UDC 621.314.12

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INTERRELATION OF A CLARKE AND FORTESCUE TRANSFORMATION FOR THE THREE-PHASE ASYMMETRICAL ELECTRICAL NETWORK

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ЗВ'ЯЗОК ПЕРЕТВОРЕННЯ КЛАРКА ТА ФОРТЕСКЬЮ ДЛЯ ТРИФАЗНОЇ НЕСИМЕТРИЧНОЇ ЕЛЕКТРИЧНОЇ МЕРЕЖІ

Purpose. Determination of the analytical interrelation of Clarke and Fortescue transformation for an asymmetric sinusoidal system of currents of a three-phase four-wire network.

Methodology. To find the way for the use of the direct, reverse and zero sequences as components of the power circulating in the intersection of the four-wire current line, a problem is set to determine the interrelation of Clarke (α-β-0) and Fortescue (1-2-0) transformations. An analysis of the order of calculation of the direct, reverse and zero sequences components is carried out for the general case and for every separate phase. Comparison of Clarke transformation for separate sequences is performed using Euler formulae in an exponential form. Analytical relations determining the components of the current and voltage of direct and reverse sequences in domain α-β-0 are obtained. The said relations are used as the basis for instantaneous power decomposition in the use of *p-q* theory. The performed numerical calculation of current and voltage components, as well as power according to *p-q* theory, confirms the obtained analytical results.

Findings. Interrelation of direct and reverse sequence conversion of voltages (currents) in domain 1-2-0 (Fortescue transformation) with voltages (currents) in domain α-β-0 (Clarke transformation) is analytically substantiated, which makes it possible to separate in the latter the components caused by the action of direct and reverse sequences.

Originality. Analytical determination of direct, reverse sequence components in domain α-β is proposed to use these components during calculation of instantaneous power according to p-q theory for four-wire lines.

Practical value. The obtained results present a part of the analysis of electric power components in electrical four-wire networks with asymmetric parameters of the mode and they can be developed for networks with non-sinusoidal voltages and currents.

Keywords: *four-wire line, power active filter, direct, reverse, zero sequences, Fortescue transformation, Clarke transformation, p-q theory of instantaneous power*

Introduction. Energy processes in electricity network electric circuits are determined by the character and value of voltage and current in the unit for which the analysis is being performed [1, 2]. For symmetric modes of the network, i. e. symmetric networks with symmetric load, the calculation and assessment of power indices are performed for one phase and afterwards the obtained results are multiplied by the number of phases [3, 4]. If asymmetric modes occur due to load or network asymmetry, energy indices for each phase are analyzed separately, and the method of symmetric components is used to assess the degree of asymmetry [5, 6]. In this case corresponding symmetric components of current and voltage with Fortescue matrices are used (1-2-0) [5]. These components are used to determine indices of electricity quality [3, 7] and corresponding components of the electric network power [2], the order of the calculation and rated values for the former are standardized. The mentioned parameters are successfully used to improve the quality of energy consumption in both scientific and engineering calculations when choosing the structure and parameters of the elements of balancing and compensating devices [1, 3, 7].

Analysis of the recent research. At the present stage of development and introduction of power transforming technology devices into electric units, the importance of solving the problems of both balancing, compensating and filtration increases. Besides, filters with constant structure meet the requirements of electricity quality in the units of electric networks less and less, especially those with sharply variable nonlinear load [1, 3, 4]. As a result, power active filters (PAF) have been developed and are being introduced [1, 3, 8]; they perform the functions of compensation and filtration. Recently, due to development of science and technology, hardware and algorithmic PAF have been actively improved [1, 3, 8]. The PAF operation algorithm is determined by the order of calculation of power components subject to compensation, and, consequently, by calculation of current for shunt PAF. Algorithms based on *p-q* theory [1, 3] and its modernization for four-wire networks – *p-q-r*-theory [1] are widely used.

Both variants provide the use of Clarke transformation (α-β-0) [9] of currents and voltages for further determination of power components. As some papers demonstrate, PAF balancing is completely absent in the first case [1], and in the second case it depends on the cause of asymmetry, i. e. asymmetry of the network, asymmetry of the load or their combination [7].

Problem statement. It is rational to find the place of the components of direct, reverse and zero sequences while determining instantaneous power using α-β-0 transformation.

Objective of the article — determination of analytical interrelation of Clarke and Fortescue transformations for asymmetric sinusoidal system of currents of a three-phase four-wire network.

Presentation of the main research and explanation of the results. The method for the analysis of unbalanced multi-phase circuits was proposed in 1918 [5]. The paper contained substantiation of a new (as to that time) point of view concerning research of three-phase devices and soon this method was known as the method of symmetric components. A three-phase system of voltages is presented

$$\mathbf{U}_{abc}=\begin{bmatrix}u_a\\u_b\\u_c\end{bmatrix}=\begin{bmatrix}U_a\sqrt{2}\sin(\omega t+\varphi_a)\\U_b\sqrt{2}\sin(\omega t+\varphi_b-2\pi/3)\\U_c\sqrt{2}\sin(\omega t+\varphi_c+2\pi/3)\end{bmatrix},\qquad(1)$$

where u_a, u_b, u_c are instantaneous values of voltages of phases a-b-c; U_a, U_b, U_c are effective values of voltage of corresponding phases; ω is angular frequency; φ_a, φ_b, φ_c are phase shifts of voltages a-b-c. In Euler's exponential form [10], taking into account complexes conjugation

$$\mathbf{U}_{abc}=\frac{\sqrt{2}}{2i}\begin{bmatrix}\dot{U}_a e^{i\omega t}-\dot{U}_a^* e^{-i\omega t}\\\dot{U}_b e^{i\omega t}-\dot{U}_b^* e^{-i\omega t}\\\dot{U}_c e^{i\omega t}-\dot{U}_c^* e^{-i\omega t}\end{bmatrix}\leftarrow\begin{bmatrix}\dot{U}_a=U_a e^{i(\varphi_a+0)}\\\dot{U}_b=U_b e^{i(\varphi_b-2\pi/3)}\\\dot{U}_c=U_c e^{i(\varphi_c+2\pi/3)}\end{bmatrix},\quad(2)$$

$\dot{U}$ is usually called u phasor in the well-known literature [10].

Determination of the components of direct, reverse and zero sequences (1-2-0) is performed on the basis of ideas stated in Fortescue's works [5] and developed in Lyon's works [6], caused by the influence of the mentioned components on the electromagnetic torque of an alternating current electric machine. In particular, in trigonometric form, components 1-2-0 are determined in the following way

$$\mathbf{U}_{120}=\begin{bmatrix}u_1\\u_2\\u_0\end{bmatrix}=\frac{1}{3}\begin{bmatrix}U_a\sqrt{2}\sin(\omega t+\varphi_a)+U_b\sqrt{2}\sin(\omega t+\varphi_b)+U_c\sqrt{2}\sin(\omega t+\varphi_c)\\U_a\sqrt{2}\sin(\omega t+\varphi_a)+U_b\sqrt{2}\sin(\omega t+\varphi_b+2\pi/3)+U_c\sqrt{2}\sin(\omega t+\varphi_c-2\pi/3)\\U_a\sqrt{2}\sin(\omega t+\varphi_a)+U_b\sqrt{2}\sin(\omega t+\varphi_b-2\pi/3)+U_c\sqrt{2}\sin(\omega t+\varphi_c+2\pi/3)\end{bmatrix},$$

which corresponds to the exponential form

$$\mathbf{U}_{120}=\frac{\sqrt{2}}{2i}\begin{bmatrix}\dot{U}_1 e^{i\omega t}-\dot{U}_1^* e^{-i\omega t}\\\dot{U}_2 e^{i\omega t}-\dot{U}_2^* e^{-i\omega t}\\\dot{U}_0 e^{i\omega t}-\dot{U}_0^* e^{-i\omega t}\end{bmatrix}\leftarrow\begin{bmatrix}\dot{U}_1=U_1 e^{i\varphi_1}\\\dot{U}_2=U_2 e^{i\varphi_2}\\\dot{U}_0=U_0 e^{j\varphi_0}\end{bmatrix}.\quad(3)$$

In this case relation of phasors a-b-c with phasors 1-2-0 is performed [5, 10], as

$$\begin{aligned}\dot{U}_1&=\frac{1}{3}\left(\dot{U}_a+\dot{U}_b a+\dot{U}_c a^{-1}\right);\\\dot{U}_2&=\frac{1}{3}\left(\dot{U}_a+\dot{U}_b a^{-1}+\dot{U}_c a\right);\\\dot{U}_0&=\frac{1}{3}\left(\dot{U}_a+\dot{U}_b+\dot{U}_c\right),\end{aligned}\qquad(4)$$

where operator a

$$\begin{aligned}a&=e^{i\frac{2\pi}{3}}=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)=-\frac{1}{2}+i\frac{\sqrt{3}}{2};\\a^2&=a^{-1}=e^{i\frac{4\pi}{3}}=e^{-i\frac{2\pi}{3}}=\\&=\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)=-\frac{1}{2}-i\frac{\sqrt{3}}{2}.\end{aligned}\qquad(5)$$

Deriving transformation matrix

$$\mathbf{T}_{120}^{abc}=\frac{1}{3}\begin{bmatrix}1&a&a^{-1}\\1&a^{-1}&a\\1&1&1\end{bmatrix},$$

expression (4) is given in the form of formula [10]

$$\mathbf{U}_{120}=\mathbf{T}_{120}^{abc}\mathbf{U}_{abc}\rightarrow\begin{bmatrix}\dot{U}_1\\\dot{U}_2\\\dot{U}_0\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1&a&a^{-1}\\1&a^{-1}&a\\1&1&1\end{bmatrix}\begin{bmatrix}\dot{U}_a\\\dot{U}_b\\\dot{U}_c\end{bmatrix}.$$

Virtually, the obtained components are voltages of the direct, reverse and zero sequences of phase a, i. e. $\mathbf{U}_{120}=\mathbf{U}_{120,\ a}$, respectively $\mathbf{T}_{120}^{abc}=\mathbf{T}_{120,a}^{abc}$. To obtain components for phases b and c it is sufficient to perform the following transformations taking into account location of vectors

$$\mathbf{U}_{120,b}=\mathbf{T}_{120,b}^{abc}\mathbf{U}_{abc}\rightarrow\begin{bmatrix}\dot{U}_{1b}\\\dot{U}_{2b}\\\dot{U}_{0b}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}a^{-1}&1&a\\a&1&a^{-1}\\1&1&1\end{bmatrix}\begin{bmatrix}\dot{U}_a\\\dot{U}_b\\\dot{U}_c\end{bmatrix};$$

$$\mathbf{U}_{120,c}=\mathbf{T}_{120,b}^{abc}\mathbf{U}_{abc}\rightarrow\begin{bmatrix}\dot{U}_{1c}\\\dot{U}_{2c}\\\dot{U}_{0c}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}a&a^{-1}&1\\a^{-1}&a&1\\1&1&1\end{bmatrix}\begin{bmatrix}\dot{U}_a\\\dot{U}_b\\\dot{U}_c\end{bmatrix}.$$

Transformations for sequences of each phase are obtained analogously

$$\mathbf{U}_{abc,1}=\mathbf{T}_{1,abc}^{abc}\mathbf{U}_{abc}\to\begin{bmatrix}\dot{U}_{a1}\\ \dot{U}_{b1}\\ \dot{U}_{c1}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & a & a^{-1}\\ a^{-1} & 1 & a\\ a & a^{-1} & 1\end{bmatrix}\begin{bmatrix}\dot{U}_{a}\\ \dot{U}_{b}\\ \dot{U}_{c}\end{bmatrix}; \quad (6)$$

$$\mathbf{U}_{abc,2}=\mathbf{T}_{2,abc}^{abc}\mathbf{U}_{abc}\to\begin{bmatrix}\dot{U}_{a2}\\ \dot{U}_{b2}\\ \dot{U}_{c2}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & a^{-1} & a\\ a & 1 & a^{-1}\\ a^{-1} & a & 1\end{bmatrix}\begin{bmatrix}\dot{U}_{a}\\ \dot{U}_{b}\\ \dot{U}_{c}\end{bmatrix}. \quad (7)$$

Performing reverse transformation as shown in [10], parameters of phases *a*, *b*, *c* are expressed through the components of direct, reverse and zero sequences of phase *a*

$$\mathbf{U}_{abc}=\mathbf{T}_{abc}^{120}\mathbf{U}_{120}\to\begin{bmatrix}\dot{U}_{a}\\ \dot{U}_{b}\\ \dot{U}_{c}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ a^{-1} & a & 1\\ a & a^{-1} & 1\end{bmatrix}\begin{bmatrix}\dot{U}_{1}\\ \dot{U}_{2}\\ \dot{U}_{0}\end{bmatrix};$$

$$\mathbf{T}_{abc}^{120}=\left[\mathbf{T}_{120}^{abc}\right]^{-1}.$$

For a numerical example we will assume phase voltages

$$u_a=360\sqrt{2}\sin(\omega t+0);$$
$$u_b=440\sqrt{2}\sin(\omega t+0-2\pi/3);$$
$$u_c=480\sqrt{2}\sin(\omega t+0.8+2\pi/3).$$

Graphic representation of these voltages in time and complex domains and their presentation in the form of symmetric components are shown in Fig. 1.

Clarke transformation is used to simplify calculations in three-phase circuits [9, 10]. It allows passing from a three-phase system to a two-phase one, or, to be more exact, to a one-phase system with two projections α-β for three basic lines and additionally a zero component *0* (it is also designated as "γ") for four-wire ones. As stated in paper [9], relation between three-phase symmetric system *a-b-c* and system α-β is established provided axis α is located on the vector of phase *a* and axis β is situated perpendicularly to α directed towards the vector of phase *b*.

Then on the basis of equations (1) and (2) and taking into account location of vectors of the three-phase system and corresponding angles between them and axes α and β, projections of each vector on the axis are determined and these projections are summed up taking the direction into consideration

$$U_{\alpha}=\frac{2}{3}\left(1\dot{U}_a-\frac{1}{2}\dot{U}_b-\frac{1}{2}\dot{U}_c\right);$$
$$U_{\beta}=\frac{2}{3}\left(0\dot{U}_a+\frac{\sqrt{3}}{2}\dot{U}_b-\frac{\sqrt{3}}{2}\dot{U}_c\right). \quad (8)$$

Zero component in system α-β-0 is calculated in the same way as zero sequence in system 1-2-0, i.e.

$$\dot{U}_0=\frac{2}{3}\left(\frac{1}{2}\dot{U}_a+\frac{1}{2}\dot{U}_b+\frac{1}{2}\dot{U}_c\right).$$

In a matrix form Clarke transformation matrix is used,

$$\mathbf{T}_{\alpha\beta0}^{abc}=\frac{2}{3}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix};$$

$$\mathbf{U}_{\alpha\beta0}=\mathbf{T}_{\alpha\beta0}^{abc}\mathbf{U}_{abc}\to\begin{bmatrix}\dot{U}_{\alpha}\\ \dot{U}_{\beta}\\ \dot{U}_{0}\end{bmatrix}=\frac{2}{3}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}\dot{U}_{a}\\ \dot{U}_{b}\\ \dot{U}_{c}\end{bmatrix}.$$

Multiplier in matrix $\mathbf{T}_{\alpha\beta0}^{abc}$ $\frac{2}{3}$ is used on the basis of materials given in [9]. Along with this variant, by reference to the balance of powers, as papers [6, 10] demonstrate, multiplier $\sqrt{\frac{2}{3}}$ is used. The transformation result is shown in Fig. 2.

Using Euler's formula for a vector that should be turned by a certain angle,

$$e^{i\Theta}z=(\cos\Theta+i\sin\Theta)(x+iy)=(x'+iy');$$
$$x'=x\cos\Theta-y\sin\Theta;$$
$$y'=x\cos\Theta+y\sin\Theta,$$

and taking into account that on a complex plane for a single vector a is

$$\Re(a)=\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2};$$
$$\Im(a)=\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2};$$
$$\Re\left(a^{-1}\right)=\cos\left(-\frac{2\pi}{3}\right)=-\frac{1}{2};$$
$$\Im\left(a^{-1}\right)=\sin\left(-\frac{2\pi}{3}\right)=-\frac{\sqrt{3}}{2},$$

for expression (4) taking into consideration (5), omitting the analysis of zero sequence

$$\dot{U}_1=\frac{1}{3}\left(\dot{U}_a-\frac{1}{2}\dot{U}_b+i\frac{\sqrt{3}}{2}\dot{U}_b-\frac{1}{2}\dot{U}_c-i\frac{\sqrt{3}}{2}\dot{U}_c\right);$$
$$\dot{U}_2=\frac{1}{3}\left(\dot{U}_a-\frac{1}{2}\dot{U}_b-i\frac{\sqrt{3}}{2}\dot{U}_b-\frac{1}{2}\dot{U}_c+i\frac{\sqrt{3}}{2}\dot{U}_c\right).$$

In the form of a real and an imaginary component, it is

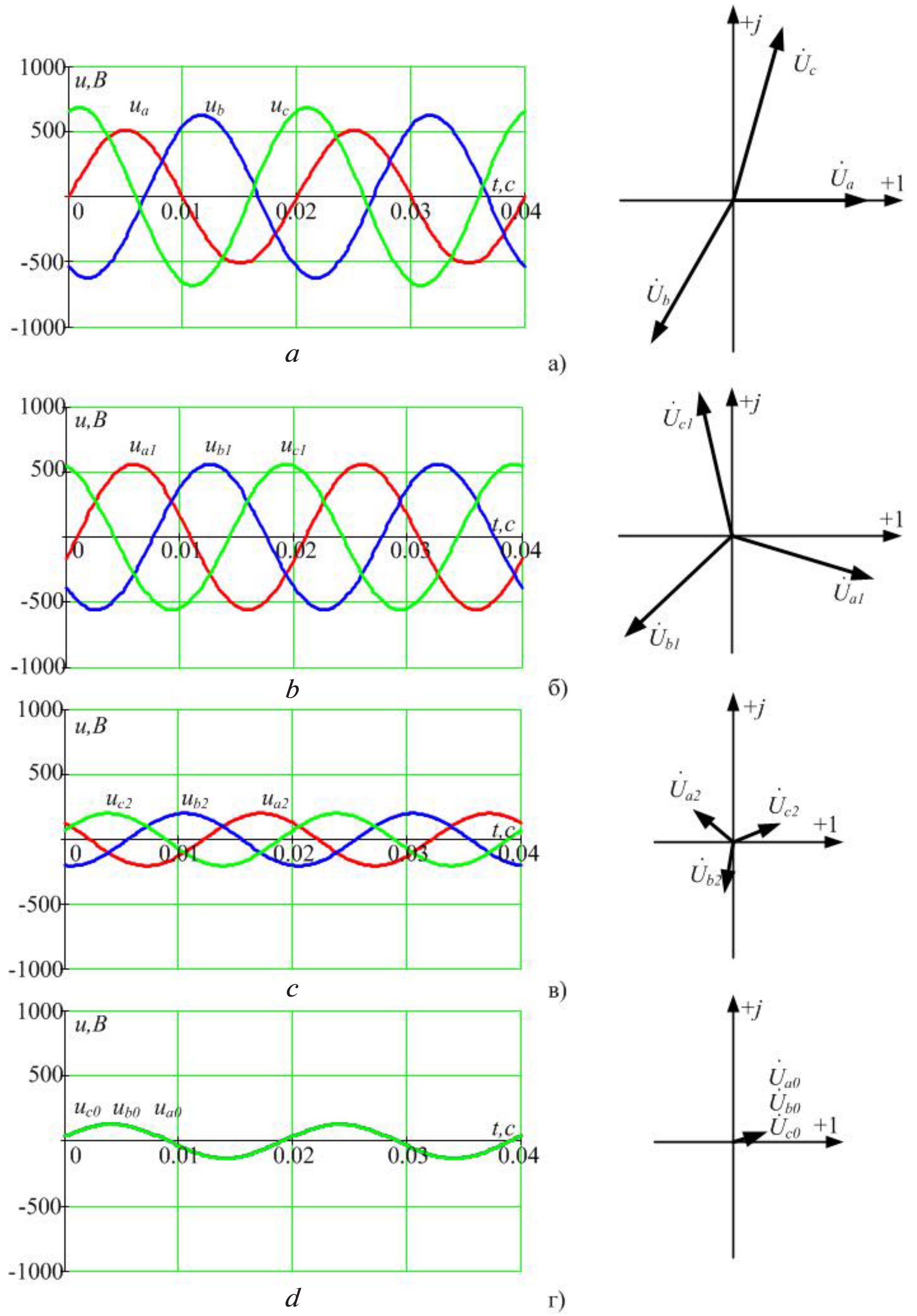

***Fig. 1.** Presentation of asymmetric system of voltages in the time and complex domains:*

a — asymmetric voltage; b — direct sequence components; c — reverse sequence components; d — zero sequence components

$$\dot U_1 = \frac{1}{3}\left[\left(\dot U_a - \frac{1}{2}\dot U_b - \frac{1}{2}\dot U_c\right) + i\left(0 + \frac{\sqrt{3}}{2}\dot U_b - \frac{\sqrt{3}}{2}\dot U_c\right)\right] = \frac{1}{2}\left[\left(\dot U_\alpha + i\dot U_\beta\right)\right];$$

$$\dot U_2 = \frac{1}{3}\left[\left(\dot U_a - \frac{1}{2}\dot U_b - \frac{1}{2}\dot U_c\right) - i\left(0 + \frac{\sqrt{3}}{2}\dot U_b - \frac{\sqrt{3}}{2}\dot U_c\right)\right] = \frac{1}{2}\left[\left(\dot U_\alpha - i\dot U_\beta\right)\right];$$

$$\dot U_0 = \frac{1}{3}\left[\dot U_a + \dot U_b + \dot U_c\right] = \dot U_0,$$

i. e. direct sequence is determined as a vector sum of a half of projection α with a half of projection β turned by 90° in positive direction, and reverse sequence — as a vector sum of a half of projection α with a half of projection β turned by 90° in negative direction. In a matrix form

$$\mathbf{U}_{120} = \mathbf{T}_{120}^{\alpha\beta0}\mathbf{U}_{\alpha\beta0} \rightarrow \begin{bmatrix}\dot U_1\\ \dot U_2\\ \dot U_0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & i & 0\\ 1 & -i & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot U_\alpha\\ \dot U_\beta\\ \dot U_0\end{bmatrix}.$$

Reverse transformation is correspondingly

$$\mathbf{U}_{\alpha\beta 0} = \mathbf{T}_{\alpha\beta 0}^{120}\mathbf{U}_{120} \rightarrow \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -i & i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{U}_\alpha \\ \dot{U}_\beta \\ \dot{U}_0 \end{bmatrix}.$$

To single out components of direct and reverse sequences in plane αβ transformation (8) of expressions (6) and (7) is performed. Omitting intermediate calculations the following is obtained

$$\mathbf{U}_{\alpha\beta,1} = \left(\mathbf{T}_{\alpha\beta}^{abc} - i\mathbf{T}_{\alpha\beta}^{abc\prime}\right)\mathbf{U}_{abc} \rightarrow \begin{bmatrix} \dot{U}_{\alpha 1} \\ \dot{U}_{\beta 1} \end{bmatrix} =$$

$$= \frac{1}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \dot{U}_a \\ \dot{U}_b \\ \dot{U}_c \end{bmatrix} - i\frac{1}{3}\begin{bmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \dot{U}_a \\ \dot{U}_b \\ \dot{U}_c \end{bmatrix};$$

$$\mathbf{U}_{\alpha\beta,2} = \left(\mathbf{T}_{\alpha\beta}^{abc} + i\mathbf{T}_{\alpha\beta}^{abc\prime}\right)\mathbf{U}_{abc} \rightarrow \begin{bmatrix} \dot{U}_{\alpha 2} \\ \dot{U}_{\beta 2} \end{bmatrix} =$$

$$= \frac{1}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \dot{U}_a \\ \dot{U}_b \\ \dot{U}_c \end{bmatrix} + i\frac{1}{3}\begin{bmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \dot{U}_a \\ \dot{U}_b \\ \dot{U}_c \end{bmatrix}.$$

Having vector presentation according to expressions (2) and (4) one passes to the time domain. All the above mentioned operations are performed with a four-wire network current.

Thus, instantaneous power for a four-wire network according to *p-q* theory [8]

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} u_0 & 0 & 0 \\ 0 & u_\alpha & u_\beta \\ 0 & u_\beta & -u_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix},$$

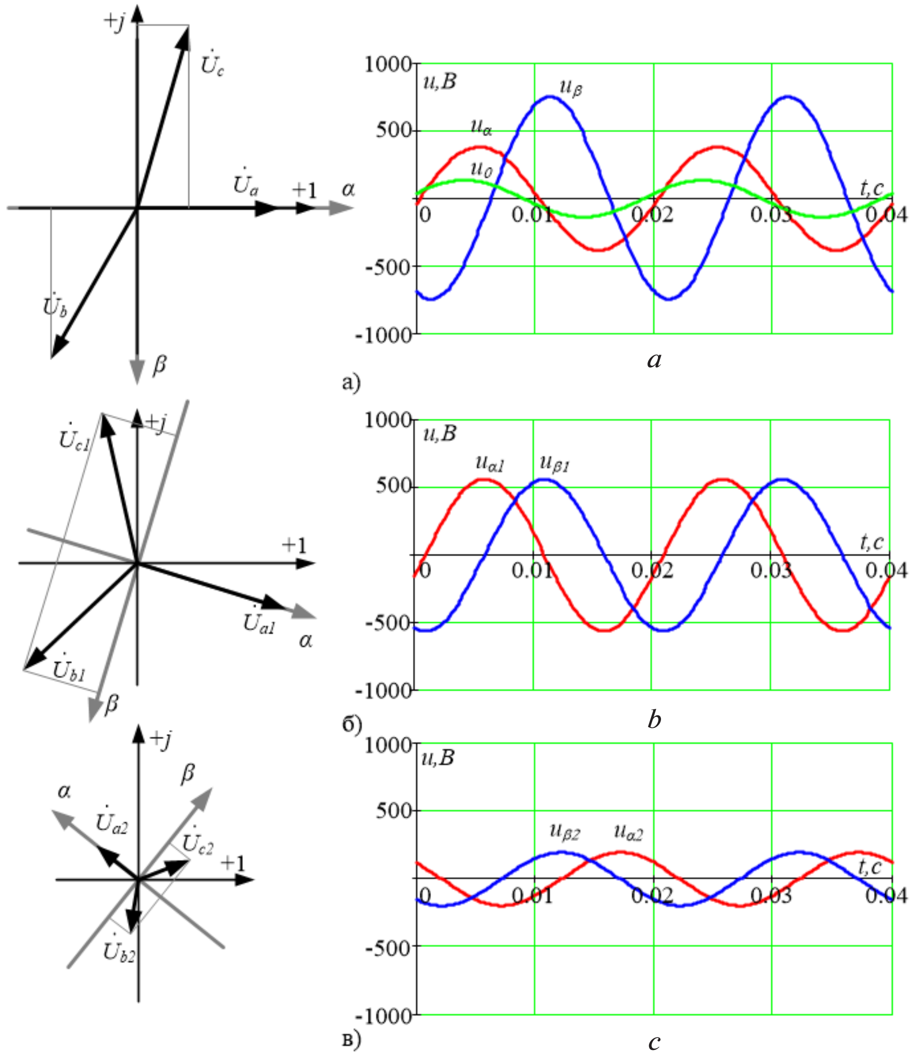

Fig. 2. Presentation of voltages using Clarke transformation:
a — voltages a-b-c; b — voltages of direct sequence αβ; c — voltages of reverse sequence αβ

can be presented taking into account the components of direct and reverse sequences

$$p_0 = u_0 i_0;$$
$$p = (u_{\alpha 1} + u_{\alpha 2})(i_{\alpha 1} + i_{\alpha 2}) + (u_{\beta 1} + u_{\beta 2})(i_{\beta 1} + i_{\beta 2});$$
$$q = (u_{\beta 1} + u_{\beta 2})(i_{\alpha 1} + i_{\alpha 2}) - (u_{\alpha 1} + u_{\alpha 2})(i_{\beta 1} + i_{\beta 2}).$$

On the basis of the obtained equation analogously to [8], the following components can be singled out

$$\begin{aligned}
p_0 &= u_0 i_0;\\
\overline{p} &= u_{\alpha 1} i_{\alpha 1} + u_{\alpha 2} i_{\alpha 2} + u_{\beta 1} i_{\beta 1} + u_{\beta 2} i_{\beta 2};\\
\tilde{p} &= u_{\alpha 1} i_{\alpha 2} + u_{\alpha 2} i_{\alpha 1} + u_{\beta 1} i_{\beta 2} + u_{\beta 2} i_{\beta 1};\\
\overline{q} &= u_{\beta 1} i_{\alpha 1} + u_{\beta 2} i_{\alpha 2} - u_{\alpha 1} i_{\beta 1} - u_{\alpha 2} i_{\beta 2};\\
\tilde{q} &= u_{\beta 1} i_{\alpha 2} + u_{\beta 2} i_{\alpha 1} - u_{\alpha 1} i_{\beta 2} - u_{\alpha 2} i_{\beta 1}.
\end{aligned} \tag{9}$$

Thus, power of the analyzed intersection of the network is divided into five components [8], among which components $\overline{p}$ and $\overline{q}$ reflect constant active and reactive powers, $\tilde{p}$ and $\tilde{q}$ components of pulsating active and reactive powers, p_0 component of zero sequence active power. In Fig. 3 for currents of phases *abc* (Fig. 3, *a*),

$$i_a = 60\sqrt{2}\sin(\omega t - 0.3);$$
$$i_b = 40\sqrt{2}\sin(\omega t - 0.3 - 2\pi/3);$$
$$i_c = 80\sqrt{2}\sin(\omega t - 1 + 2\pi/3).$$

Taking into account their α-β-0 transformation (Fig. 3, *b*) and voltages (Fig. 2), power components calculated according to (9) are created in time domain (Fig. 3, *c, d*). It should be mentioned that zero sequence power p_0 (Fig. 3, *c*) does not change the sign, i. e. it is active.

Analyzing equation system (9), attention should be paid to the structure of the components: constant active power consists of products of the like voltages and currents of the like sequences; variable active power consists of products of the like voltages and currents of unlike sequences; constant inactive power consists of products of unlike voltages and currents of the like sequences; variable inactive power consists of multitude of unlike voltages and currents of unlike sequences.

Conclusions and recommendations for further research.

1. Direct and reverse transformations of voltages (currents) in domain 1-2-0 (Fortescue transformation) with voltages (currents) in domain α-β-0 (Clarke transformation) have been analytically substantiated, which makes it possible to single out components caused by the action of direct and reverse sequences in the latter transformation.

2. Analytic determination of components of direct, reverse sequences in domain α-β has been proposed for their use in calculation of instantaneous power according to *p-q* theory for four-wire lines.

3. Using the obtained sequences of currents and voltages in domain α-β-0 according to the concept of *p-q* theory, the components of power have been deter-

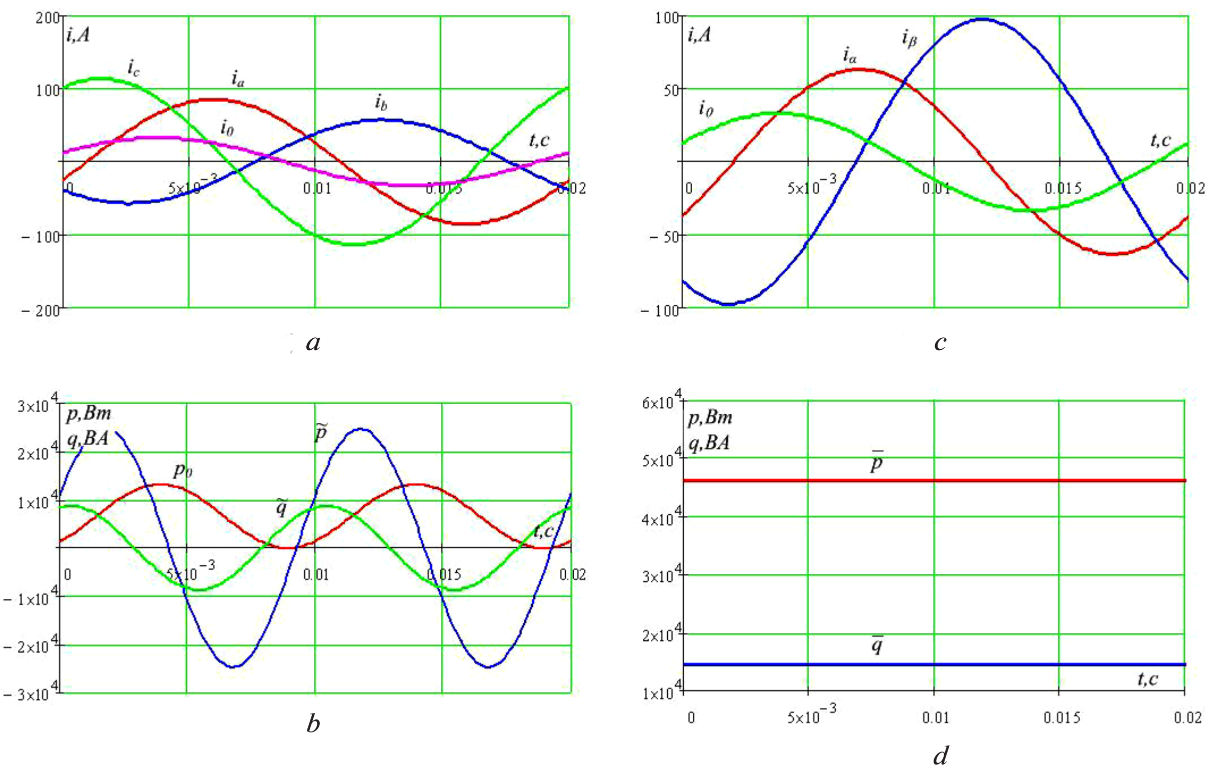

Fig. 3. Determination of power components:
a — currents a-b-c; b — variable components of power; c — currents α-β-0; d — constant components of power

mined taking into account the distribution of multipliers characterized by a certain unique order in each of the components. The obtained theoretical results have been confirmed by numerical calculations.

4. The obtained results present the basis for correction of algorithms of power components in assessment and compensation in electric four-wire networks at nonsinusoidal and asymmetric parameters of the mode.

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Мета. Визначення аналітичного зв'язку перетворення Кларка та Фортескью для несиметричної синусоїдальної системи струмів трифазної чотирипровідної мережі.

Методика. Для пошуку шляху використання прямої, зворотної та нульової послідовностей як складових потужності, що циркулює в перетині чотирипровідної електричної лінії, поставлено задачу визначення зв'язку перетворення Кларка (α-β-0) та Фортескью (1-2-0). Проведено аналіз порядку розрахунку складових прямої, зворотної та нульової послідовностей у загальному випадку та для кожної з фаз окремо. Виконане співставлення перетворення Кларка для окремих послідовностей, використовуючи формули Ейлера в показовій та експоненційній формах. Отримані аналітичні співвідношення, що визначають складові струму й напруги прямої та зворотної послідовностей в області α-β-0. Зазначені співвідношення покладено як підґрунтя для декомпозиції миттєвої потужності при використанні p-q-теорії. Виконаний чисельний розрахунок складових струму та напруги, а також потужності згідно з p-q-теорією, підтверджує отримані аналітичні результати.

Результати. Аналітично обґрунтовано зв'язок прямого та зворотного перетворення напруг (струмів) в *о*бласті 1-2-0 (перетворення Фортескью) з напругами (струмами) в області α-β-0 (перетворення Кларка), що дозволяє в останній відокремити складові, викликані дією прямої та зворотної послідовностей.

Наукова новизна. Запропоноване аналітичне визначення складових прямої, зворотної послідовностей в області α-β для використання цих складових під час розрахунку миттєвої потужності за p-q-теорією для чотирипровідних ліній.

Практична значимість. Отримані результати є складовою аналізу компонент потужності електричної енергії в електричних чотирипровідних мережах при несиметричних параметрах режиму та можуть бути розвинуті для мереж із несинусоїдальними напругами та струмами.

Ключові слова: *чотирипровідна лінія, силовий активний фільтр, пряма, зворотна, нульова послідовності, перетворення Фортескью, перетворення Кларка, p-q-теорія миттєвої потужності*

Цель. Определение аналитической связи преобразования Кларка и Фортескью для несимметричной синусоидальной системы токов трехфазной четырехпроводной сети.

Методика. Для поиска пути использования прямой, обратной и нулевой последовательностей как составляющих мощности, циркулирующей в пересечении четырехпроводной электрической линии, поставлена задача определения связи преобразования Кларка (α-β-0) и Фортескью (1-2-0). На основании анализа последовательности расчета составляющих прямой, обратной и нулевой последовательностей в общем случае и для каждой из фаз отдельно, и сопоставление с преобразованием Кларка для отдельных последовательностей, получены аналитические соотношения, которые позволяют определить составляющие прямой и обратной последовательностей в области α-β-0. Такой способ представления составляющих тока и напряжения позволяет выполнить декомпозицию мгновенной мощности при использовании *p-q*-теории. Выполнен численный расчет составляющих тока и напряжения, а также мощности по *p-q*-теории, который подтвердил полученные аналитические выражения.

Результаты. Аналитически обоснована связь прямого и обратного преобразования напряжений (токов) в области 1-2-0 (преобразование Фортескью) с напряжениями (токами) в области α-β-0 (преобразование Кларка), что позволяет в последней отделить составляющие, вызванные действием прямой и обратной последовательностей.

Научная новизна. Предложено аналитическое определение составляющих прямой, обратной последовательностей в области α-β для использования этих составляющих при расчете мгновенной мощности по *p-q*-теории для четырехпроводных линий.

Практическая значимость. Полученные результаты являются составляющей анализа компонент мощности электрической энергии в электрических четырехпроводных сетях при несимметричных параметрах режима и могут быть использованы для сетей с несинусоидальными напряжениями и токами.

Ключевые слова: *четырехпроводная линия, силовой активный фильтр, прямая, обратная, нулевая последовательности, преобразования Фортескью, преобразования Кларка, p-q-теория мгновенной мощности*

Рекомендовано до публікації докт. техн. наук Д. Й. Родькіним. Дата надходження рукопису 06.10.15.

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A TRIGGER METHOD BASED ON MAGNETIC INDUCTION IN METAL-ENCLOSED SPACE

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ПУСКОВИЙ МЕТОД НА ОСНОВІ МАГНІТНОЇ ІНДУКЦІЇ В МЕТАЛОЗАМКНЕНОМУ ПРОСТОРІ

Purpose. As the application environment of storage test technology is getting worse and worse, the trigger signal with the function of wake becomes the key to the success of the test. In this paper, a new method that induces changes of the intensity of magnetization to implement trigger is proposed. This method is very useful for solving the harsh environment of metal-enclosed space.

Methodology. We design a kind of thin film coils which are placed inside the metal shell. It can induct the slight changes in the magnetic field, and output corresponding induction electromotive force which will be amplified to meet requirements of the trigger signal.

Findings. By theoretical and experimental analysis, this method can be used to realize the trigger with the metal-enclosed space. Moreover, it can improve the reliability of the trigger by several times to generate the trigger signal.

Originality. We designed a controlled variable magnetic field, a flexible thin-film coil and a signal processing unit. These can achieve the metal confined space of the trigger problem. The research on this aspect has not been found at present.

Practical value. We also increase the reliability of the trigger design, and the optimization of the system can be used to achieve the best form of trigger. This provides a certain way and method to extend the application field of storage test technology.

Keywords: *storage test, metal-enclosed space, thin film coil, electromagnetic induction, magnetic induction, trigger signal*