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L. M. Vasiliev<sup>1</sup>, Dr. Sc. (Tech.), Prof.,  
D. L. Vasiliev<sup>1</sup>, Cand. Sc. (Tech.),  
M. G. Malich<sup>2</sup>, Cand. Sc. (Tech.), Assoc. Prof.,  
O. O. Anhelovskiy<sup>3</sup>, Cand. Sc. (Tech.)

1 – Institute of Geotechnical Mechanics. N. S. Polyakova National Academy of Sciences of Ukraine, Dnipro, Ukraine, e-mail: office.igtm@nas.gov.ua

2 – National Metallurgical Academy of Ukraine, Dnipro, Ukraine, e-mail: n22051957m@gmail.com

3 – Public Joint Stock Company “Krasnodonugol”, Krasnodon, Ukraine, e-mail: Aleksandr Angelovskii@krasnodon cool.com

## ANALYTICAL METHOD FOR CALCULATING AND CHARTING “STRESS–DEFORMATION” PROVIDED LONGITUDINAL FORM OF DESTRUCTION OF ROCK SAMPLES

Л. М. Васильєв<sup>1</sup>, д-р техн. наук, проф.,  
Д. Л. Васильєв<sup>1</sup>, канд. техн. наук,  
М. Г. Маліч<sup>2</sup>, канд. техн. наук, доц.,  
О. О. Ангеловський<sup>3</sup>, канд. техн. наук

1 – Інститут геотехнічної механіки імені Н. С. Полякова Національної академії наук України, м. Дніпро, Україна, e-mail: office.igtm@nas.gov.ua

2 – Національна металургійна академія України, м. Дніпро, Україна, e-mail: n22051957m@gmail.com

3 – Публічне акціонерне товариство „Краснодонвугілля“, м. Краснодон, Україна, e-mail: Aleksandr Angelovskii@krasnodon cool.com

## АНАЛІТИЧНИЙ МЕТОД РОЗРАХУНКУ Й ПОБУДОВИ ДІАГРАМИ „НАПРУЖЕННЯ–ДЕФОРМАЦІЯ“ ПРИ ПОЗДОВЖНІЙ ФОРМІ РУЙНУВАННЯ ЗРАЗКІВ ГІРСЬКИХ ПОРІД

**Purpose.** Disclosure of the formation mechanism of limiting the deformation of rock samples provided their longitudinal form of destruction.

**Methodology.** Methods are based on the solving the problem of destruction of rock samples on the basis of Coulomb strength criterion, an improved view of the contact friction. An analytical method has been developed for calculating the limit strength of the sample knowing three experimental indices of rock properties: shear strength of the material, coefficients of internal friction and contact friction and geometrical parameters of peaks in the sample cracks. On the basis of the developed method we determine the specific stress on the sample still under the load in the process of development of cracks. Knowing coordinate values of one or two cracks of the sample at any given moment load-carrying part is determined which is equal to the original area minus the last part, which appeared from under the load. Knowing the limit specific stress on the carrier part of the sample, the amount of deformation that corresponds to the limit specific stress with respect to the magnitude of the initial area is determined by Hooke’s law.

**Findings.** An analytical method is developed for calculating and charting “stress–deformation” of rock samples in their longitudinal form destruction and the mechanism of formation of this form is revealed.

**Originality.** For the first time the mechanism of self-organization by Bridgman paradoxical form of destruction has been disclosed and the method for constructing the “stress–deformation” chart has been given. The longitudinal form of the destruction is self-organized by internal and contact friction, in accordance with the generalized Coulomb strength criterion, which takes into account the contact friction, and local mechanics limit state of the material at the crack peak.

**Practical value.** Accessibility of calculating and charting “stress–deformation” on the basis of three parameters: limits shear strength of the material, coefficients of internal and contact friction as well as elasticity modulus of the material do not require involving complicated pressure equipment any more, but can be identified in the laboratory production companies where information on indicators of rock strength can be efficiently used.

**Keywords:** *rock sample, tensile strength, destruction, crack, “stress–deformation” chart*

**Introduction.** One of the important information characteristics necessary for controlling the stress-strain state of the rock massif, increasing the efficiency of crushing and grinding processes in various disintegrator machines is the “normal stress-longitudinal strain” diagram of the over-breaking destruction of rock samples. Such a diagram is determined on special presses pos-

sessing the necessary properties, under which the stresses and displacements in their elements for a given load are small in comparison with those in the samples. Such unique press equipment is available only in several scientific and research centers of the country (M. S. Polyakov Institute of Geotechnical Mechanics under the National Academy of Sciences of Ukraine, Institute of Physics of Mining Processes under the National Academy of Sciences of Ukraine). They require highly skilled

maintenance and are away from the consumer, where we just need timely information about the properties of the rocks. There is therefore a need to develop a method for the theoretical construction of the mentioned diagrams knowing performance properties of rocks defined by simpler methods available to mining companies.

**Unsolved aspects of the problem.** Longitudinal form of destruction of rock samples refers to anomalous types of destruction.

Well-known scientist-structural analyst Bridgman called this kind of rock destruction paradoxical. Professor A. P. Filin explained why this kind of destruction was attributed to the paradoxical in the book "Applied Mechanics of a Solid Deformable Body". He wrote that "in the sample of the rock there are cracks, in the main, parallel to the direction of compression. In the direction perpendicular to the planes of such cracks, there are no normal tensile stresses (as well as no stress at all)".

An attempt to explain the longitudinal form of separation by some scientists is in contradiction with the conditions of compression of the specimen by contact friction and the absence of tensile stresses in the transverse direction.

The researchers describe the calculation of the limiting state of materials by means of a certain criterion (hypothesis) of destruction. When comparing the results of theoretical calculations with experimental data, it was noted that for each hypothesis there is a region of stressed states in which the theory is most consistent with experiment. This circumstance led to the idea of the expediency of dividing the limiting surface into a series of belts, one of which may be, for example, a cylinder, the other – a cone, and so on.

The concept of the impossibility of describing the limiting state of the material by one equation is most clearly expressed in the combined theory of Davidenkov-Friedman strength. This theory is based on the following main points:

1. Depending on the nature of the stressed state, the material can break down both from normal stresses (brittle fracture or breakage by detachment) and from tangential stresses (plastic fracture-shearing failure).

2. For each material, there is a stress-strain relation independent of the type of stress state between stresses and deformations in the coordinates. For plastic materials, the final ordinate of the deformation curve, which is the ultimate resistance to shear, is the material constant.

Therefore, the disclosure of the mechanism of self-organization of the longitudinal form of fracture of rock samples is of great scientific importance. In addition, the longitudinal form of failure, in contrast to other forms of failure, in which a jump-like form of load dependencies on deformation is observed, has an ever increasing character until complete destruction.

We have written several articles on the self-organization of three forms of destruction: truncated-wedge, wedge, diagonal, which are formed due to the maximum effective tangential stresses by the Coulomb criterion, supplemented by taking into account contact friction. In

this paper, we propose to explain this anomalous case of failure due to the mentioned stresses.

**Objectives of the article.** The article aims at development of an analytical method for calculating and constructing stress-strain diagrams, in particular an unusual, longitudinal form of fracture of rock samples.

Input data of the analytical method are four indicator properties available for the operational definition including limit of the material shear strength, internal and contact friction coefficients, the elastic modulus of the material.

**Presentation of the main research.** Longitudinal destruction of rock samples related to anomalous types of destruction, the so-called "paradoxical destruction" (according to Bridgman), was perhaps the first to gain a physical interpretation in our work on the substantiation of the mechanism of fracture of rock samples along a uniaxial compressive load [1]. Difficulty of the explanation of this phenomenon (cracking along compressive load) from the viewpoint of the common theory of body deformation is that in the direction perpendicular to the plane of fracture, there is no normal tensile (and generally) there are no tensions. Researchers express different assumptions on this issue. Most of them are inclined to think that the appearance of longitudinal cracks is due to the margin, perpendicular to the direction of longitudinal loads. But then it is not clear where tensile strains of separation appear from, when, on the contrary, the sample is compressed at the ends with the forces of contact friction, and no tensile force is present. The assumption of the presence of the separation of rocks occurring under compression is anomalous and confuses many researchers. According to the accepted notions of a solid strength, the gap should not occur if there are no tensile strains, although the view of the presence of the separation is supported by some of the authorised scientists in the field of general mechanics of the body. We proceed from the proposition that all forms of destruction of rock samples obtained by Baron during the experimental determination of the strength coefficients on the scale of M. M. Protodyakonov by testing drill cores for crushing as early as 1958, including longitudinal ones, are formed by maximum effective tangential stresses (TMETS) by the Coulomb criterion with the occurrence of contact friction. This was shown by us while solving the problem of the destruction of rocks by compression under their wedge and truncated-wedge form of destruction [1]. The criterion states that the failure occurs on the sliding trajectories on which the effective shear tensions has a maximum value and reaches the limit resistance of the material to pure shear, to be specific, in the limit state of the material before crack formation. The latest works there are the methods of calculating and charting "strain-longitudinal strain" for the truncated wedge and wedge shapes of destruction of samples. In this article, we give a method for calculating and constructing a rather complex diagram of a longitudinal form of fracture without separation concept.

Let us chart TMETS scheme in case of deformation of the sample with the height and the length  $\alpha_1$  (Fig. 1).

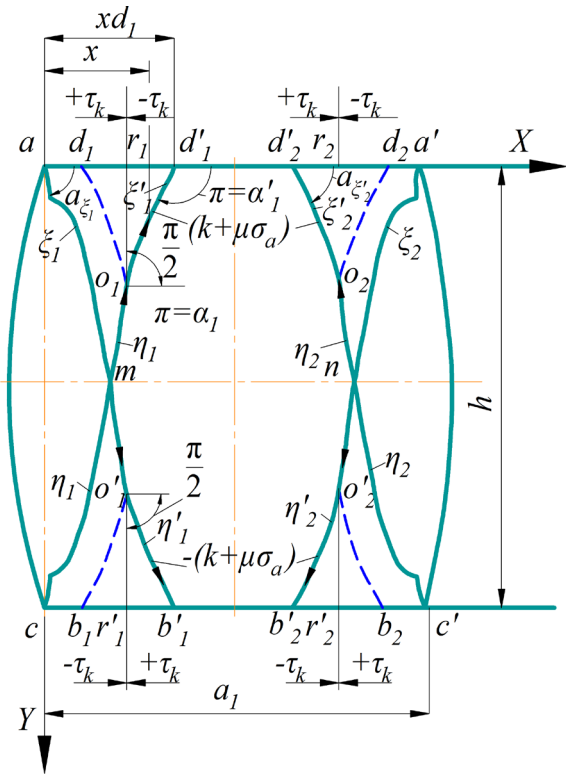


Fig. 1. The initial scheme of formation of TMETS in the sample at the longitudinal form of destruction

It is known that TMETS  $\xi'_1$  and  $\xi'_2$  and TMETS  $\eta_1$  and  $\eta_2$  equivalent develop from the corners of the samples.

Let us send the contact shear strain  $\tau_k$  against lateral deformation. To construct TMETS on the entire cross section of the sample, the  $XY$  axes centre is located in its upper left corner. We accept the rule of tangential strains signs: if the external normal to the site does not coincide with the direction of one of the coordinate axes, the positive shear strain is directed along the corresponding axis, if the outward normal to the site is in the positive direction of the axis, the positive shear strain has the opposite direction. The angle between the direction of the tangent to TMETS at a given point and measured clockwise is denoted with  $a$ . Let us consider one TMETS  $\xi'_1$  coming out of the left corner. TMETS  $\xi'_2$ , starting from the point  $a'$  is symmetrical. The values of signs  $\tau_k$  are shown in Fig. 1.

Let us recall the physics of contact friction action on equilibrium triangles on TMETS  $\xi_1$  at an angle  $a_{\xi_1}$  greater than  $\frac{\pi}{2}$  (Fig. 2, *b*) according to the article [1], in which we developed the concept of a longitudinal fracture of samples based on the strength criteria Coulomb, improved with our view of the contact friction. It was shown that at  $\alpha_{\xi_1} = \frac{\pi}{2}$  the point  $o'_1$  the influence of contact shear strain disappears (Fig. 2, *b*), because it changes the direction of the latter [1].

Point  $o'_1$  is singular. When  $\alpha_{\xi_1} > \frac{\pi}{2}$  the turn of TMETS  $\xi_1$  appears. In this case, in the bottom left quarter of the sample (Fig. 2, *b*) the contact friction pulls

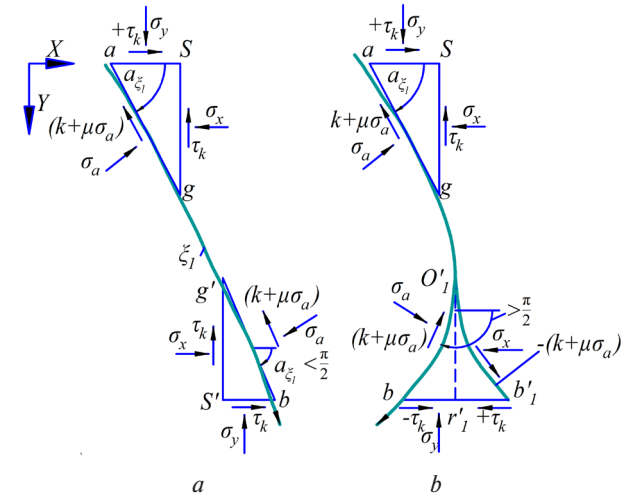


Fig. 2. Equilibrium triangles at TMETS  $\xi_1$ , given that:

$$a - \alpha_{\xi_1} < \frac{\pi}{2}; b - \alpha_{\xi_1} > \frac{\pi}{2}$$

triangle  $o'_1r'_1b'_1$  from TMETS  $\xi_1$ , on the contrary of condition (Fig. 2, *a*), where it is pressed to TMETS  $\xi_1$  this triangle. Consequently, the left TMETS  $\xi_1$  rotates in the lower part and moves to the right and cross to the  $o'_1b'_1$  line. On this line the contact shear strain influences the equilibrium triangle. This situation arises in the upper part of the sample on  $\eta_1$  TMETS. Here the question arises: where will a crack develop? The crack will obviously develop in the area in which the absolute value of the vertical normal strain  $\sigma_y$  required for the development of cracks, will have the lowest value.

Now we write the formula to determine the normal strain  $\sigma_y$  for the left TMETS from [1] works with parameter changes  $b_b, b_d, \beta_b, \beta_d, k_b, k_d$  to the parameters  $b_m, \beta_m$  and  $k_m$

$$\sigma_{y_{\xi_1(n)}} = \frac{1}{\mu} \left[ \frac{k_n (1 + \sin \rho \sqrt{1 - b_{\xi_1(n)}^2}) \times \exp(2\mu(\beta_{\xi_1(n)} + \beta_m))}{1 - \sin \rho \sqrt{1 - b_m^2}} - k_m \right] \quad (1)$$

It is important to note that previously the parameters  $b_b, \beta_b, k_b$  were calculated from the value of the coefficient of friction at the point  $b$  on the lower plane or parameters  $b_d, \beta_d, k_d$  at the point  $d$  on the upper plane. With the longitudinal form of the destruction, the values of these parameters are determined by tangential strains at the intersection of different TMETS. These options are common to overlapping TMETS. Therefore, we denote the parameters  $b_m, \beta_m, k_m$  with the index  $m$ , and their values will be stipulated with the appropriate conditions along the lines  $d'_1c$  and  $d'_2c$  with the fracture downward and along the lines  $ab'_1$  and the line  $b'_2a'$  with the fracture upwards (Fig. 1). We consider the development of a crack on TMETS  $\eta_1$ . With regard to that trajectory parameters justified expressions signs of the equation (1) in the above-mentioned works.

We write parameters from the work [1] into general formula (1) with subsequent clarification. The angle of TMETS  $\eta_1$  is determined by the formula

$$\alpha_{\eta} = \frac{3\pi}{4} - \rho/2 - \beta_{\eta}, \quad (2)$$

where  $\rho$  is the angle of internal friction;  $\beta_{\eta}$  is the angle of rotation of TMETS  $\eta_1$  from the contact friction,

$$\beta_{\eta} = -\frac{1}{2} \operatorname{arctg} \frac{b_{\eta} \cos \rho}{\sin \rho - \sqrt{1 - b_{\eta}^2}}; \quad (3)$$

$$b_{\eta} = \frac{f_k \left(1 - \frac{2Y}{h}\right) \cdot \sigma_{y_{\eta}} \left(1 + \frac{2f_k X}{h}\right)}{k_n + \mu \sigma_{y_{\eta}} \left(1 + \frac{2f_k X}{h}\right)}, \quad (4)$$

where  $f_k$  is the coefficient of contact friction between the sample and the loading plate;  $Y$  is the ordinate value of the crack tip;  $\sigma_{y_{\eta}}$  is the vertical strain at the crack tip;  $k_n$  is limit of the material shear strength;  $\mu = \operatorname{tg} \rho$  is the internal friction coefficient.

We should emphasize that the coefficient of contact friction on the upper surface has a plus sign and at the bottom – the minus sign.

To know the parameters included into the formula (1), we should determine the conditions, when there is TMETS twist from the action of contact shear strains (Fig. 2). This occurs when the strains are zero at points  $o_1$  and  $o_2$ . The point  $o_1$  is formed according to the formula (2)  $\alpha_{\eta} = \frac{3\pi}{4} - \rho/2 - \beta_{\eta} = \frac{\pi}{2}$  if TMETS develops upwardly from point  $c$  to point  $d$  and enters the upper surface of the contact. Then  $\beta_{\eta}$ , in which ta form of longitudinal fracture appears, will be calculated as

$$\beta_{\eta} = \frac{\pi}{4} - \rho/2.$$

There should be defined  $b_{\eta}$  with  $\beta_{\eta} = \left(\frac{\pi}{4} - \rho/2\right)$

from the (3). From here we obtain

$$\operatorname{ctg} \rho = \frac{b_{\eta} \cos \rho}{\sin \rho - \sqrt{1 - b_{\eta}^2}}.$$

After the conversion, we have

$$b_{\eta} = \frac{\cos^2 \rho}{1 + \sin^2 \rho} = \frac{f_k \left(1 - \frac{2Y}{h}\right) \cdot \sigma_{y_{\eta}}}{k_n + \mu \sigma_{y_{\eta}}}. \quad (5)$$

According to the expression (5) we can established that a longitudinal fracture at  $\mu = 1.0$  occurs at approximately  $f_{\sigma} \geq |0.45|$ .

Since at the points  $o_1$  and  $o_2$  (Fig. 2) the shear strain of friction changes sign, in the points of parameters  $b_{\eta} = 0, \beta_{\eta} = 0$ . This condition reduces the normal strain  $\sigma_y$ , necessary for the formation of cracks at  $d'_1$  and  $d'_2$  when we review their development from the top down

respectively with TMETS  $\xi'_1$  and  $\eta_1$  and  $\xi'_2$  and  $\eta_2$  points or  $c'_1$  and  $c'_2$  at TMETS  $\eta'_1$  and  $\xi_1$  or  $\eta'_2$  and  $\xi_2$  with the development of cracks upwards. At points  $o_1$  and  $o_2$  or points  $o'_1$  and  $o'_2$  the voltage and parameters  $b_0$  and  $\beta_0$  are common to different TMETS  $\xi'_1$  and  $\eta_1$  and  $\xi'_2$  and  $\eta_2$  (Fig. 2, b).

So, when  $|\beta_{\xi'_1}| \geq \left(\frac{\pi}{4} - \rho/2\right)$  and  $Y < 0.5h$  at TMETS  $\xi'_1$  in formula b  $m = (1)$ :  $b_m = 0; \beta_m = 0$ , and

$$k_m = \frac{(k_n + \mu \sigma_{y_{\xi'_1}}) \cdot (1 - \sin \rho)}{(1 + \sin \rho)}.$$

The angle TMETS  $\xi'_1$  is determined by formula

$$\alpha_{\xi'_1} = \frac{\pi}{4} + \rho/2 - \beta_{\xi'_1}.$$

Parameter  $\beta_{\xi'_1}$  is determined by the equation (3), wherein the parameter  $b_{\xi'_1}$  is calculated using the formula (4) with a negative  $f_k$  sign, since shear strains on TMETS  $\xi'_1$  has changed the direction from positive to negative.

For TMETS  $\eta_1$  with

$$|\beta_{\eta_1}| \leq \left(\frac{\pi}{4} - \rho/2\right); \quad b_m = \frac{\cos^2 \rho}{1 + \sin^2 \rho}; \quad \beta_m = \left(\frac{\pi}{4} - \rho/2\right).$$

The angle TMETS  $\eta_1$  is defined by the formula

$$\alpha_{\eta_1} = \frac{3\pi}{4} - \rho/2 + \beta_{\eta_1}.$$

To calculate the  $\beta_{\eta_1}$  and  $b_{\eta_1}$  parameters we use the same corrected formulas (3), (4), and

$$k_m = \frac{(k_n + \mu \sigma_{y_{\eta_1}}) (1 - \sin \rho \sqrt{1 - b_{\eta_1}^2})}{(1 + \sin \rho \sqrt{1 - b_m^2}) \exp(2\mu(2\beta_m))}.$$

Finally, we need to decide what values of parameters  $bm$  and  $m\beta$  has to be taken at  $Y > 0.5h$ :  $\beta_m = \frac{\pi}{4} - \rho/2$  or

from the condition  $b_m = \frac{f_k \cdot \sigma_{y_{\eta_1}}}{k_n + \mu \sigma_{y_{\eta_1}}}$  at  $X = 0$  in the lower

left corner. When  $b_{\eta_1} \geq \left(\frac{\pi}{4} - \rho/2\right)$  and  $Y > 0.5h$  TMETS  $\xi_1$  is also formed  $\xi_1$ , since in this case required voltage  $\sigma_y$  becomes the smallest compared to TMETS  $\eta_1$ .

Therefore, if  $\beta_{\xi_1} \geq \left(\frac{\pi}{4} - \rho/2\right)$  and  $y > 0.5h$

$$b_m = \frac{f_k \cdot \sigma_{y_{\xi_1}}}{1 + \sin^2 \rho}.$$

Formulas (3, 4) retain the same form, and

$$\alpha_{\xi_i} = \frac{\pi}{4} + \rho/2 + \beta_{\xi_i}.$$

$$\varepsilon = \frac{p}{E},$$

Thus, all presented expressions are needed to determine the normal strain on the left  $\xi_1$  and  $\eta_1$  by the equation (1). Due to the symmetry, the same voltage values will be on the right side and on TMETS  $\xi_2$  and  $\eta_2$ . In addition, the same pattern will be to solve the problem upwards. Then we get a complex trajectory effective shear strains in the form of (Fig. 1) during their formation from the top down and the bottom up. The cracks will develop from the top and bottom towards each other.

The relative bearing pad is defined by the left and right TMETS from the formula

$$S = \frac{a_1 - 2(X_{d'_i} - X)}{a_1}.$$

Fig. 3 shows the dependence of the strain  $\sigma_y, \sigma_x, p, S$  along the lines of  $d'_1c$  and  $d'_2c'$ . Dependencies  $\sigma_y, \sigma_x, p$ , have increasing character and  $-S$  decreasing, the intensity of the relative pad grounds in the initial period is low.

Now we have to define specific effort, taking into account the output of the sample from under the load and build the “strain-strain” diagram. Specific force is determined by the formula according to the distribution of contact normal strains by L. Prand for the entire sample, taking into account symmetric TMETS in their development from top to bottom according to the formula

$$p = 2\sigma_{\xi_i(n)} \left( \begin{array}{l} \left( 0,5\dot{a}_1 + 0,25\dot{a}_1^2 \frac{f_k}{h} \right) - \\ - \left( X_{d'_i} + X_{d'_i}^2 \frac{f_k}{h} \right) + \\ + \left( X + X^2 \frac{f_k}{h} \right) \end{array} \right) / (a_1 - 2X_{d'_i} + 2X).$$

The strain is determined by Hooke’s law

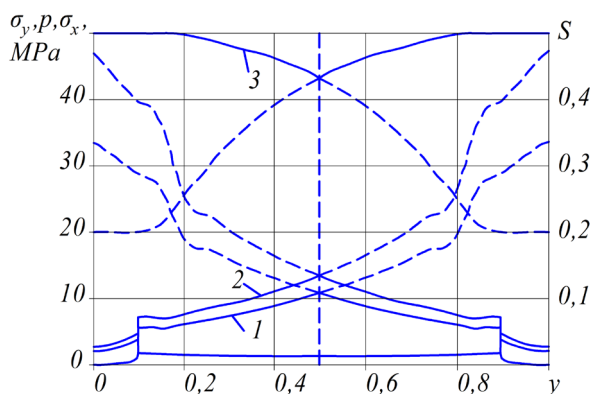


Fig. 3. Dependence of the voltage  $\sigma_y$  (1) at the crack tip, specific efforts  $p$  (2) and the relative pad  $S$  (3) from the vertical axis for  $\rho = 45^\circ; f_k = 0.5; k_n = 1.0 \text{ MPa}$

and the normal strain on the exorbitant curve

$$\sigma_c = p \cdot S,$$

a is analytically derived destruction parts of the sample

Fig. 4 shows the calculated “normal strain-longitudinal strain” chart until the middle of the sample, as the sample is divided into parts with counter developing cracks. Beyond-limit curves of the “voltage – longitudinal strain” diagrams have all the time increasing character before complete destruction of the sample (Fig. 4), as  $L$ . Baron observed in experimental studies. Since strain  $\sigma_c$  has increasing character and exceeds the voltage required for the development of cracks, all four cracks are going to meet each other, probably, in turn, as the rocks do not have exactly the same values of the indicators of physical and mechanical properties. Increasing pattern of the voltage  $\sigma_c$  is explained due to the low intensity of the output of the sample from under the load, i.e. to a small change in the parameter  $S$  in the initial period (Fig. 3).

With the development of the four cracks  $d'_1m, b'_1m, d'_2n, b'_2n$  in pairs toward each other (Fig. 1), the sample splits into three parts (Fig. 5, a).

The central fragment has the form of an elongated ellipse with a long axis. For comparison, we present the results of experimental observations by L. Baron, which qualitatively confirm the valid set of theoretical positions (Fig. 5, b).

**Conclusion.** We developed the mechanism of self-destruction of the elongated shape of rock samples on the basis of the criterion of maximum effective shear strains by Coulomb, improved by taking into account the contact friction. The method of calculating and charting “tension – the longitudinal deformation” was developed. It is shown that the normal strain of the beyond-limit curve has a rising character up to the complete destruction of the samples into three parts, which is confirmed by experimental studies of L. Baron.

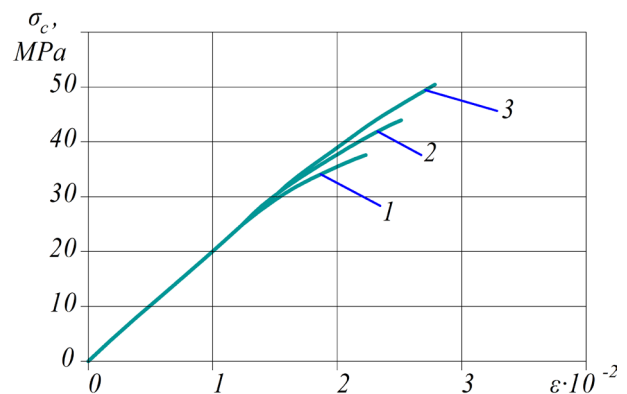


Fig. 4. Beyond-limit curves of the “Voltage-longitudinal strain” chart with longitudinal fracture when  $k_n = 1.0 \text{ MPa}$  and  $f_k = 0.5$ ;  
1 –  $\rho = 40^\circ$ ; 2 –  $\rho = 45^\circ$ ; 3 –  $\rho = 50^\circ$

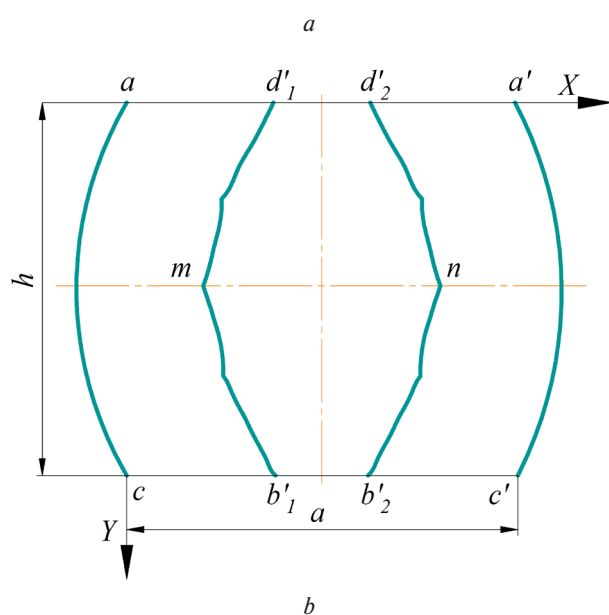


Fig. 5. Centrals parts of the samples with longitudinal form of destruction:

*a* – analytically derived destruction parts of the sample;  
*b* – experimentally obtained central fragments by L. Baron

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**Мета.** Розкриття механізму формування поза-межного деформування зразків гірських порід при їх подовжній формі руйнування.

**Методика.** Побудована на вирішенні задачі руйнування зразків гірських порід на підставі критерію міцності Кулона, удосконаленого врахуванням контактної тертя. Розроблено аналітичний метод розрахунку межі міцності зразка при знанні чотирьох експериментальних показників властивостей гірських порід: межі опору матеріалу зсуву, коефіцієнтів внутрішнього й контактної тертя, модулю пружності матеріалу та геометричних параметрів зразка. На підставі розробленого методу визначені питомі зусилля на тій, що не вийшла з-під навантаження частині зразка у процесі роз-

витку тріщини. Знаючи в кожен момент значення координат вершини однієї або двох тріщин, визначається несуча частина матеріалу зразка, що дорівнює первісній площі останнього за вирахуванням частини, яка вийшла з-під навантаження. На базі граничних питомих зусиль на несучу частину зразка визначається за законом Гука величина деформації, що відповідає граничному питомому зусиллю по відношенню до величини початкової площини.

**Результати.** Розроблено аналітичний метод розрахунку й побудови діаграм „напруження–деформація“ зразків гірських порід при їх подовжній формі руйнування та розкрито механізм утворення цієї форми.

**Наукова новизна.** Уперше розкрито механізм самоорганізації парадоксальної за Бріджменом форми руйнування й надано метод побудови діаграми „напруження–деформація“. Подовжня форма руйнування самоорганізується внутрішнім і контактним тертям відповідно до узагальненого критерію міцності Кулона, що враховує контактне тертя, і механіки локального граничного стану матеріалу у вершині тріщини.

**Практична значимість.** Доступність розрахунку й побудови діаграм „напруження–деформація“ на підставі чотирьох показників: опір матеріалу породи зсуву, коефіцієнтів внутрішнього й контактної тертя, модулю пружності матеріалу, що можуть бути без залучення складного пресового устаткування визначені в лабораторіях виробничих підприємств, де інформація щодо показників міцності порід може бути оперативним використана.

**Ключові слова:** гірська порода, межа міцності, руйнування, тріщина, діаграма „напруження–деформація“

**Цель.** Раскрытие механизма формирования за-предельного деформирования образцов горных пород при их продольной форме разрушения.

**Методика.** Построена на решении задачи разрушения образцов горных пород на основании критерия прочности Кулона, усовершенствованного учетом контактного трения. Разработан аналитический метод расчета предела прочности образца при знании четырех экспериментальных показателей свойств горных пород: предела сопротивления материала сдвигу, коэффициентов внутреннего и контактного трения, модуля упругости материала и геометрических параметров образца. На основании разработанного метода определены удельные усилия на не вышедшей из-под нагрузки части образца в процессе развития трещин. При знании в каждый момент значения координат вершини одной или двух трещин, определяется несущая часть материала образца, которая равна первоначальной площади последнего за вычетом части, вышедшей из-под нагрузки. При знании предельных удельных усилий на несущей части образца определяется по закону Гука величина деформации, которая соответствует предельному удель-

ному усилию по отношению к величине первоначальной площадки.

**Результаты.** Разработан аналитический метод расчета и построения диаграмм „напряжение–деформация“ образцов горных пород при их продольной форме разрушения и раскрыт механизм образования этой формы.

**Научная новизна.** Впервые раскрыт механизм самоорганизации парадоксальной по Бриджмену формы разрушения и дан метод построения диаграммы „напряжение–деформация“. Продольная форма разрушения самоорганизовывается внутренним и контактным трением в соответствии с обобщенным критерием прочности Кулона, учитывающим контактное трение, и механики локального предельного состояния материала в вершине трещины.

**Практическая значимость.** Доступность расчета и построения диаграмм „напряжение–деформация“ на основании четырех показателей: сопротивление материала породы сдвигу, коэффициентов внутреннего и контактного трения, модуля упругости материала, которые могут быть без привлечения сложного прессового оборудования определены в лабораториях производственных предприятий, где информация о показателях прочности пород может быть оперативно использована.

**Ключевые слова:** горная порода, предел прочности, разрушение, трещина, диаграмма „напряжение–деформация“

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