

определены особенности и оценены показатели работы рассматриваемых систем.

Результаты. Установлено, что улучшение показателей работы исследуемых систем возможно путем расширения скользящего окна и увеличения количества циклов декодирования по сравнению с дифференциальными системами с LDPC-кодированием и дифференциальным детектированием с преобразованием.

Научная новизна. Доказано, что предлагаемая схема может применяться для решения проблемы ухудшения рабочих показателей дифференциальных систем с LDPC-кодированием и дифференциальным детектированием с преобразованием.

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Практическая значимость. Предлагаемая схема может использоваться в системах беспроводной связи, когда когерентное детектирование дорого или неосуществимо.

Ключевые слова: LDPC-код, дифференциальная система с LDPC-кодированием, дифференциальное детектирование с преобразованием, многосимвольное дифференциальное детектирование, SISO-модуль, выходящая характеристика

Рекомендовано до публікації докт. техн. наук
В.І. Корнієнком. Дата надходження рукопису
28.04.14.

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PERFORMANCE ANALYSIS IN A CALL CENTER WITH CALLS' ABANDONMENT AND OPTIONAL FEEDBACK

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АНАЛІЗ ПРАЦЕЗДАТНОСТІ КОЛЛ-ЦЕНТРА З ФУНКЦІЯМИ ПОВОДЖЕННЯ З НЕТЕРПЛЯЧИМИ КЛІЄНТАМИ ТА ОПЦІЙНОГО ЗВОРОТНОГО ЗВ'ЯЗКУ

Purpose. With the increase of the call center business and the equipment update, the Interactive Voice Response Units (IVRU) become widely used in call centers. This research investigated two kinds of call problems (abandonment and optional feedback) and the role of the service channel (servers and the IVRU) in a call center. We have obtained some important performance measures, which are very helpful for optimizing call centers.

Methodology. We formulated the call center with the Interactive Voice Response Units by a two-stage queuing system. Applying the queuing theory, we discussed a call center with the customers' impatience, optional feedback, and part shut-down of the servers.

Findings. We first get the systems' Q-matrix, and then by using the Structured Gaussian Elimination method, we obtained the idle probability, the average number of customers in the second-level queue, the leaving probability due to customers' impatience and some other performance measures.

Originality. We made a study of a call center made up of trunk lines, interactive voice response units (IVRU) and agents. We discussed a partial closing rule, call abandonment and feedback in the center. The research on this aspect has not been found at present.

Practical value. We have also considered the fact that some customers who are dissatisfied with the service may return for service, and they may return to the Interactive Voice Response Units or the servers or both. Our model is more reasonable and close to widely used call centers.

Keywords: call center, partial closing rule, call abandonment, optional feedback, the Interactive Voice Response Units

Introduction. A call center is a place where agents handle a large volume of incoming and outgoing calls for various purposes. Call center business is developing rapidly in the past few years [1–9]. Based on queuing theory, some call centers in real life are discussed; this kind of analysis

can be more accurate and closer to our life. Several researchers [3, 4] considered call abandonment and retrial. They obtained the stationary distribution of the system and other performance measures, but they did not consider the call center with the Interactive Voice Response Units (IVRU). Customers who enter the call center with the IVRU can receive the automatic service by the IVRU first,

and then choose service from agents. Srinivasan [5] and Wang [6] both discussed call centers with IVRU, they obtained the explicit expressions for the stationary distribution of their investigated systems. Zhang [7] further consider retrial phenomenon in call centers. Chen [8] surveyed a call center with a closing rule and impatient customers.

When the customers who are dissatisfied with the service return for service, and they can choose the Interactive Voice Response Units or the servers or both. In this research we investigated the combination of the calls (abandonment and optional feedback) and the role of the service channel (servers and the IVRU) in a call center as the research on this aspect had not been found.

Based on the queuing model, we discussed a call center with the customers' impatience, optional feedback, and part shutdown of the servers. Our model is more reasonable and alike to widely used call centers. The remainder of the paper is organized as follows. In Section 2, we present the model description. In Section 3, we develop some significant performance measures for the system in the steady state. In Section 4, some numerical examples are given, and conclusions are presented at the end of the paper.

Model description. In this paper, we consider a call center made up of telephone trunk lines, an automatic call distributor (ACD) together with the Interactive Voice Response Units (IVRU) and servers (or agents). Readers can see the operation of a call center in fig. 1. The first one represents the IVRU processor, the second queue represents the calls, which will be served by agents. Assuming: (1) The call center consists of N trunk lines, S agents ($S \leq N$). This IVRU processor can handle at most N jobs at a time, where N represents the total number of truck lines available. The arrival process of incoming calls is a Poisson process with the constant rate λ . Calls enter the call center whenever a trunk line is available; otherwise, it is lost. Once a trunk line is seized, the call is served by the IVRU first; then the call may leave the call center with probability $1-p$ or be routed to an available agent with probability p . If all agents are busy, the call is queued at the ACD until one agent is free; while waiting for the agents, calls may abandon the queue if their waiting time becomes unreasonably long. Customers' patience time is an exponential distribution with the rate θ . The processing times of the IVRU are independent and identically distributed exponential random variables with rate μ_1 , and the agents' service times are independent and identically distributed exponential random variables with rate μ_2 . (2) The partial closing rule is as follows: Once there are no calls, the system will shut down $S-C$ servers (or agents) simultaneously for a random time ($1 \leq C \leq S \leq N$), the shutdown time is exponential distributed with rate η . If any trunk line is seized before the end of the shutdown, $S-C$ servers return to the system. Otherwise, these servers take another shutdown and continue until they find calls in the trunk lines. (3) The call which have been serviced by the agents may hold the truck line and releases the agent (feedback to the IVRU with probability r_2 , or to the ACD with probability r_1), or may releases both the truck line and the agent simultaneously (leaving the call center)

with probability $r_0 = 1 - r_1 - r_2$. (4) All above random variables are mutually independent.

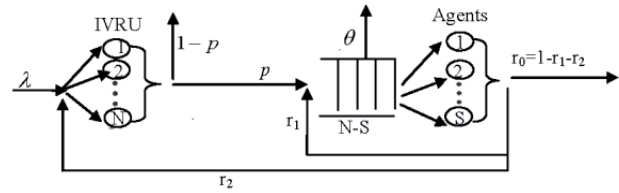


Fig.1. Call Center Model with Impatient Customers and Optional Feedback.

Main results. The call served by the IVRU is regarded as the first-level queue while the call that is served by agents or queued at the ACD is regarded as the second-level queue. Let

$M(t)$:= The calls' number of the first-level queue at time t ;

$N(t)$:= The calls' number of the second-level queue (Including the waiting and being serviced calls) at time t ($M(t) + N(t) \leq N, t \geq 0$);

$J(t)$:= Servers statuses at time t , where $J(t) = 0$ means C servers are open or $J(t) = 1$ means all servers are open.

Because the system has a finite number of states and all its random variables are independent and exponentially distributed, so $\{M(t), N(t), J(t) : t \geq 0\}$ is the irreducible finite state Markov chain. The state space S is defined as

$$S = \{(m, n, j) | 0 \leq m, n \leq N, 0 \leq m + n \leq N, j = 0, 1\}.$$

Because of the finite state space (contains $[(N+1) \cdot (N+2) - 1]$ states), the steady-state can be reached. Therefore, we can find the stationary distribution of the system by QBD approximations. For more details of this method, readers can see Kawanishi K. (2008) in [9]. We denote $\pi_{(m,n,j)}$:= a steady-state probability when the system has m calls in the first-level queue and n calls in the second-level queue and the servers in state j ,

$$(0 \leq m, n \leq N, 0 \leq m + n \leq N, j = 0, 1);$$

$$\pi_0 = (\pi_{(0,0,0)}, \pi_{(0,1,0)}, \pi_{(0,1,1)}, \dots, \pi_{(0,N,0)}, \pi_{(0,N,1)});$$

$$\pi_i = (\pi_{(i,0,0)}, \pi_{(i,0,1)}, \pi_{(i,1,0)}, \pi_{(i,1,1)}, \dots, \pi_{(i,N-i,0)}, \pi_{(i,N-i,1)});$$

$$(i = 1, 2, \dots, N).$$

Then the stationary probability vector π can be defined by $\pi = (\pi_0, \pi_1, \pi_2, \dots, \pi_N)$, which is uniquely determined by solving the equation $\pi Q = \mathbf{0}$ with the normalization condition $\pi \mathbf{1}^T = 1$ numerically, where $\mathbf{0}$ is a row vector of zeroes, $\mathbf{1}^T$ is a column vector of ones, and Q is the infinitesimal generator, which has the block tridiagonal form as follows.

Numerical results. In this section, we will investigate the impact of various parameters on the performance measures that have already been obtained in Section 3. In particular, we mainly demonstrate how the arrival rate λ and the parameter p affect the performance measures of the system by some numerical examples.

Case1: Given $\mu_1^{-1} = 1/2 \text{ min}$, $\mu_2^{-1} = 2 \text{ min}$, $r_1 = 0.2$, $\lambda^{-1} = 1 \text{ min}$, $\eta^{-1} = 1/2 \text{ min}$, $\theta^{-1} = 1/3 \text{ min}$, $p = 0.2$, $r_2 = 0.2$, $N = 3$, $S = 2$, $C = 1$. We can present system performance measures in details:

(i) Stationary probability:

$\pi = (0.35991, 0.0121246;$
 $0.177481, 0.000352705, 0.0488102, 7.75494 \times 10^{-6};$
 $0.00262514, 0.0806708, 0.108722, 0.00486972;$
 $0.0946558, 0.000162854, 0.0249001, 0.0127408;$
 $0.0370873, 0.00106455, 0.0250741, 0.0016059;$
 $0.00713442).$

(ii) The idle probability $P_0 = \pi_{(0,0,0)} = 0.35991$.

(iii) The average number of customers in the second-level queue: $E(N_{Agents}) = 0.00315621$.

(iv) The system busy probability: $P_B = 0.0625749$.

(v) The leaving probability due to customers' impatience: $P_{IL} = 0.00228401$.

Case 2: Given $\mu_1^{-1} = 1/2 \text{ min}$, $\mu_2^{-1} = 1/2 \text{ min}$, $r_1 = 0.1$, $\lambda^{-1} = \text{min}$, $\eta^{-1} = 1/2 \text{ min}$, $\theta^{-1} = 1/3 \text{ min}$, $r_2 = 0.2$, $N = 3$, $S = 2$, $C = 1$.

We study the changes of the parameter p on the performance measures by four figures.

It can be seen from fig. 2–5 that when parameter p increases, the idle probability P_0 decreases, while the average number of the waiting customers in the second-level queue $E(N_{Agents})$, all lines busy probability P_B and the leaving probability due to impatience P_{IL} increase.

Case3: Given $\mu_1^{-1} = 1/6 \text{ min}$, $\mu_2^{-1} = 1/4 \text{ min}$, $p = 0.9$, $\eta^{-1} = 1/2 \text{ min}$, $\theta^{-1} = 1/3 \text{ min}$, $r_2 = 0.2$, $r_1 = 0.1$, $N = 3$, $S = 2$, $C = 1$.

We study the changes of the arrival rate λ on the following four performance measures by four figures.

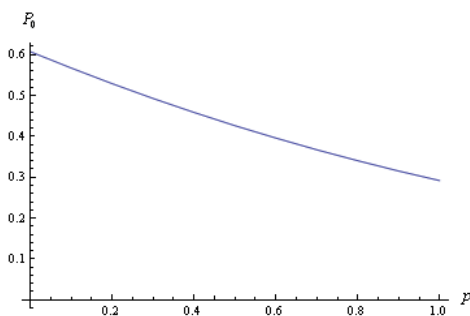


Fig. 2. The idle probability P_0 versus p

It can be seen from fig. 6–9: when the arrival rate λ increases, the idle probability P_0 decreases, but all lines busy probability P_B and the average number of the waiting customers in the second-level queue $E(N_{Agents})$ increase. When

the arrival rate λ is large enough, the system performance measures become stable.

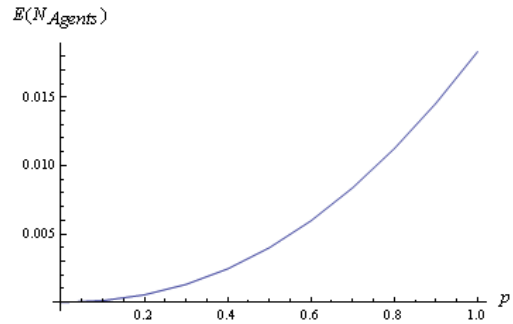


Fig. 3. The average number in the second-level queue $E(N_{Agents})$ versus p

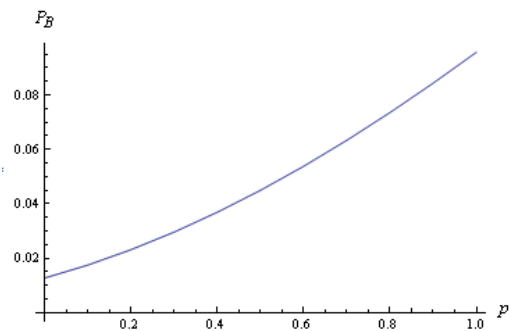


Fig. 4. All lines busy probability P_B versus p

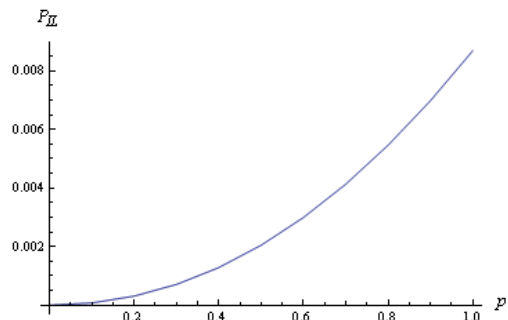


Fig. 5. The leaving probability due to impatience P_{IL} versus p

Case4: Given $\lambda^{-1} = 1/2 \text{ min}$, $\mu_1^{-1} = 5 \text{ min}$, $p = 0.5$, $\theta^{-1} = 1 \text{ min}$, $\eta^{-1} = 1 \text{ min}$, $r_2 = 0$, $N = 3$, $S = 2$, $C = 1$.

We study the changes of the parameter r_1 on the following four performance measures with $\mu_2 = 0.1$ and $\mu_2 = 0.5$, respectively.

It can be seen from fig. 10–13 that when parameter r_1 increases, the idle probability P_0 decreases, but the average number of the waiting customers in the second-level queue $E(N_{Agents})$ and all lines busy probability P_B and the leaving probability due to impatience P_{IL} increase. Because all calls

should use the lines, the system performance measures are influenced by the agents' service rate μ_2 .

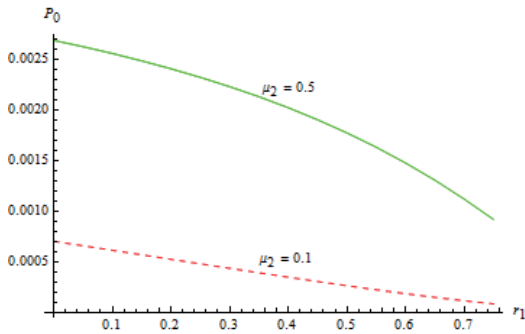


Fig. 10. The idle probability P_0 versus r_1

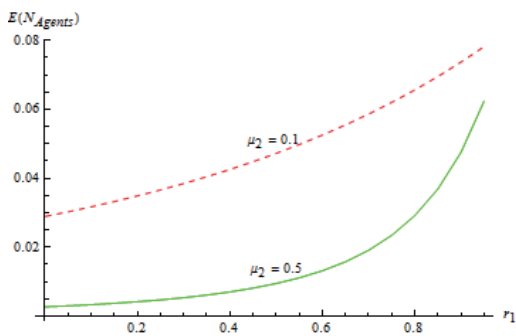


Fig. 11. The average number in the second-level queue

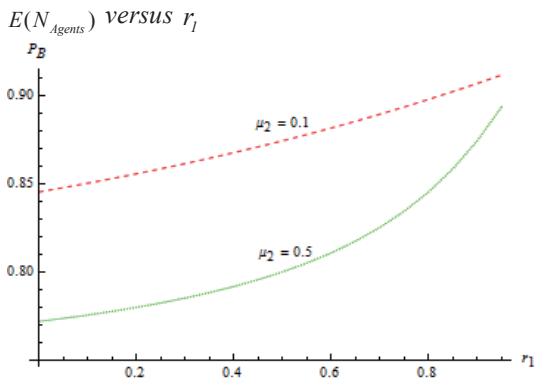


Fig. 12. All lines busy probability P_B versus r_1

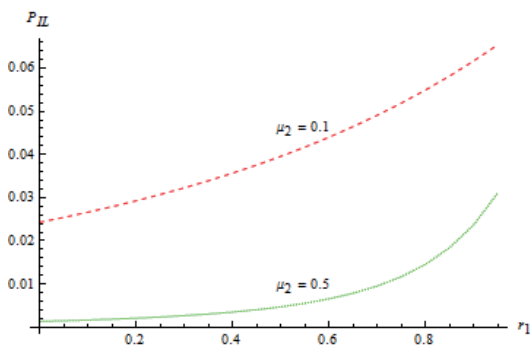


Fig. 13. The leaving probability due to impatience P_L versus r_1

Conclusions. In this paper, we have studied a call center made up of trunk lines, interactive voice response units (IVRU) and agents. We discussed a partial closing rule, call abandonment and feedback in the center. We have obtained the idle probability, the average number of customers in the second-level queue, the leaving probability due to customers' impatience and some other performance measures. Some numerical examples on how the design parameters affect some important performance measures were also investigated.

Acknowledgements. This research is partially sponsored by NSFA (KJ2013B162, KJ2013B165, KJ2014A174), CPSF (2015M572327). It is also sponsored by Chaohu University Scientific Research Fund.

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Мета. У міру збільшення кількості вхідних викликів у колл-центри та їх переоснащення, система інтерактивних голосових меню (IVRU) знаходить усе більш широке вживання. У даній статті розглянуті два питання: відхід необслуговуваних клієнтів із системи та опційний зворотний зв'язок; проблеми, що виникають при роботі колл-центрів; роль каналу обслуговування (серверів і IVRU). Отримані важливі показники, що сприяють оптимізації колл-центрів.

Методика. Створений нами колл-центр із системою інтерактивних голосових меню заснований на двоступінчастій системі масового обслуговування. З точки зору теорії масового обслуговування, нами вивчені питання, що пов'язані з нетерпінням клієнтів, опційним зворотним зв'язком і можливістю виключення окремих серверів.

Результати. Отримавши Q-матрицю системи, методом структурованого виключення Гауса (Structured Gaussian Elimination), ми визначили вірогідність того, що система вільна, середнє число клієнтів у черзі 2-ого рівня, вірогідність відходу необслуговуваних клієнтів через довге чекання та інші показники працездатності системи.

Наукова новизна. Нами розроблений колл-центр з магістралями, системою інтерактивних голосових меню (IVRU) та агентами (Agents), розглянуте правило часткового виключення колл-центра, а також питання, що пов'язані з нетерпінням клієнтів і зворотним зв'язком від них. Подібні дослідження доки не зустрічаються.

Практична значимість. На нашу думку, клієнти, що не задоволені обслуговуванням, можуть повертатися до колл-центру, або до системи інтерактивних голосових меню, або до серверів, або до обох разом. У порівнянні з аналогами, наша модель доцільніша та більше схожа на широко використовувані колл-центри.

Ключові слова: *колл-центр, правило часткового виключення, нетерпіння, опційний зворотний зв'язок, система інтерактивних голосових меню*

Цель. По мере увеличения количества входящих вызовов в колл-центры и их переоснащения, система интерактивных голосовых меню (IVRU) находит всё более широкое применение. В данной статье рассмотрены два вопроса: уход необслуженных клиентов из системы, опционная обратная связь; проблемы, возникающие при

работе колл-центров; роль канала обслуживания (серверов и IVRU). Получены важные показатели, способствующие оптимизации работы колл-центров.

Методика. Созданный нами колл-центр с системой интерактивных голосовых меню основан на двухступенчатой системе массового обслуживания. С точки зрения теории массового обслуживания, нами изучены вопросы, связанные с нетерпением клиентов, опционной обратной связью и возможностью выключения отдельных серверов.

Результаты. Получив Q-матрицу системы, методом структурированного исключения Гаусса (Structured Gaussian Elimination), мы определили вероятность того, что система свободна, среднее число клиентов в очереди 2-ой степени, вероятность ухода необслуженных клиентов из-за долгого ожидания и другие показатели работоспособности системы.

Научная новизна. Нами разработан колл-центр с магістралями, системою інтерактивних голосових меню (IVRU) и агентами (Agents), рассмотрено правило частичного выключения колл-центра, а также вопросы, связанные с нетерпением клиентов и обратной связью от них. Подобные исследования пока не встречаются.

Практическая значимость. По нашему мнению, клиенты, недовольные обслуживанием, могут возвращаться к колл-центру, либо к системе интерактивных голосовых меню, либо к серверам, либо к обоим вместе. По сравнению с аналогами, наша модель целесообразнее и более похожа на широко используемые колл-центри.

Ключевые слова: *колл-центр, правило частичного выключения, нетерпение, опционная обратная связь, система интерактивных голосовых меню*

Рекомендовано до публікації докт. техн. наук В.І. Корнієнком. Дата надходження рукопису 07.05.14.