ЕКОНОМІКА ТА УПРАВЛІННЯ

UDC 330.4:338.24:519.86

V. M. Kuznichenko, Cand. Sc. (Phys.-Math.), Assoc. Prof.,
V. I. Lapshyn, Dr. Sc. (Phys.-Math.), Prof.

Kharkiv Institute of Finance of the Kyiv National University of Trade and Economics, Kharkiv, Ukraine, e-mail: kuznichenkovm@i.ua

GENERALIZED DEFICIT MODEL OF EXCHANGE FOR CONTINUOUS PROCESSES WITH EXTERNAL MANAGEMENT

В. М. Кузніченко, канд. фіз.-мат. наук, доц.,
В. І. Лапшин, д-р фіз.-мат. наук, проф.

Харківський інститут фінансів Київського національного торговельно-економічного університету, м. Харків, Україна, e-mail:kuznichenkovm@i.ua

УЗАГАЛЬНЮЮЧА ДЕФІЦИТНА МОДЕЛЬ ОБМІНУ ДЛЯ БЕЗПЕРЕРВНИХ ПРОЦЕСІВ ІЗ ЗОВНІШНІМ УПРАВЛІННЯМ

Purpose. A method for constructing a deficit-having model of exchange based on the Markov chain theory for continuous processes with an external management is demonstrated. The results of the continuous and discrete models are identical at the moments of time proportional to the time step of discrete process.

Methodology. The Markov Chain theory, method of Z – transformation are used for determining vectors of the supply and demand between those taking part in the exchange processes. The intensity matrices are defined from transformation of logarithmic function of corresponding transition matrices into polynomials. This is done with help of the Cayley-Hamilton theorem.

Findings. The developed deficit-having model of exchange is investigated. The method of constructing a deficit-having model of exchange is demonstrated on the example. The discrete model is used for constructing the continuous model. The vectors of the supply and demand in the exchange process are determined.

Originality. Previously the method for constructing a deficit-having model of exchange on the basis of the Markov Chain theory for continuous processes with an external management was presented briefly without examples. On the example of three partners of exchange process the transition from the deficit-free model to the deficit-having one is shown.

Practical value. The continuous model can be used in different economic systems for studying and planning exchange processes. Such systems can be exemplified by a linear model of international trade, money and commodity streams between the performers of regional budgetary projects. The developed model of exchange allows us to analyse separately the supply and demand between partners, to determine the balance of the exchange process, and increases the management level. At further development the built model of exchange can be used for the queuing systems with an external management.

Keywords: *model of exchange, continuous processes, external management, Markov Chain*

Introduction. The economic development of nations, their interactions, lead to continuous production.The distribution and exchange of goods (consumer goods, resources, capital, workforce, material, industrial products etc.) are the important constituent of economic integration processes.

Due to this, models of exchange in economics (as well as their development) are always of great interest both for the discrete and for continuous planning (discrete and continuous processes).

One of approaches to study the exchange processes was developed based on the mathematical theory of random processes-Markov chains (see the monographs of Howard R. A., Kemeny J. G. and Snell J. L., Zhluktenko, Begun [1], Sokolov and Chistyakov [2], Kuznichenko, Lapshyn and Stetsenko.

It gives us possibilities not only to determine the equilibrium states of systems, but also to study the dynamics of the discrete transition from the initial state to the equilibrium state.

The deficit-free and deficit-having models both without and with external management, which were based on the Markov chain theory, were developed and used only for discrete processes. In the deficit-having model the processes of the "supply" and "demand" elements of the exchange process are examined separately.

Objectives of the article. The goal of the present article is the demonstration of the deficit-having model of exchange for continuous processes with external management developed by the authors, which would generalize the discrete model and the deficit continuous model.

Originality of the research. A generalized continuous model without an external management, which takes into account both deficit-free and deficit-having cases, was analysed by the authors of the present article Kostenko, Kuznichenko, Lapshyn (2014), in addition a deficit-free continuous model with control was considered in [3] (without determination of an external management – the vectors of the administrative redistribution of the resources). A method for constructing a deficit-having model for continuous processes with external control was presented briefly in [4].

Presentation of the main research. The necessary condition for the construction of the continuous model involves its transition to the discrete model at the moments of time proportional to the time step of the discrete process.

In order to determine the proper way to approach the creation of the continuous model with management (that fulfils the necessary conditions), let us first analyse the discrete model developed by the authors Kostenko, Kuznichenko, and Lapshyn (2014).

The deficit-having discrete model with control is described with the following model recurrence equations with the matrices $L_i \quad i=\overline{1.2}$ of transition probabilities for the ergodic Markov chains (supply and demand) and has the following form

$$\bar{r}_i(n+1) = \bar{r}_i(n)L_i + \bar{f}_i, \cdots n = 0,1,2,\ldots, i = 1.2, \quad (1)$$

where $\bar{r}_1(n) = \bar{q}(n)$ is the demand budget distribution vector between the m members of the system after the n^{th} step, $\bar{r}_2(n) = \bar{p}(n)$ is the supply budget distribution vector between the m members of the system after the n^{th} step and $\bar{f}_i = (f_{i1}, f_{i2}, \ldots, f_{im})$ are the vectors of the administrative redistribution (evening-out) of their resources ($\sum_{k=1}^{m} f_{ik} = 0$).

We will call the matrix $L_1 = B_1^{'}B_2$ the demand matrix, while the matrix $L_2 = B_2B_1^{'}$ is called the supply matrix. The transposed stochastic matrix $B_1^{'}$ is the "transition matrix" from the demand vector $\bar{q}$ to the supply vector $\bar{p}$. The stochastic matrix B_2 is the "transition matrix" from the supply vector $\bar{p}$ to the demand vector $\bar{q}$.

In order to fully define the Markov chain, it is necessary to state the initial distribution of the random value as well (the vector of initial distribution of exchanged goods $\bar{r}(0) = \bar{p}(0) = \bar{q}(0)$).

Let us analyse the created model by looking at an example: the exchange process between three members in a deficit-having case (Table).

In order to study the behaviour of the Markov chain analytically (while taking into account the external control) until the transition into a stationary state, let us apply the method for the z transformation to the equation (1), taking $\bar{r}_i(n) \longleftrightarrow R_i(z)$.

The matrix L_1 and L_2 are obtained from Table

$$L_1 = B_1^{'}B_2 = \begin{pmatrix} \frac{3}{8} & \frac{1}{2} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} * \begin{pmatrix} \frac{3}{8} & \frac{1}{8} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{27}{64} & \frac{13}{64} & \frac{3}{8} \\ \frac{13}{32} & \frac{7}{32} & \frac{3}{8} \\ \frac{3}{8} & \frac{3}{16} & \frac{7}{16} \end{pmatrix};$$

$$L_2 = B_2B_1^{'} = \begin{pmatrix} \frac{3}{8} & \frac{1}{8} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix} * \begin{pmatrix} \frac{3}{8} & \frac{1}{2} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{27}{64} & \frac{3}{8} & \frac{13}{64} \\ \frac{3}{8} & \frac{7}{16} & \frac{3}{16} \\ \frac{13}{32} & \frac{3}{8} & \frac{7}{32} \end{pmatrix}.$$

Let us construct the matrix $(I - zL_1)$ for the demand matrix $L_1 = B_1^{'}B_2$

$$(I - zL_1) = \begin{pmatrix} 1-\frac{27z}{64} & -\frac{13z}{64} & -\frac{3z}{8} \\ -\frac{13z}{32} & 1-\frac{7z}{32} & -\frac{3z}{8} \\ -\frac{3z}{8} & -\frac{3z}{16} & 1-\frac{7z}{16} \end{pmatrix}.$$

The determinant of this matrix is equal to

$$\Delta = \det(I - zL_1) = (1-z)(1-\frac{z}{16})(1-\frac{z}{64}).$$

Meanwhile, the inverse matrix has the following form

$$\left(I - zL_1\right)^{-1} = \frac{1}{(1-z)} \begin{pmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \end{pmatrix} + \frac{1}{(1-\frac{z}{16})} \begin{pmatrix} \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{3}{5} \end{pmatrix} + \frac{1}{(1-\frac{z}{64})} \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Table

Distribution of deficit-having exchange between three members

	S_1	S_2	S_3	Sum	
S_1	270	90	360	x_1	720
S_2	360	180	180	x_2	720
S_3	90	90	180	x_3	360
Sum	$\dot{x}_1$	$\dot{x}_2$	$\dot{x}_3$	D = 1800	
	720	360	720		

Putting what we have found into (1) at $i = 1$, we see that $R_1(z) = (\overline{r_1}(0) + \frac{z}{(1-z)}\overline{f_1})(I - zL_1)^{-1}$.

Let us apply the inverse transformation to $R_1(z)$ and find the analytic solution to the demand equation

$$\overline{q}(n) = \overline{q}(0) + \left[\begin{pmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \end{pmatrix} + \left(\frac{1}{16}\right)^n \begin{pmatrix} \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{3}{5} \end{pmatrix} + \left(\frac{1}{64}\right)^n \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] +$$

$$+\overline{f_1}\left[n\begin{pmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \end{pmatrix} + \frac{16}{15}(1-\left(\frac{1}{16}\right)^n)\begin{pmatrix} \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{3}{5} \end{pmatrix} + \frac{64}{63}(1-\left(\frac{1}{64}\right)^n)\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}\right], \quad (2)$$

where $\overline{q}(0)$ is the vector of initial budget distribution.

Let us recall that in order to prevent the demand process from diverging, the following condition is required to be satisfied for the components of the vector $\overline{f_1} = (f_{11}, f_{12}, f_{13})$

$$f_{11} + f_{12} + f_{13} = 0. \quad (3)$$

In order to determine the form of the control vector $\overline{f_1}$ when planning the limit state of the demand vector as, for instance, $\overline{q^*} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, we must put this vector into the equation (2) instead of $\overline{q}(n)$ and solve the system (2, 3) at $n \to \infty$

$$\begin{cases} (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = (\frac{2}{5}, \frac{1}{5}, \frac{2}{5}) + (f_{11}, f_{12}, f_{13}) \times \\ \times\left[\begin{pmatrix} \frac{64}{225} & \frac{32}{225} & -\frac{32}{75} \\ \frac{64}{225} & \frac{32}{225} & -\frac{32}{75} \\ -\frac{32}{75} & -\frac{16}{75} & \frac{16}{25} \end{pmatrix} + \begin{pmatrix} \frac{64}{189} & -\frac{64}{189} & 0 \\ -\frac{128}{189} & \frac{128}{189} & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]. \\ f_{11} + f_{12} + f_{13} = 0 \end{cases}$$

Solving this system, we find the vector $\overline{f_1}$

$$\overline{f_1} = (-\frac{13}{192}, \frac{25}{192}, -\frac{3}{48}).$$

Following procedures for determination $\overline{q}(n)$ we obtain the following expression for $\overline{p}(n)$

$$\overline{p}(n) = \overline{p}(0)\left[\begin{pmatrix} \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix} + \left(\frac{1}{16}\right)^n \begin{pmatrix} \frac{4}{15} & -\frac{2}{5} & \frac{2}{15} \\ -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ \frac{4}{15} & -\frac{2}{5} & \frac{2}{15} \end{pmatrix} + \left(\frac{1}{64}\right)^n \begin{pmatrix} \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 \\ -\frac{2}{3} & 0 & \frac{2}{3} \end{pmatrix}\right] +$$

$$+\overline{f_2}\left[n\begin{pmatrix} \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix} + \frac{16}{15}(1-\left(\frac{1}{16}\right)^n)\begin{pmatrix} \frac{4}{15} & -\frac{2}{5} & \frac{2}{15} \\ -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ \frac{4}{15} & -\frac{2}{5} & \frac{2}{15} \end{pmatrix} + \frac{64}{63}(1-\left(\frac{1}{64}\right)^n)\begin{pmatrix} \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 \\ -\frac{2}{3} & 0 & \frac{2}{3} \end{pmatrix}\right], \quad (4)$$

where $\overline{p}(0)$ is the vector of initial budget distribution.

Solving this system

$$\begin{cases} (\frac{1}{3},\frac{1}{3},\frac{1}{3}) = (\frac{2}{5},\frac{2}{5},\frac{2}{5}) + (f_{21}, f_{22}, f_{23}) \left[\begin{pmatrix} \frac{64}{225} & -\frac{32}{75} & \frac{32}{225} \\ -\frac{32}{75} & \frac{16}{25} & -\frac{16}{75} \\ \frac{64}{225} & -\frac{32}{75} & \frac{32}{225} \end{pmatrix} + \begin{pmatrix} \frac{64}{189} & 0 & -\frac{64}{189} \\ 0 & 0 & 0 \\ -\frac{128}{189} & 0 & \frac{128}{189} \end{pmatrix} \right], \\ f_{21} + f_{22} + f_{23} = 0 \end{cases}$$

we find the vector

$$\overline{f_2} \ \ (\overline{p^*} = (\frac{1}{3},\frac{1}{3},\frac{1}{3}));$$

$$\overline{f_2} = (-\frac{13}{192}, -\frac{3}{48}, \frac{25}{192}).$$

Now we can describe a continuous-in-time Markov process with a transition intensity matrix $A_i, i = 1.2$ with components a_{ij}, the diagonal elements of which must be specified additionally (the sum of the components in each row of the matrix A must be equal to zero).

The continuous deficit-having model is described by the system of linear differential equations

$$\frac{d}{dt}\overline{r_i}(t) = \overline{r_i}(t)A_i + \overline{g_i}, \cdots i = 1.2. \qquad (5)$$

Here $L_i = e^{A_i}$ (or, equivalently, A_i = ln L_i), while the following condition is fulfilled for the vector $\overline{g_i} = (g_{i1}, g_{i2}, g_{i3}):\ g_{i1} + g_{i2} + g_{i3} = 0$. As it is shown, the control vectors in the deficit-free and deficit-having models are different [3, 4]. Using the Laplace transform in (5) $\overline{r_i}(t)$ were found with $\overline{g_i} = \overline{f_i}$. Vectors $\overline{r_i}(t)$ did not coincide with $\overline{r_i}(n)$ (n = t; (2, 4)).

The intensity matrix A_1 is found using the standard methods based on the Hamilton-Kelly theorem and its derivatives (see the monograph by Gandmakher [5]). The characteristic polynomial of the matrix L_1 has the following form

$$\Delta_{L_1}(\lambda) = \begin{vmatrix} \frac{27}{64} - \lambda & \frac{3}{8} & \frac{13}{64} \\ \frac{3}{8} & \frac{7}{16} - \lambda & \frac{3}{16} \\ \frac{13}{32} & \frac{3}{8} & \frac{7}{32} - \lambda \end{vmatrix} =$$

$$= -(\lambda - 1)(\lambda - \frac{1}{16})(\lambda - \frac{1}{64}).$$

All of the roots of the characteristic polynomial are simple. Therefore, the characteristic polynomial is the same as the minimal nullifying $\psi(\lambda) = \Delta_{L_1}(\lambda)$. The spectrum of the matrix L_1, which we will call Λ_{L_1}, is equal to

$$\Lambda_{L_1} = \left\{ \frac{1}{64}, \frac{1}{16}, \ 1 \right\}.$$

The function $f(\lambda) = \ln(\lambda)$ is defined on the spectrum of the matrix L_1. If the function $f(\lambda)$ is defined on the spectrum of the matrix L_1 and $g(\lambda)$ is any polynomial that coincides with $f(\lambda)$ on the spectrum of the matrix L_1 (as in, $f(\Lambda_{L_1}) = g(\Lambda_{L_1})$), then by definition we have the following

$$f(L_1) = g(L_1).$$

This polynomial can be obtained by a variety of methods. In our case, the polynomial $g(\lambda)$ of the lowest order, that is uniquely defined on the spectrum of the matrix L_1, has the following form

$$g(\lambda) = c\lambda^2 + b\lambda + a.$$

Let us compose the following system of linear algebraic equations

$$\begin{cases} f(1) = g(1) = 0 = a + b + c \\ f(\frac{1}{16}) = g(\frac{1}{16}) = 4\ln(\frac{1}{2}) = a + \frac{b}{16} + \frac{c}{256} \\ f(\frac{1}{64}) = g(\frac{1}{64}) = 6\ln(\frac{1}{2}) = a + \frac{b}{64} + \frac{c}{4096} \end{cases}.$$

We solve the system of linear algebraic equations and find the unknown coefficients

$$a = \frac{704}{105}\ln(\frac{1}{2});\ b = -\frac{320}{7}\ln(\frac{1}{2});\ c = \frac{4096}{105}\ln(\frac{1}{2}).$$

Knowing the coefficients a, b and c, we can find the matrix A_1

$$A_1 = \ln(L_1) = (\frac{704}{105}I - \frac{320}{7}L_1 + \frac{4096}{105}L_1^2)\ln(\frac{1}{2}) = \begin{pmatrix} \frac{46}{15} & -\frac{22}{15} & -\frac{8}{5} \\ -\frac{44}{15} & \frac{68}{15} & -\frac{8}{5} \\ -\frac{8}{5} & -\frac{4}{5} & \frac{12}{5} \end{pmatrix} \ln(\frac{1}{2}) = \begin{pmatrix} -\frac{46}{15} & \frac{22}{15} & \frac{8}{5} \\ \frac{44}{15} & -\frac{68}{15} & \frac{8}{5} \\ \frac{8}{5} & \frac{4}{5} & -\frac{12}{5} \end{pmatrix} \ln(2).$$

The intensity matrix A_2 is found in an analogous way

$$A_2 = \ln(L_2) = (\frac{704}{105}I - \frac{320}{7}L_2 + \frac{4096}{105}L_2^2)\ln(\frac{1}{2}) = \begin{pmatrix} \frac{46}{15} & -\frac{8}{5} & -\frac{22}{15} \\ -\frac{8}{5} & \frac{12}{5} & -\frac{4}{5} \\ -\frac{44}{15} & -\frac{8}{5} & \frac{68}{15} \end{pmatrix} \ln(\frac{1}{2}) = \begin{pmatrix} -\frac{46}{15} & \frac{8}{5} & \frac{22}{15} \\ \frac{8}{5} & -\frac{12}{5} & \frac{4}{5} \\ \frac{44}{15} & \frac{8}{5} & -\frac{68}{15} \end{pmatrix} \ln(2).$$

From (5) ($\overline{g_i} = \overline{f_i}X_i$, $\overline{r_i}(t) = \overline{r_i}(n), \cdot\, n = t$) the matrices X_i are defined

$$X_1 = \begin{pmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \end{pmatrix} + \frac{64}{15}\begin{pmatrix} \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ \frac{4}{15} & \frac{2}{15} & -\frac{2}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{3}{5} \end{pmatrix} \ln 2 + \frac{128}{21}\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{2}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} \ln 2;$$

$$X_2 = \begin{pmatrix} \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix} + \frac{64}{15}\begin{pmatrix} \frac{4}{15} & -\frac{2}{5} & \frac{2}{15} \\ -\frac{2}{5} & \frac{3}{5} & -\frac{1}{5} \\ \frac{4}{15} & -\frac{2}{5} & \frac{2}{15} \end{pmatrix} \ln 2 + \frac{128}{21}\begin{pmatrix} \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 0 \\ -\frac{2}{3} & 0 & \frac{2}{3} \end{pmatrix} \ln 2.$$

Therefore, the control vectors in the continuous and discrete models are different. The proposed method allows us to determine one vector through the other, and facilitates the transition from the continuous model to the discrete model for a given exchange process.

Conclusions. A deficit-having exchange model for continuous processes with external control is demonstrated. The results of the continuous and discrete models are identical at the moments of time proportional to the time step of discrete process.

For planning the exchange processes the discrete model can be used. The deficit-having continuous model which can be constructed based on the discrete model separates the processes of the "supply" and "demand" elements and increases the management level.

Therefore, the exchange model we have constructed for continuous processes with external control allows us not only to study, but also to correct (through a collegial organ) the interaction of partners while executing international trade agreements or regional projects (and exchange processes in other closed systems) for the duration of the entire process, as well as to obtain information regarding the state of the system at any moment in time.

At further development the built model of exchange can be used for the queuing systems with external control.

References.

1. Zhuktenko, V.I. and Begun, A.V., 2005. *Stochastic models in economics*. Kiev: KNEU.

2. Sokolov, G.A. and Chistiakov, N.A., 2005. *Probability. Controlled Markov Chains in the economy.* Moscow.

3. Kuznichenko, V.M. and Lapshyn, V.I., 2015. Probabilistic approach to the continuous model in the economy. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 5, pp. 137–141.

4. Kostenko, E., Kuznichenko, V.M. and Lapshyn, V.I., 2016. Deficit model of exchange for continuous processes with external control, *Applied Economics Letters* [e-journal]. pp. 1–4. doi: 10.1080/13504851.2016.1205717.

5. Gandmakher, F.R., 2010. *Theory of matrices*. Moscow: Fizmatlit.

Мета. Продемонструвати метод побудови дефіцитної моделі обміну на основі теорії ланцюга Маркова для безперервних процесів із зовнішнім управлінням, для яких результати безперервних і дискретних моделей однакові в моменти часу, пропорційні часовому кроку дискретного процесу.

Методика. Теорія ланцюгів Маркова, метод Z-перетворення використовуються для визначення векторів станів попиту й пропозиції між тими, хто бере участь у процесі обміну. Матриці інтенсивності визначаються з перетворення логарифмічної функції відповідних матриць переходу в поліноми. Це робиться за допомогою теореми Гамільтона-Келі.

Результати. Досліджена дефіцитна модель обміну. На прикладі продемонстровано метод розробки дефіцитної моделі обміну. Дискретна модель використовується для побудови неперервної моделі. Визначені вектори станів попиту й пропозиції у процесі обміну.

Наукова новизна. Раніше метод побудови дефіцитної моделі обміну на основі теорії ланцюгів Маркова для безперервних процесів із зовнішнім управлінням був коротко представлений без прикладів. На прикладі трьох партнерів обмінного процесу показано перехід від бездефіцитної до дефіцитної моделі.

Практична значимість. Безперервна модель може бути використана в різних економічних системах для дослідження й планування процесів обміну. Прикладом таких систем можуть бути лінійна модель міжнародної торгівлі, модель грошових і товарних потоків між виконавцями регіональних бюджетних проектів. Розроблена модель обміну дозволяє аналізувати окремо попит і пропозиції між партнерами, визначити баланс процесу обміну, а також підвищує рівень управління. При подальшому розвитку побудована модель обміну може бути використана для систем масового обслуговування з зовнішнім управлінням.

Ключові слова: *модель обміну, безперервні процеси, зовнішнє управління, ланцюг Маркова*

Цель. Продемонстрировать метод построения дефицитной модели обмена на основе теории цепи Маркова для непрерывных процессов с внешним управлением, для которых результаты непрерывных и дискретных моделей одинаковы в моменты времени, пропорциональные часовому шагу дискретного процесса.

Методика. Теория цепей Маркова, метод Z-преобразования используются для определения векторов состояний спроса и предложения между теми, кто участвует в процессе обмена. Матрицы интенсивности определяются при преобразовании логарифмической функции соответствующих матриц перехода в полиномы. Это делается с помощью теоремы Гамильтона-Кэли.

Результаты. Исследована дефицитная модель обмена. На примере продемонстрирован метод разработки дефицитной модели обмена. Дискретная модель используется для построения непрерывной модели. Определены векторы состояний спроса и предложения в процессе обмена.

Научная новизна. Ранее метод построения дефицитной модели обмена на основе теории цепей Маркова для непрерывных процессов с внешним управлением был представлен кратко без примеров. На примере трех партнеров обменного процесса показан переход от бездефицитной к дефицитной модели.

Практическая значимость. Непрерывная модель может быть использована в разных экономических системах для исследования и планирования процессов обмена. Примером таких систем могут быть линейная модель международной торговли, модель денежных и товарных потоков между исполнителями региональных бюджетных проектов. Разработанная модель обмена позволяет анализировать отдельно спрос и предложения между партнерами, определить баланс процесса обмена, а также повышает уровень управления. При дальнейшем развитии построенная модель обмена может быть использована для систем массового обслуживания с внешним управлением.

Ключевые слова: *модель обмена, непрерывные процессы, внешнее управление, цепь Маркова*

Рекомендовано до публікації докт. фіз.-мат. наук В. М. Кукліним. Дата надходження рукопису 08.09.16.