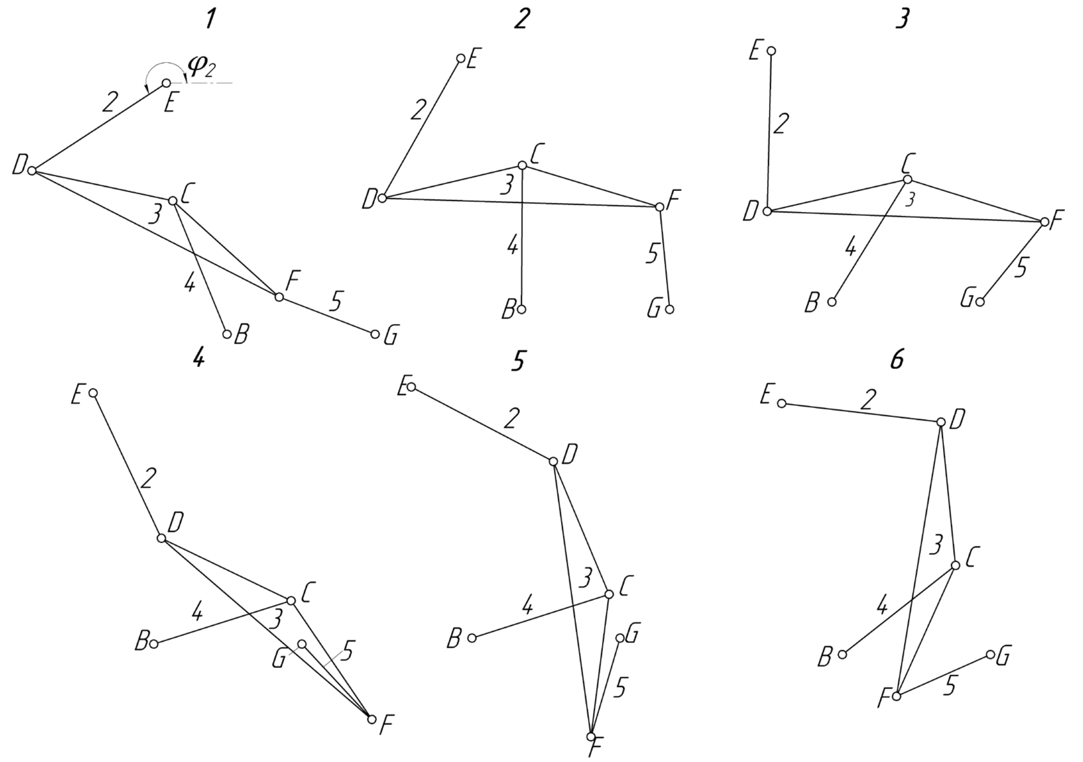

Fig. 3. Assemblies of 3[rd] class four-link group:

1 – φ_2 = 3.72 rad; 2 – φ_2 = 4.229 rad; 3 – φ_2 = 4.668 rad; 4 – φ_2 = 5.15 rad; 5 – φ_2 = 5.812 rad; 6 – φ_2 = 6.156 rad

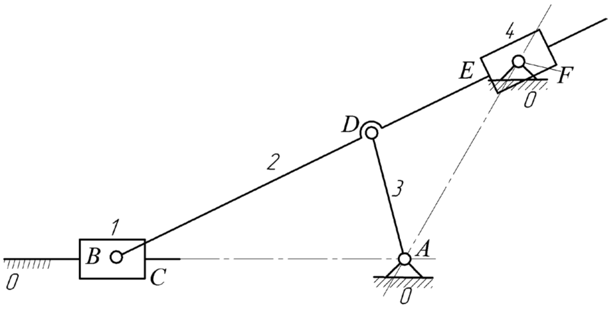

Fig. 4. Scheme of 3[rd] class group with two prismatic kinematic pairs:

1 is slider; 2 is connection rod; 3 is balance arm; 4 is rotating guide; A, B, D, F are turning pairs; C and E are prismatic kinematic pairs

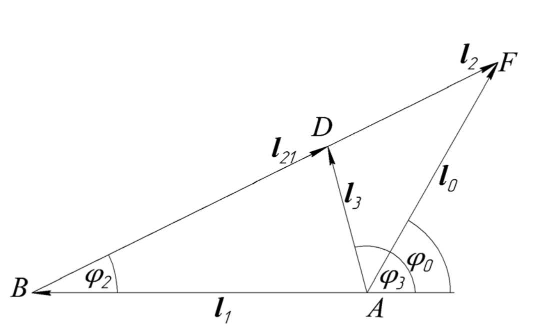

Fig. 5. Vector interpretation of the group links

ing slider B-position; $l_{21} = l_{BD}$; $l_2 = l_{BF}$; $l_3 = l_{AD}$. Vector module is $l_0 = \sqrt{x_F^2 + y_F^2}$. Angle is $\varphi_0 = \mathrm{arctg}\left(\dfrac{y_F}{x_F}\right)$.

The group contains two independent vector contours; closeness equations for them are

$$l_1 + l_{21} = l_3;$$

$$l_3 + l_2 - l_{21} = l_0.$$

Express vectors with the help of complex numbers.

$$l_1 + l_{21}e^{\varphi_2 i} = l_3 e^{\varphi_3 i};$$

$$l_3 e^{\varphi_3 i} + l_2 e^{\varphi_2 i} - l_{21}e^{\varphi_2 i} = l_0 e^{\varphi_0 i}.$$

The system consisting of two nonlinear equations involves two unknown angles φ_2 and φ_3. To preclude angle φ_3 represent equation two as follows

$$l_3 e^{\varphi_3 i} = l_{21}e^{\varphi_2 i} - l_2 e^{\varphi_2 i} + l_0 e^{\varphi_0 i}.$$

It is known from a theory of complex numbers that the equation

$$l_3 e^{-\varphi_3 i} = l_{21}e^{-\varphi_2 i} - l_2 e^{-\varphi_2 i} + l_0 e^{-\varphi_0 i},$$

is equivalent to previous one. Multiply the two equations term by term. On the left hand side we obtain l_3^2; on the right hand side we obtain polynom with one unknown angle φ_2.

After elementary transformation have been effected and universal trigonometric substitution have been applied

$$\sin\varphi_2 = \frac{2\,\mathrm{tg}\dfrac{\varphi_2}{2}}{1 + \mathrm{tg}^2\dfrac{\varphi_2}{2}};$$

$$\sin\varphi_2 = \frac{1-\mathrm{tg}^2\dfrac{\varphi_2}{2}}{1+\mathrm{tg}^2\dfrac{\varphi_2}{2}},$$

we obtain sextic concerning $x = \mathrm{tg}\dfrac{\varphi_2}{2}$. The equation is $k_1x^6 + k_2x^5 + k_3x^4 + k_4x^3 + k_5x^2 + k_6x + k_7 = 0$. Fig. 6 demonstrates a fragment of Mathcad document representing formulas to determine all the seven polynomial coefficients. Coefficient one is $k_1 = 0$ if point 7 ordinate is equal to zero to be impossible as the group cannot exist. It means that both for the group with two prismatic pairs and for group with turning pairs only polynom is of the sixth degree.

Thus, determination of assembling variants for the concerned group is to find roots of 6[th] class polynom. For this purpose, Mathcad program provides specific function called polyroots(v) where v is a vector of polynom coefficients. Fig. 7 explains relevant fragment of Mathcad program document.

After polynom roots have been estimated it is time to initiate determination of values of angles

$$\varphi_{2i} = 2\,\mathrm{arctg}\,x_i.$$

Table demonstrates values of angles φ_{2i} corresponding to values x_i.

It follows from the group existence condition that $0 < \varphi_2 < \pi$. Thus, in terms of concerned structural group possible assembling number is four (Fig. 8).

Conclusions. Geometry analysis of three-link structural group of 3[rd] class where two links contain prismatic kinematic pairs of 5[th] class has been carried out. It has been shown that for such a group assembling determination is to estimate real roots of 6[th] class polynom. Dependences of polynom coefficients on geometric parameter of the group have been developed.

In the context of the analyzed group it has been determined that maximum assembling number is four.

It has been demonstrated that current computer facilities make it possible to determine numerically both the number of possible solutions and solutions themselves without initial approximation deducing.

$$k_1 := \frac{1}{4}y_F^2;\quad k_2 := -l_{21}y_F + l_0\cos(\varphi_0)y_F;$$
$$k_3 := l_{21}^2 + \frac{3}{4}y_F^2 + l_0^2 - l_3^2 - 2l_0\sin(\varphi_0)y_F - 2l_0\cos(\varphi_0)l_{21};$$
$$k_4 := 4l_0\sin(\varphi_0)l_{21} - 2l_{21}y_F;$$
$$k_5 := l_{21}^2 + \frac{3}{4}y_F^2 + l_0^2 - l_3^2 + 2l_0\cos(\varphi_0)l_{21} - 2l_0\sin(\varphi_0)y_F;$$
$$k_6 := -l_{21}y_F - l_0\cos(\varphi_0)y_F;\quad k_7 := \frac{1}{4}y_F^2.$$

Fig. 6. Fragment 1 of Mathcad document

$$v := (k_1\quad k_6\quad k_5\quad k_4\quad k_3\quad k_2\quad k_1)^T$$
$$v^T = (0.122\quad -0.525\quad 0.08\quad 0.84\quad -0.28\quad -0.315\quad 0.122)$$
$$\mathrm{polyroots}(v)^T = (-1.237\quad -1.082\quad 0.274\quad 0.808\quad 1.403\quad 2.405)$$

Fig. 7. Estimation of polynom roots

Table

The values of x_i and φ_{2i}

Parameter	Values of quantities					
x_i	–1.237	–1.082	0.274	0.808	1.403	2.405
φ_{2i}	–102.09	–94.52	30.62	77.90	109.06	134.85

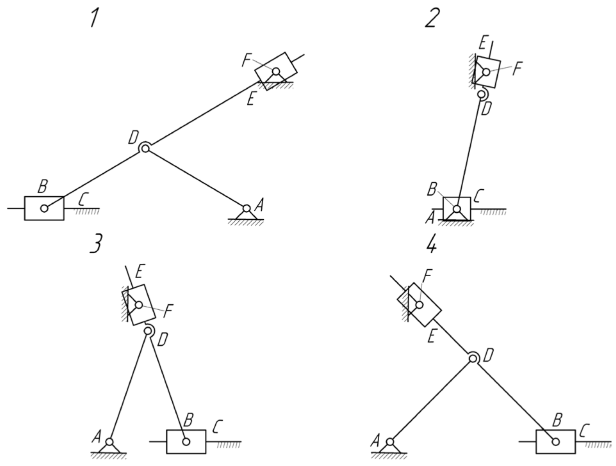

Fig. 8. Possible assembling of the group are:

1 – $\varphi_2 = 30.62°$; *2* – $\varphi_2 = 77.90°$; *3* – $\varphi_2 = 109.06°$; *4* – $\varphi_2 = 134.85°$

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Мета. Встановити вплив наявності поступальних пар п’ятого класу на можливі варіанти складань структурної групи третього класу у плоских важільних механізмах, що застосовуються в сучасній техніці.

Методика. У роботі виконано аналітичне дослідження можливих складань триповодкової структурної групи третього класу, у двох поводках якої є поступальні кінематичні пари. Дослідження проведено за допомогою багатофункціонального пакета прикладних програм Mathcad.

Результати. Виявлені можливі варіанти взаємного розташування ланок групи третього класу при фіксованих положеннях її зовнішніх кінематичних пар. Отримано поліном шостого ступеня й визначені всі його коефіцієнти. Результат говорить про те, що й для такої структурної групи з двома поступальними парами, як і для групи тільки з обертальними парами, поліном має шосту ступінь.

Показано, що й для розглянутого варіанту групи пошук складань зводиться до знаходження дійсних коренів полінома шостого ступеня.

Отримані залежності коефіцієнтів полінома від геометричних параметрів групи.

Наукова новизна. Полягає у визначенні кількості можливих складань чотириланкової структурної групи третього класу, у складі якої два поводки мають поступальні кінематичні пари п’ятого класу. Для дослідженої структурної групи встановлено, що максимальне число складань дорівнює чотирьом.

Практична значимість. Результати дослідження можуть бути використані при проектуванні механізмів нових машин при виборі конкретного варіанту складання структурної групи третього класу, що відповідає найкращим чином поставленому завданню. Пошук можливих складань структурних груп третього класу можна здійснювати чисельно за допомогою сучасних комп’ютерних технологій, що дозволяє визначати й кількість можливих рішень, і самі рішення без завдання початкових наближень.

Ключові слова: *складання механізму, структурна група, геометричний аналіз, пошук коренів поліному*

Цель. Установить влияние наличия поступательных пар пятого класса на возможные варианты сборок структурной группы третьего класса в плоских рычажных механизмах, применяемых в современной технике.

Методика. В работе выполнено аналитическое исследование возможных сборок трехповодковой структурной группы третьего класса, в двух поводках которой имеются поступательные кинематические пары. Исследование проведено с помощью многофункционального пакета прикладных программ Mathcad.

Результаты. Найдены возможные варианты взаимного расположения звеньев группы третьего класса при фиксированных положениях её внешних кинематических пар. Получен полином шестой степени и определены все его коэффициенты. Результат говорит о том, что и для такой структурной группы с двумя поступательными парами, как и для группы только с вращательными парами, полином имеет шестую степень.

Показано, что и для рассмотренного варианта группы поиск сборок сводится к нахождению действительных корней полинома шестой степени.

Получены зависимости коэффициентов полинома от геометрических параметров группы.

Научная новизна. Заключается в определении количества возможных сборок четырехзвенной структурной группы третьего класса, в составе которой два поводка имеют поступательные кинематические пары пятого класса. Для исследованной структурной группы установлено, что максимальное число сборок равно четырем.

Практическая значимость. Результаты исследования могут быть использованы при проектировании механизмов новых машин при выборе конкретного варианта сборки структурной группы третьего класса, соответствующей наилучшим образом поставленной задаче. Поиск возможных сборок структурных групп третьего класса можно осуществлять численно с помощью современных компьютерных технологий, что позволяет определять и количество возможных решений, и сами решения без задания начальных приближений.

Ключевые слова: *сборки механизма, структурная группа, геометрический анализ, поиск корней полинома*

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