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MATHEMATICAL MODEL OF ONE-AXIS INCLINOMETER TRANSDUCER OF ZENITH AND SIGHT ANGLES

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МАТЕМАТИЧНА МОДЕЛЬ ДВОСТУПЕНЕВОГО ІНКЛІНОМЕТРИЧНОГО ПЕРЕТВОРЮВАЧА ЗЕНІТНОГО ТА ВІЗИРНОГО КУТІВ

Purpose. The study has two purposes: development of mathematical model of one-axis inclinometer transducer of zenith and sight angles designed in the form of extended floating tube, balanced by the different and with floatation miss in viscous fluid filling in the transducer case; examination of pointing errors and development of the techniques that reduce them.

Methodology. The research used the methods of comparative analysis and mathematical modeling.

Findings. Mathematical model of one-axis inclinometer transducer of zenith and sight angles on vibrating base has been developed. Sensing inclinometer device has been made as extended cylindrical floating tube balanced by different and on floatation miss in viscous fluid filling in transducer case. Measurement range of zenith and sight angles is $0 \div 360^0$. The formulae permitting to evaluate measurement errors of zenith, sight angles and floating tube movement on the axis has been obtained.

Originality. For the first time we have developed the mathematical model of inclinometer transducer with two degrees of freeness: longitudinal axial movement and rotary movement about symmetry axis. Measurement range of zenith and sight angles is $0 \div 360^0$.

Practical value. The resulted mathematical model may be accepted as a basis for inclinometer transducers with float structure intended to control dimensional orientation of a borehole during drilling process.

Keywords: *error, one-axis transducer, zenith angle, sight angle, inclinometer, measurement accuracy, different and floatation miss, mathematical model*

State of the art. Over the last years oil-and-gas producing industry of Ukraine has been developed rapidly. New technologies of deep drilling of oil and gas boreholes with bottom-hole depth from 5,000 m to more are above. Whereas the boreholes building conditions are considerably complicated, therefore this hole making technologies are improved, they provide material changes of drilling process performance [1].

There are kinds of drilling which are of specific importance, they are directional and horizontal drilling, additional boreholes drilling in cluster drilling technique caused by dense drilling in old deposits, out-of-the-way marshlands, seas and oceans shelf zones, under industrial facilities, power supply and transport utilities, built-up areas.

In directional drilling orientation parameter check-out is carried out by inclinometer transducers is meant for measuring the azimuth, zenith angle and whipstock orientation angle during the drilling process.

Analysis of researches and publications. Analysis of well-known works [2] shows that in order to measure zenith angle and angle of inclinometer whipstock orientation angle such structures are proposed as transducer structures made on the base of one-axis gimbal suspensions with external and internal frames or one-axis inclinometer transducers.

Technical features of transducers in many instances estimate the accuracy of the hole wellbore passage in accordance with designed trajectory and borehole entering the required part of producing zone.

Thus, actual engineering problem is the improvement of inclinometer transducers which ensure production intensification, refinement of oil and gas production, cost saving.

The object of the article is to describe new structure of inclinometer transducer of zenith and sight angles, which has two degrees of freeness: longitudinal axial movement and rotary movement about symmetry axis, the development of its mathematical model, errors study and identification of the techniques of their reduction.

Basic material. It is proposed for orientation monitoring to use inclinometer transducer with two degrees of freeness: longitudinal axial movement and rotary movement about symmetry axis, measuring range is $0 \div 360^0$ [3].

Such transducer consists of a pendulum component in the form of extended cylindrical floating tube 1, balanced by a different in viscous fluid and on floatation miss. Centre of gravity of the float is displaced relative to rotation axis.

During device inclinations relatively to vertical, the float while rotating is set in the plane of inclination which is measured by the transducer of rotation angle 2.

Axial movement of the float is measured by displacement transducers 3. The transducers generate electrical signals functionally through trig dependences connected with zenith and sight angles.

We shall make the mathematical model of such transducer. We shall introduce connected with the Earth fixed coordinate system, fig.1, $R_0\,(O\xi\eta\zeta)$, directing the axis $O\zeta$ vertically and down the centre of the Earth. Axes $O\xi$ and $O\eta$ will be placed in horizontal plane.

Centre of gravity and centre of volume of the float are superposed with its symmetry plane.

Vector projection of accelerated gravity force $\bar{g}$ in system of coordinate $O\xi\eta\zeta$

$$\bar{g}_{R_0} = \begin{Vmatrix} 0 \\ 0 \\ g \end{Vmatrix}.$$

We shall connect directional borehole with coordinate system $R\ (OX\,Y\,Z\,)$, directing the axis OZ at longitudinal axis of the borehole and obtained $O\xi\eta\zeta$ from system of coordinate by rotation about the axis $O\eta$ anticlockwise on zenith angle θ with rotation matrix [4]

$$\mathbf{A}_\theta = \begin{Vmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{Vmatrix}.$$

We shall connect transducer enclosure with system of coordinate $R_1\,(OX_1Y_1Z_1)$ obtained by rotation on zenith ang-le φ about the axis OZ with the matrix

$$\mathbf{A}_\varphi = \begin{Vmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{Vmatrix}.$$

System of coordinate $R_2\ (OX_2Y_2Z_2)$ is connected with the sensing element (float) of the transducer and logical coordinates $(O_1X_1^*Y_1^*Z_1^*)$, about the axis OZ_1^* on angle β, connected with the matrix

$$\mathbf{A}_\beta^T = \begin{Vmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{Vmatrix}.$$

System of coordinate $R_1^*\ (O_1X_1^*Y_1^*Z_1^*)$ is obtained by axial movement OZ_1 of the system $(O\,X_1\,Y_1\,Z_1)$ to the value Z.

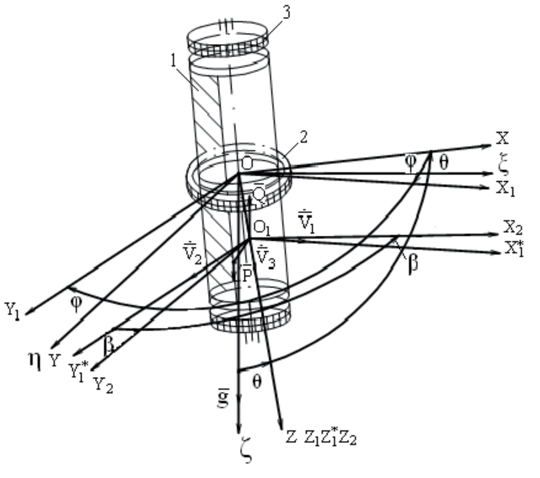

Fig. 1. Systems of Coordinate to derive the mathematical model of one-axis transducer of zenith and sight angles of inclinometer: 1 – extended cylindrical floating tube; 2 – transducer of float rotation angle; 3 – transducers of float movement; $O\xi\eta\zeta$ – fixed coordinate system connected with the Earth; $OXYZ$ – coordinate system connected with directional borehole; $OX_1Y_1Z_1$ – system of coordinate connected with transducer enclosure; $OX_2Y_2Z_2$ – system of coordinate con-nected with sensing element of the transducer; $O_1X_1^*Y_1^*Z_1^*$ – system of coordinate obtained by axial movement OZ_1 of the system $OX_1Y_1Z_1$

According to d'Alembert principle [5] we shall generate the mathematical model of such transducer on moving base.

We shall sum up the moments acting relatively to the rotation axis of the float

$$M_I + M_D + M_{YST.} + M_{ft.} + M_{\ddot{\tau}_2} = 0, \tag{1}$$

where M_I – inertia from the float weight P and stati lift Q applied to the float; M_D – damping moment of the float during its rotation in viscous fluid; $M_{YST.}$ – static moment from the float weight and buoyant force applied to the float; $M_{ft.}$ – frictional moment in float rotation axes; $M_{\ddot{\tau}}$ – vibrational acceleration moment.

At determining the inertia we have regard to that the float is attached by: $M_{\mathrm{Im}} = J\cdot\ddot{\beta}$ – the moment from inertial forces of the float mass, $M_{\mathrm{Im}1} = J_1\cdot\ddot{\beta}$ – the moment from inertial forces of fluid mass added to the float [6], then

$$M_I = (J + J_1)\cdot\ddot{\beta}, \tag{2}$$

where $J = m\cdot r^2$; m – float mass; r – distance to the centre of cylindrical float mass.

Damping moment of the float during its rotation in viscous fluid

$$M_D = K_d \cdot \dot{\beta}, \tag{3}$$

where $K_d = \frac{2 \cdot \pi \cdot R^3 \cdot l_f \cdot \mu}{981 \cdot h}$ – coefficient of damping of the float during its rotation in viscous fluid (this formula was represented by E. A. Nikitin, A. A. Balashova Design of differentiating and integrating gyroscopes and accelerometers); l_f – float length; R – gap midradius; h – gap between float and frame; μ – coefficient of dynamic viscosity of fluid.

Hence, float damping moment during its rotation in viscous fluid

$$M_D = \frac{2 \cdot \pi \cdot R^3 \cdot l_F \cdot \mu}{981 \cdot h} \dot{\beta}. \tag{4}$$

Frictional moment in float rotation axes

$$M_{ft.} = \pm M_{ft.} \frac{\dot{\beta}}{|\dot{\beta}|}. \tag{5}$$

Projections of the moment $\overline{M}_{Y\,ST.}$ acting relative to the float rotation axis and originating by the float load are determined on the reference mark R_2 from vector equality

$$\begin{gathered}\|M_{Y\,ST}\| = \|\tilde{r}_{12}\| \times A_\beta^T \cdot A_\varphi \cdot A_\theta \cdot \overline{P}_{R_0} - \\ - \|\tilde{\rho}_{12}\| \times A_\beta^T \cdot A_\varphi \cdot A_\theta \cdot \overline{Q}_{R_0},\end{gathered} \tag{6}$$

where $\tilde{r}_{12} = \begin{Vmatrix} 0 & -c & b \\ c & 0 & a \\ -b & -a & 0 \end{Vmatrix}$ – alternate matrix of radius vector of float application point weight in the reference mark R_2; $\tilde{\rho}_{12} = \begin{Vmatrix} 0 & f & -e \\ -f & 0 & -d \\ e & d & 0 \end{Vmatrix}$ – alternate matrix of radius vector of application point of static lift in the reference mark R_2; $\overline{P}_{R0} = m\|0,\ 0,\ g\|$ – vector of float weight; $\overline{Q}_{R0} = m_1\|0,\ 0, -g\|$ – vector applied to the float by static lift; m_1 – mass of the fluid added to the float.

We think that vibration is linear and it is not accompanied by angular oscillations: $\bar{\tau}_1(t) = \|X_1(t), Y_1(t), Z_1(t)\|$. Considering that transducer case moves according to harmonic law relatively to the axes OX_1^*, OY_1^*, OZ_1^* with various amplitudes but the same frequency originated by drilling tool

$$X_1(t) = A_1 \sin \omega t;$$
$$Y_1(t) = A_2 \sin \omega t;$$
$$Z_1(t) = A_3 \sin \omega t,$$

then the vector of vibrational acceleration is in the form

$$\ddot{\bar{\tau}}_1(t) = \|\ddot{X}_1(t), \ddot{Y}_1(t), \ddot{Z}_1(t)\| = \|\dot{V}_1, \dot{V}_2, \dot{V}_3\|;$$
$$\ddot{\bar{\tau}}_1(t) = \|-\omega^2 \cdot A_1 \sin \omega t, -\omega^2 \cdot A_2 \sin \omega t, -\omega^2 \cdot A_3 \sin \omega t)\|.$$

Projection of vibrational acceleration vector in the reference mark R_2

$$\ddot{\bar{\tau}}_2 = A_\beta^T \cdot \ddot{\bar{\tau}}_1.$$

Vector of vibrational moment acting on sensing device is calculated in the following way [7]

$$M_{\ddot{\bar{\tau}}_3} = \sum_{i=1}^{n} r_i \cdot \left(m_i \cdot \ddot{\bar{\tau}}_2\right). \tag{7}$$

Substituting (2–7) for (1) we can get the equation of inclinometer transducer, it describes float movement relatively to rotation axis. If the centre of gravity and centre of volume are superposed with its symmetry plane i.e. $b = c = d = e = f = 0$, the equation is written the following way

$$\begin{gathered}(m \cdot r^2 + J_1) \cdot \ddot{\beta} + K_d \cdot \dot{\beta} + g\ \text{am} \cdot \sin(\phi\text{-}\beta) \sin\theta + \\ + am \cdot \left(\sin\beta \cdot \dot{V}_1 + \cos\beta \cdot \dot{V}_2\right) = \pm M_{ft.} \frac{\dot{\beta}}{|\dot{\beta}|}.\end{gathered} \tag{8}$$

Analogous we shall derive the equation of float axial movement OZ_1 with regard to external forces acting on the float.

Projection of the force F_Z originated by float load during axial movement OZ_1 can be designated by the equation

$$\|F_Z\| = A_\theta \cdot \overline{P}_{R_0} - A_\theta \cdot \overline{Q}_{R_0}. \tag{9}$$

The equation of inclinometer transducer describing float axial movement OZ_1 is written in the following way

$$\begin{gathered}\Delta m \cdot \ddot{Z} + K_d \cdot \dot{Z} + C \cdot Z + \\ + \Delta m \cdot \mathrm{g} \cos\theta + \Delta m \cdot \dot{V}_3 = \pm F_{ft.} \frac{\dot{Z}}{|\dot{Z}|},\end{gathered} \tag{10}$$

where $\Delta m = (m - m_1)$ – reserve buoyancy of float (this formula was represented by K.L. McClure Theory of inertial navigation); C – rate of "electrical" spring of feedback; $Z, \dot{Z}, \ddot{Z}$ – axial OZ_1 movement, velocity and acceleration of the float; $F_{ft.}$ – frictional force in axes of supports during float movement.

Mathematical model of one-axis inclinometer transducer of zenith and sight angles describing transducer performance on a movable base is written in the following way

$$\begin{cases} (m\cdot r^2+J_1)\cdot\ddot{\beta}+K_d\cdot\dot{\beta}+g\,\mathrm{a}\cdot\mathrm{m}\cdot\sin(\phi\text{-}\beta)\sin\theta+ \\ +a\cdot m\cdot\left(\sin\beta\cdot\dot{V}_1+\cos\beta\cdot\dot{V}_2\right)=\pm M_{ft.}\dfrac{\dot{\beta}}{|\dot{\beta}|} \\ \Delta m\cdot\ddot{Z}+K_d\cdot\dot{Z}+C\cdot Z+ \\ +\Delta m\cdot\mathrm{g}\cos\theta+\Delta m\cdot\dot{V}_3=\pm F_{ft.}\dfrac{\dot{Z}}{|\dot{Z}|} \end{cases}. \quad (11)$$

Mathematical model of one-axis inclinometer transducer on a fixed base is obtained from the system of equations (11) having put $\dot{V}_1=\dot{V}_2=\dot{V}_3=0$

$$\begin{cases} (m\cdot r^2+J_1)\cdot\ddot{\beta}+K_d\cdot\dot{\beta}+g\cdot\mathrm{a}\cdot\mathrm{m}\cdot\sin(\phi\text{-}\beta)\cdot\sin\theta=\pm M_{ft.}\dfrac{\dot{\beta}}{|\dot{\beta}|} \\ \Delta m\cdot\ddot{Z}+K_d\cdot\dot{Z}+C\cdot Z+\Delta m\cdot\mathrm{g}\cdot\cos\theta=\pm F_{ft.}\dfrac{\dot{Z}}{|\dot{Z}|} \end{cases}. \quad (12)$$

Static behavior of inclinometer transducer of zenith and sight angles can be obtained from the system of equations (12) have put $\ddot{\beta}=\dot{\beta}=0$, $\ddot{Z}=\dot{Z}=0$. Hence

$$\begin{cases} g\,\mathrm{a}\cdot\mathrm{m}\cdot\sin(\phi\text{-}\beta)\sin\theta-M_{ft.}\dfrac{\dot{\beta}}{|\dot{\beta}|}=0 & (13) \\ C\cdot Z+\Delta m\cdot\mathrm{g}\cos\theta-F_{ft.}\dfrac{\dot{Z}}{|\dot{Z}|}=0 & (14) \end{cases}$$

The equation (13) shows measurement error of sight angle determined by setting moment of the float

$$\Delta\varphi=\frac{\pm M_{ft.}\dfrac{\dot{\beta}}{|\dot{\beta}|}}{g\,\mathrm{a}\cdot\mathrm{m}\cdot\sin\theta}. \quad (15)$$

The equation (14) shows axial OZ_1 movement of the float in the form

$$Z=\frac{\pm F_{ft.}\dfrac{\dot{Z}}{|\dot{Z}|}}{C}-\frac{\Delta m\cdot\mathrm{g}}{C}\cdot\cos\theta. \quad (16)$$

Whence it follows that the transducer generates not only movement functionally connected with zenith angle but the error determined by force of dry friction.

For inclinometer transducer which has minimum forces of dry friction in the axes of supports we can get static behavior of the transducer

$$\varphi=\beta;$$

$$Z=-\frac{\Delta m\cdot\mathrm{g}}{C}\cdot\cos\theta;$$

$$\theta=\arccos\left(-\frac{C\cdot Z}{\Delta m\cdot\mathrm{g}}\right).$$

Conclusion. For the first time we have proposed the structure of sensing device of zenith and sight angles on the base of pendulum in the form of extended cylindrical float with two degrees of freeness: longitudinal axial movement and rotary movement about symmetry axis.

The float is placed into cylindrical frame with close gap filled in with viscous fluid and it is balanced by a different and on floatation miss. Measurement range of zenith and sight angles is $0\div360^0$.

According to d'Alembert principle we have obtained mathematical model of one-axis transducer of zenith and sight angles of inclinometer on vibrating and fixed bases with regard to inertial forces and moments acting relative to the float rotation axis and axial movement of the float which permits to study its errors.

References / Список літератури

1. Bulatov, A.I., Demikhov, V.I. and Makaremko, P.P. (1998), *Control protsesov bureniya neftyanyx i gazovyax skvazhin* [Control of Drilling Oil and Gas Wells], "Nedra", Moscow, Russia.

Булатов А.И. Контроль процессов бурения нефтяных и газовых скважин / Булатов А.И., Демихов В.И., Макаренко П.П. – М.: ОАО Издательство „Недра“, 1998. – 345с.

2. Kovshov, G.N., Alimbekov, R.I. and Zhiber, A.V. (1998), *Inklinometry (Osnovy teorii i proektirovaniya)* [Inclinometers (Basic Theory and Design)], Gilem, Ufa, Russia.

Ковшов Г.Н. Инклинометры (Основы теории и проектирования) / Ковшов Г.Н., Алимбеков Р.И., Жибер А.В. – Уфа: Гилем, 1998. – 380 с.

3. Kovshov, G.N., Ryzhkov, I.V. and Zhivtsova, L.I. (2013), Patent for useful model 78852, Ukraine, MPK[7] E 21 B 47/022. *Datchik zenitnogo i vizirnogo kutiv*, (Ukraine), Patentee DVNZ "Prydniprovska State Academy of Civil Engineering and Architecture", No. u 201206932, applied on June 6, 2012, published on April 10, 2013, Bulletin no.7.

Патент на корисну модель 78852 України, МПК[7] E 21 B 47/022. Датчик зенітного і візирного кутів / Ковшов Г.М., Рижков І.В., Живцова Л.І. (Україна); заявник і патентовласник ДВНЗ „Придніпровська державна академія будівництва та архітектури“. – № u 201206932; заявл. 06.06.12; опубл. 10.04.13, Бюл.№7.

4. Markeev, A.P. (1999), *Teoreticheskaia mekhanika.* [Theoretical Mechanics], CheRo, Moscow, Russia.

Маркеев А.П. Теоретическая механика / Маркеев А.П. – М.: ЧеРо – 1999. – 572с.

5. Borisov, A.V. and Mamaev, I.S. (2001) *Dinamika tverdogo tela.* [Dynamics of Solid Body], SRC "Regular and chaotic dynamics", Izhevsk, Russia.

Борисов А.В. Динамика твердого тела / А.В. Борисов, И.С. Мамаев – Ижевск: НИЦ „Регулярная и хаотическая динамика“, 2001. – 384с.

6. Ikrin, V.A. (2004), *Soprotivlenie materialov s elementami teorii uprugocti i plastinchnosti.* [Strength of Materials with the Elements of Theory of Elasticity and Plasticity], ACB, Moscow, Russia.

Икрин В.А. Сопротивление материалов с элементами теории упругости и пластичности / Икрин В.А. – М.: Изд. АСВ – 2004. – 424 с.

7. Seregin, V.V. (2007), *Prikladnaya teoriya i printsypy postroeniya giroskopicheskikh sistem* [Applied Theory and Aufbau Principles of Gyroscopic Systems], StPSU ITMO, St. Petersburg, Russia.

Серегин В.В. Прикладная теория и принципы построения гироскопических систем / Серегин В.В. – СПб.: СпбГУ ИТМО. – 2007. – 78с.

Мета. Розробка математичної моделі двоступеневого інклінометричного перетворювача зенітного та візирного кутів, виконаного у вигляді подовженого циліндричного поплавка, врівноваженого по диференту й на залишкову вагу у в'язкій рідині, що заповнює корпус перетворювача. Дослідження похибок і виявлення прийомів їх зниження.

Методика. У роботі використовувалися методи порівняльного аналізу та математичного моделювання.

Результати. Розроблена математична модель двоступеневого інклінометричного перетворювача зенітного та візирного кутів на вібруючій основі. Первинний інклінометричний перетворювач виконаний у вигляді подовженого циліндричного поплавка, врівноваженого по диференту й на залишкову вагу у вязкій рідині, що заповнює корпус перетворювача. Діапазон вимірювання зенітного та візирного кутів $0 \div 360^0$. Отримані вирази, що дозволяють оцінити похибки у вимірюванні зенітного, візирного кутів і при переміщенні поплавка уздовж осі.

Наукова новизна. Уперше розроблена математична модель інклінометричного перетворювача, що має два ступені свободи: переміщення за поздовжньою віссю та обертальне навколо осі симетрії. Діапазон вимірювання зенітного та візирного кутів $0 \div 360^0$.

Практична значимість. Полягає в розробці математичної моделі, покладеної в основу інклінометричних перетворювачів на основі поплавкової конструкції для контролю просторового положення свердловини у процесі буріння.

Ключові слова: *похибка, двоступеневий перетворювач, зенітний кут, візирний кут, інклінометр, точність вимірювання, диферент і залишкова вага, математична модель*

Цель. Разработать математическую модель двухстепенного инклинометрического преобразователя зенитного и визирного углов, выполненного в виде удлиненного цилиндрического поплавка, уравновешенного по дифференту и на остаточный вес в вязкой жидкости, заполняющей корпус преобразователя. Исследование погрешностей и выявление приемов их снижения.

Методика. В работе использованы методы сравнительного анализа и математического моделирования.

Результаты. Разработана математическая модель двухстепенного инклинометрического преобразователя зенитного и визирного углов на вибрирующем основании. Первичный инклинометрический преобразователь выполнен в виде удлиненного цилиндрического поплавка, уравновешенного по дифференту и на остаточный вес в вязкой жидкости, заполняющей корпус преобразователя. Диапазон измерения зенитного и визирного углов $0 \div 360^0$. Получены выражения, позволяющие оценить погрешности в измерении зенитного, визирного углов и при перемещении поплавка вдоль оси.

Научная новизна. Впервые разработана математическая модель инклинометрического преобразователя, имеющего две степени свободы: перемещение по продольной оси и вращательное вокруг оси симметрии. Диапазон измерения зенитного и визирного углов $0 \div 360^0$.

Практическая значимость. Заключается в разработке математической модели, положенной в основу инклинометрических преобразователей на основе поплавковой конструкции для контроля пространственного положения скважины в процессе бурения.

Ключевые слова: *погрешность, двухстепенной преобразователь, зенитный угол, визирный угол, инклинометр, точность измерения, дифферент и остаточный вес, математическая модель*

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