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RBF NEURAL NETWORKS OPTIMIZATION OF THE CONTROL OVER THE CLASS OF STOCHASTIC NONLINEAR SYSTEMS WITH UNKNOWN PARAMETERS

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ОПТИМІЗАЦІЯ КОНТРОЛЮ НЕЛІНІЙНИХ СТОХАСТИЧНИХ СИСТЕМ З НЕВІДОМИМИ ПАРАМЕТРАМИ ЗА ДОПОМОГОЮ РАДІАЛЬНО-БАЗИСНИХ НЕЙРОМЕРЕЖ

Purpose. There are two types of uncertainties, reducible and irreducible. The uncertainties not only include the uncertainty of the noise signal, but also include the uncertainty of system parameters. In the research, the control problem of the stochastic nonlinear systems with unknown parameters was considered. The neural network was used to solve the control and learning problem in a class of nonlinear systems with unknown parameters. The objective is to control the stochastic, single-input single-output, affine nonlinear system.

Methodology. RBF neural networks were used to approximate the nonlinear function. The optimal linear parameters requiring learning appeared linearly in the output equation.

Findings. The estimated value is substituted into the control law. Then the weight is optimized to make the control achieve the compromise between control and learning performance.

Originality. The controller developed can control the system output towards the desired output, and it can learn the unknown parameters in the system. The controller is implemented easily.

Practical value. The simulation results have proved that the control law obtained is effective, and it is better than the previous control strategy.

Keywords: *radial basis function, neural network, control law, stochastic nonlinear, control problem, unknown parameters*

Introduction. The world is full of uncertainties. There are two types of uncertainties, reducible and irreducible. The uncertainties not only include the uncertainty of the noise signal, but also include the uncertainty of system parameters. These make the control problem not simply use a deterministic model to describe and control in actual industrial processes, as well as social, economic and other fields [1, 2].

There exist two types of learning strategies to reduce reducible uncertainties, active leaning and passive learning [2, 3]. Except for a few ideal situations, an optimal control usually pursues two often conflicting objectives: to drive the system toward the desired state, and to perform active learning to reduce the systems reducible uncertainty, as pointed by Feldbaum A.A. in his seminal work more than thirty-five years ago [1]. The dual roles of an optimal control, optimization, and estimation, in common situations, cannot be separated. This coupling between optimization and estimation makes an analytical form of optimal control, in most situations, unattainable. Previous efforts in dual control have thus mainly been devoted to the development of certain suboptimal solution schemes, such as certainty equivalence scheme and open-loop feedback control, by passing this essential feature of coupling in dual control [4]. Most resulting suboptimal control laws are of a nature of passive learning, since the function of future active probing of the control is purposely deprived in order to achieve an analytical attainability in the solution process. Recent work on a class of dual control problems where there exists a parameter uncertainty in the observation equation of the linear quadratic Gaussian problem. An analytical active dual control law is derived by a variance minimization approach [2].

In addition, in recent years, with the rapid development of the neural network, it has been effectively used in nonlinear system identification and control. Compared to the other nonlinear identification methods, neural network does not depend on model function, and not need to know the mathematical relationship between input and output in the nonlinear system (measured system) [5, 6].

In this research, the neural network is used to solve the control and learning problem in a class of nonlinear systems with unknown parameters.

Problem statement. ***Control object.*** The objective is to control the stochastic, single-input single-output, affine nonlinear system. Its general form is

$$y(k) = H[x(k-1)] + G[x(k-1)]u(k-1) + e(k-1). \quad (1)$$

Where $u(k)$ is control signal, namely system input; $y(k)$ is system output. $e(k)$ is a zero-mean Gaussian noise signal with known variance σ_e^2. $H[x(k-1)]$ and $G[x(k-1)]$ are unknown nonlinear functions. And $x(k-1)$ includes known system states, and its form is $x(k-1) = [y(k-n), ..., y(k-1), u(k-1-m), \cdots u(k-2)]$, and $m \le n$.

Assume that $y_r(k)$ is desired output, $I(k)$ is available system input and output information from the beginning to k time, and the form is

$$I(k) = [y(k), \cdots, y(1), u(k-1), \cdots, u(0)].$$

Objective function. For the stochastic system, controller aims at system output $y(k)$ that tracks desired output $y_r(k)$. Thus, the objective function is defined as

$$J = E\{[y(k+1) - y_r(k+1)]^2 \mid I(k)\}. \quad (2)$$

Here, $E\{\cdot \mid I(k)\}$ is mathematical expectation.

For the objective function (2), because dynamic programming cannot obtain analytical solutions, a modified objective function is proposed from a random sub-optimal point of view

$$(P) \bar{J} = J_c + \exp^{-J_i}, \quad (3)$$

here

$$J_c = E\{[y(k+1) - y_r(k+1)]^2 + qu^2(k) \mid I(k)\};$$

$$J_i = E\{[\varepsilon^2(k+1) \mid I(k)\},$$

here, $\varepsilon(k+1) = y(k+1) - \hat{y}(k+1), \hat{y}(k+1)$ is system estimated output.

On the basis of the original object function J, J_c adds a $qu^2(k)$. q is a weight, clearly, higher q induces a penalty for a large control signal, and reflects that in practice the control amplitude needs to be constrained. It is called control objec-tive. And J_i expresses learning requirements of unknown no-nlinear functions, is called learning object. Therefore, using objective function (3), the controller has a dual property, can implement the balance between control and learning.

RBF neural network. First, the design of controller needs to learn the nonlinear functions. RBF neural networks can be used to approximate the nonlinear function [7]. The structure of RBF neural network is shown in Fig. 1.

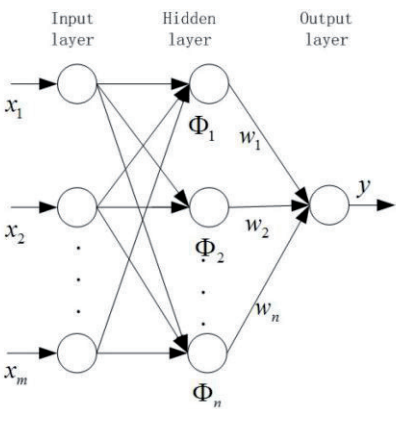

Fig. 1. Structure of RBF neural network

RBF neural network can be used to approximate functions $H[x(k-1)]$ and $G[x(k-1)]$. Assume that network approximation errors are negligibly small, RBF neural network can be used to approximate nonlinear function and network's linear parameter vectors are equal to optimal value ω_H^*, and ω_G^*.

$$H[x(k-1)] = \omega_H^* \Phi_H[x(k-1)];$$

$$G[x(k-1)] = \omega_G^* \Phi_G[x(k-1)],$$

here $\Phi_H[x(k-1)]$, $\Phi_G[x(k-1)]$ are Gaussian basis function vectors, whose element is given by

$$\Phi_{H_i}[x(k-1)] = \exp\left\{\frac{-\left\|\mathbf{x} - \mathbf{m}_{H_i}\right\|^2}{2\sigma_H^2}\right\};$$

$$\Phi_{G_i}[x(k-1)] = \exp\left\{\frac{-\left\|\mathbf{x} - \mathbf{m}_{G_i}\right\|^2}{2\sigma_G^2}\right\}.$$

Here $\mathbf{m}_{H_i}, \mathbf{m}_{G_i}$ are center of the basis function and σ_H^2, σ_G^2 are variances.

The basis functions are centered on points of a regular square sampling mesh where the mesh spacing and variance of basis functions are chosen a priori, so that only linear parameters are unknown, need to be learned.

Hence from (1), it follows that

$$\omega^*(k+1) = \omega^*(k);$$

$$y(k) = \omega^{*T}(k)\Phi[x(k-1)] + e(k-1),$$

where

$$\omega^*(k) = [\omega_H^*(k), \omega_G^*(k)];$$

$$\Phi[x(k-1)] = [\Phi_H[x(k-1)], \Phi_G[x(k-1)]u(k-1)].$$

The optimal linear parameters requiring learning appear linearly in output equation so that well-established technique based on Kalman filtering [8] can be used if we assume that initial optimal parameter vector ω_0^* has a Gaussian distribution with mean $\mathbf{m}$ and covariance R_0.

Defining $\hat{\omega}(k) = E\{\omega(k) \mid I(k)\}$ and using Kalman filter theory [8], we obtain the following recursive parameter learning rules

$$\hat{\omega}(k+1) = \hat{\omega}(k) + K(k)\varepsilon(k);$$

$$K(k) = \frac{P(k)\Phi[x(k-1)]}{\Phi^T[x(k-1)]P(k)\Phi[x(k-1)] + \sigma^2};$$

$$P(k+1) = P(k)I - K(k)\Phi^T[x(k-1)]P(k).$$

Here $K(k)$ is filter gain and $P(k+1)$ is covariance with initial conditions $\hat{\omega}(0) = m, P(0) = R_0$. $\varepsilon(k) = y(k) - \hat{\omega}^T(k)\Phi \times$ $\times[x(k-1)]$ denotes the innovation at time k.

Control law. The problem *(P)* is very difficult to be solved directly, so a multiobjective optimization problem (MOP) is constructed as follows

$$\min_{u(k)} [J_c(k), -J_i(k)].$$

Defination 1. Suppose that $\hat{u}(k)$ is the feasible solution of problem (MOP), if there is no feasible solution $u(k)$, which makes

$$J_c(u(k)) \le J_c(\hat{u}(k));$$

$$J_i(u(k)) \geq J_i(\hat{u}(k)).$$

At least one inequality was strictly established. And $\hat{u}(k)$ is the non-inferior solution of the problem (MOP).

Theorem 1. The optimal solution of problem *(P)* must be in the non-inferior solution set of problem (MOP).

Proof. Reduction to Absurdity. Suppose that $u^*(k)$ is the optimal solution of the problem *(P),* but is not noninferior solution of the problem (MOP). Then, there is a feasible solution $u^*(k)$, which makes

$$J_c(k)|_{\{u(k)\}} \leq J_c(k)|_{\{u^*(k)\}};$$

$$J_i(k)|_{\{u(k)\}} \geq J_i(k)|_{\{u^*(k)\}}.$$

At least one inequality strictly established. Obviously, $J(k)$ monotonically increase on $J_c(k)$, monotonically decrease on $J_i(k)$. And then according to Defination 1, it is obtained that

$$J(k)|_{\{\hat{u}(k)\}} \leq J(k)|_{\{u^*(k)\}}.$$

There is a conflict between this inequality and u's optimality. Therefore, the optimal solution $u^*(k)$ of the problem *(P)* is in noninferior solutions set of the problem (MOP).

This theorem shows that finding optimal solution of the problem *(P),* only focuses on noninferior solutions set of the problem (MOP). The noninferior solutions set can be obtained from the following Lagrange problem (LOP).

$$(LOP)\min_{u(k)}[J_c(k) + \lambda J_i(k)].$$

It is relatively easy to optimize this problem with respect to $u(k)$ by differentiation and equating to zero, using Kalman filter equation, which results in the following control law.

$$u^*(k) = \frac{(y_r(k+1) - \hat{H}^2(k+1))\hat{G}^2(k+1)}{\hat{G}^2(k+1) + q + (1+\lambda)v_{GG}} - \frac{(1+\lambda)v_{GH}}{\hat{G}^2(k+1) + q + (1+\lambda)v_{GG}}. \quad (4)$$

Here, $\hat{H}[\cdot]$ and $\hat{G}[\cdot]$ are respectively $[x(k), \hat{\omega}_H(k+1)]$ and $[x(k), \hat{\omega}_G(k+1)]$. $v_{GH} = \Phi_G^T[x(k)]P_{GH}(k+1)\Phi_H[x(k)]$ and $v_{GG} = \Phi_G^T[x(k)]P_{GG}(k+1)\Phi_G[x(k)]$. P_{GH} and P_{GG} are submatrices of matrix $P(k+1)$, and matrix $P(k+1)$ is repartitioned as

$$\begin{pmatrix} P_{HH}(k+1) & P_{GH}^T(k+1) \\ P_{GH}(k+1) & P_{GG}(k+1) \end{pmatrix}.$$

In the control law, parameter λ is acting as a weight.

The case $\lambda = -1$ and $q = 0$, corresponds to a controller de-signed on a heuristic certainty equivalence basis [2]. The pa-rameter estimates $\hat{\omega}(k)$ are used as if they were the optimal parameters ω^* by replacing the actual nonlinear system func-tions, completely disregarding the approximation uncertainty.

The case $\lambda = 0$ is equivalent to a cautious controller with the disadvantages normally associated with this kind of suboptimal controller [4]. Strong emphasis is given to the uncertainty of the parameter estimates and the controller is very cautious on using them.

The case $-1\langle\lambda\langle 0$ provides a compromise between two extremes, being neither too cautious nor too bold. This is similar to Simon's controller. It takes λ as a fixed value in the whole control process, and λ may not be the optimal compromise [9]. So we need to find the optimal compromise λ.

Theorem 2. The corresponding λ^* non-inferior solution is the problem *(P)* of the optimal solution, then

$$\lambda^*(k) = -\exp^{-J_i(k)}|_{u^*(k)}.$$

Proof. The optimal solution $(\hat{u}(k,\lambda))$ of problem (LOP) substitutes into the objective function of problem *(P).* If λ^* is optimal, then

$$\frac{dJ_c(k)}{d\lambda}|_{\lambda^*} - \exp^{-J_i(k)}\frac{dJ_i(k)}{d\lambda}|_{\lambda^*} = 0.$$

For problem (LOP), corresponding to the optimal solution at λ_3, according to [10], it has

$$\frac{dJ_c(k)}{d\lambda}|_{\lambda^*} + \lambda^*\frac{dJ_i(k)}{d\lambda}|_{\lambda^*} = 0.$$

Combined the above two equations, we can obtain $\lambda^*(k) = -\exp^{-J_i(k)}|_{u^*(k)}$.

This theorem gives the optimal weight factor at each stage.

Then, we will derive the searching optimal modified formulas, to seek out problem (*P*)'s optimal solution in problem (MOP)'s non-inferior solution set.

For problem (*P*), the gradient of $\bar{J}(k)$ is

$$\nabla\bar{J}(k) = \left[\frac{\partial\bar{J}(k)}{\partial J_c(k)}, \frac{\partial\bar{J}(k)}{\partial J_i(k)}\right]^T = [1, -\exp^{-J_i(k)}]^T.$$

Assuming $\omega = [1,\lambda]^T$, we construct the following direction vector

$$V(\omega) = [V_1(\omega), V_2(\omega)]^T = -\nabla\bar{J}(k) + \frac{\omega^T\nabla\bar{J}(k)}{\omega^T\omega}\omega.$$

According to Cauchy-Schwarz in equality, it has

$$\nabla\bar{J}(k)V(\omega) = -\left\|-\nabla\bar{J}(k)\right\|^2 + \frac{(\omega^T\nabla\bar{J}(k))^2}{\omega^T\omega} \leq 0.$$

This shows that $V(\omega)$ is a $\bar{J}(k)$'s descent direction. Obviously, $V(\omega) = 0$, it shows that the optimal condition is established, namely $\lambda = -\exp^{-J_i(k)}$.

Supposing that sth iteration's λ denotes λ^s. Using the algorithm of gradient search, the formula is

$$\lambda^{s+1} = \lambda^{s} - \mu V_2(\omega). \quad (5)$$

Where, μ is the searching step length. $V_2(\omega) = 0$ ends searching λ, means we have find the optimal dual control. And the optimal dual control is named by Weight Optimal Dual Control (WODC).

To sum up, the procedures of solving the control signal $u(k)(k = 0,1,\cdots,N-1)$ are:

Step 1: Using Kalman filter equation, calculate RBF networks parameter vectors $\hat{\omega}$, estimate variance $P(k)$.

Step 2: Given a λ, solving problem (LOP), we can obtain control $u(k)$ from equation (4).

Step 3: Judging $\left|\lambda + \exp^{-J_i(k)}\right| \le \varepsilon$, if control $u(k)$ meetinequality, it means $u(k)$ is optimal control, and continue next step. Otherwise, λ need to be modified using equation (5), then switch Step 2.

Step 4 : Judging $k = N-1$, if it meet the condition, procedure is end; if not, $k = k+1$, and switch Step 2.

Simulation analysis.

The simulation plant is given by

$$y(k+1) = \sin(3y(k)) + \cos(y(k)) + \\ + (3 + 2\cos(y(k)))u(k) + e(k).$$

Where the noise variance $\sigma^2 = 0.01$ and $x(k) = y(k)$. $H[x(k)] = \sin(3x(k)) + \cos(x(k))$ and $G[x(k)] = 3 + 2\times \times\cos(x(k))$ respectively is an unknown nonlinear function. The reference input is obtained by sampling a unit amplitude, 0.1Hz square wave filtered by transfer function $1/(s+1)$. A Gaussian RBF network is implemented in this example. For unknown output amplitude and priori knowledge, the H network is chosen to have Gaussian basis functions of variance 1 placed on a mesh of spacing 1, whilst the G network basis functions have variance of 3.6 and a mesh spacing of 2. The Kalman filter initial parameter covariance was set to $P(0) = 100I$.

We can compare the proposed algorithm and former algorithms on learning and control performance by a loss function. And the form is

$$J' = \frac{1}{M}\sum_{j=1}^{M}\frac{1}{N}\sum_{t=1}^{N}[y(t) - y_r(t)]^2.$$

Where, N is simulation step, and is equal to 100. M is the number of Monte Carlo trial, and is equal to 200.

Fig. 2 shows that cautious, dual, and weight dual control output curves respectively. Obviously, after about 10 steps, WODC tracks the reference output; oscillation amplitude is small, short duration; while, at the DC and CC it lasts about 17 steps, output tracks the reference output.

Based on the results shown in Table, we can clearly prove the proposed algorithm has a remarkable improvement comparing with the former algorithms.

Conclusion. The research was focused on the unknown parameters of nonlinear stochastic system of learning and control. The weight optimal dual control algorithm was proposed. RBF neural networks are used to approximate nonlinear functions online. The estimated value is substituted into the control law. Then the weight is optimized to make the control achieve the optimal compromise between control and learning performance. The simulation results showed that the proposed control algorithm is better than the former.

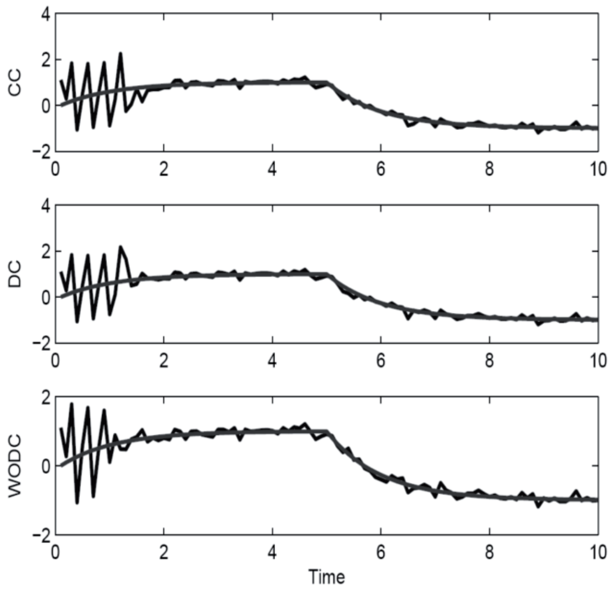

Fig. 2. Reference output and actual output

Table

Comparison of loss function

	CC	DC	WODC
Average value	14.9299	13.6879	11.5245
Minimum	4.4922	4.1830	3.1547

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References / Список літератури

1. Alpcan, T., Shames, I., Cantoni, M. and Nair, G., 2013. Learning and information for dual control. In: *Proc. of the 19th Asian Control Conference*, pp. 1–6, Istanbul, June 2013.

2. Alpcan, T. and Shames, I., 2015. An Information-based learning approach to dual control. *IEEE Neural Networks and Learning Systems*, vol. 26, no.11, pp. 2736–2748.

3. Kronander, K. and Billard, A., 2015. Passive interaction control with dynamical systems. *IEEE Robotics and Automation Letters*, vol. 1, no.1, pp. 106–113.

4. Zacekova, E., Privara, S., Vana, Z., Cigler, J. and Ferkl, L., 2013. Dual control approach for zone model predictive control. In: IEEE, *Proc. of Control Conference*, pp. 1398–1403, Europe, July 2013.

5. C. L. P. Chen, Yan-Jun Liu and Guo-Xing Wen, 2014. Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems. *Cybernetics*, vol. 44, pp. 583–593.

6. Tieshan Li, Zifu Li, Dan Wang, Chen, C.L.P., 2015. Output-feedback adaptive neural control for stochastic nonlinear time-varying. *Neural Networks and Learning Systems*, vol. 26, pp. 1188–1201.

7. Yunpeng Pan and Jun Wang, 2012. Model predictive control of unknown nonlinear dynamical systems based on recurrent neural networks. *IEEE Industrial Electronics*, vol. 59, no.8, pp. 3089–3101.
8. Talel, B. and Ben Hmida, F., 2013. Fuzzy Kalman filter for nonlinear stochastic systems. In: IEEE, *10th International Multi-Conference on Systems, Signals & Devices*, pp.1–7, Hammamet, 18–21 March 2013.
9. Wei-Yu Chiu, Bor-Sen Chen, Poor, H.V., 2013. A *multi-objective approach for source estimation in fuzzy networked system*s. *IEEE Circuits and Systems I: Regular Papers*, vol. 60, no.7, pp. 1890–1900.
10. Zhao, J, Wei, H, Zhang, C, Li, W, Guo, W and Zhang, K., 2015. Natural gradient learning algorithms for RBF networks. *Neural Computation*, vol. 27, no.2, pp. 481–505.

Мета. Існує два типи невизначеності – та що усувається та не усувається. Невизначеність включає не лише невизначеність сигналу перешкод, але й невідомість параметрів системи. У роботі розглянуте завдання управління стохастичними нелінійними системами з невідомими параметрами. Для вирішення завдання контролю та навчання класу нелінійних систем з невідомими параметрами використана нейронна мережа. Мета полягає в контролі стохастичної, однорідної, нелінійної системи з одним входом і одним виходом.

Методика. Апроксимація нелінійної функції була проведена за допомогою радіально-базисної нейронної мережі. Оптимальні лінійні параметри, що вимагають визначення, послідовно (лінійно) з'являються в рівнянні виходу.

Результати. Розрахункове значення підставляється до алгоритму управління. Потім вага оптимізується для досягнення системою контролю компромісу між показниками контролю та навчання.

Наукова новизна. Розроблений контролер дозволяє скорегувати вихід (вихідні характеристики) системи до бажаного та визначити невідомі параметри системи. Запропонований контролер легкий у використанні.

Практична значимість. Результати моделювання показали, що розроблений алгоритм управління ефективний і перевершує стратегію контролю, що раніше використовувалася.

Ключові слова: *радіальна базисна функція, нейронна мережа, алгоритм управління, стохастичний, нелінійний, завдання управління, невідомі параметри*

Цель. Существует два типа неопределенности – устранимая и неустранимая. Неопределенность включает не только неопределенность сигнала помех, но и неизвестность параметров системы. В работе рассмотрена задача управления стохастическими нелинейными системами с неизвестными параметрами. Для решения задачи контроля и обучения класса нелинейных систем с неизвестными параметрами использована нейронная сеть. Цель заключается в контроле стохастической, однородной, нелинейной системы с одним входом и одним выходом.

Методика. Аппроксимация нелинейной функции была проведена посредством радиально-базисной нейронной сети. Требующие определения оптимальные линейные параметры последовательно (линейно) появляются в уравнении выхода.

Результаты. Расчетное значение подставляется в алгоритм управления. Затем вес оптимизируется для достижения системой контроля компромисса между показателями контроля и обучения.

Научная новизна. Разработанный контроллер позволяет скорректировать выход (выходные характеристики) системы до желаемого и определить неизвестные параметры системы. Предложенный контроллер легок в применении.

Практическая значимость. Результаты моделирования показали, что разработанный алгоритм управления эффективен и превосходит ранее использовавшуюся стратегию контроля.

Ключевые слова: *радиальная базисная функция, нейронная сеть, алгоритм управления, стохастический, нелинейный, задача управления, неизвестные параметры*

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