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# AN IMPROVED PRINCIPAL COMPONENT ANALYSIS METHOD BASED ON WAVELET DENOISING PREPROCESSING FOR MODAL PARAMETER IDENTIFICATION

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# ПОКРАЩЕНИЙ МЕТОД АНАЛІЗУ ГОЛОВНИХ КОМПОНЕНТ, ЗАСНОВАНИЙ НА ПОДАВЛЕННІ ШУМУ ЗА ДОПОМОГОЮ ВЕЙВЛЕТ-ПЕРЕТВОРЕННЯ, З МЕТОЮ ІДЕНТИФІКАЦІЇ МОДАЛЬНИХ ПАРАМЕТРІВ

**Purpose.** Accurate identification of modal parameters is an important prerequisite for structural health monitoring and damage identification.

**Methodology.** Wavelet analysis is one of the most advantageous methods because it has the ability to represent the local features of the signal in time and frequency domain. The modal parameter identification effectively achieved using principal component analysis (PCA), can be regarded as a type of system recognition.

**Findings.** Because PCA is sensitive to Gaussian measurement noise, the authors propose a novel method that combines wavelet denoising with PCA. The technique was applied to modal parameter identification.

**Originality.** The signals are decomposed into wavelets with several layers, and the resulting wavelet coefficients are preprocessed according to a threshold. The signals are then reconstructed to reduce the effect of noise. The research on this aspect has not been found at present.

**Practical value.** Simulation results for beams show that the proposed method is able to recognize the main modal shapes and eigenfrequencies. Additionally, it can improve the precision of the identified modal parameters and extract some previously lost modes.

**Keywords:** *modal parameter identification, principal component analysis (PCA), wavelet denoising, system recognition, wavelet analysis, Gaussian measurement noise*

**Introduction.** Modal parameters are definitive characteristics of dynamic analysis of a mechanical structure, and modal parameter identification has been developed as a vital approach for solving vibration problems in engineering structures. The modal parameters of a system include eigenfrequencies, modal damping ratios, and modal shapes, amongst others. Eigenfrequencies and modal shapes determine, respectively, the resonant frequency of the dynamic system and the mode of vibration of the structure in resonance, and are thus regarded as two of the most valuable modal parameters. Accurate identification of modal parameters is most important in structural damage recognition and health monitoring. Several methods can be used to identify modal parameters. Traditional time-domain identification methods use a linear fit or calculate an eigenvalue. In the late 1980s, researchers began to study operational modal analysis. Modal parameters of a structure can also be determined using the eigen parameters of the coefficient matrix of the autoregressive exogenous excitation model [1]. Bakir, P. G., Eksioglu, E. M. and Alkan, S., (2012) studied reliability analysis of the complex mode indicator function and Hilbert transform techniques for operational modal analysis [2], while Bai, J., Yan, G. and Wang, C. (2013) proposed a method based on manifold learning.

Principal component analysis (PCA) is a multivariate analysis method that has been extensively applied to large multidimensional data sets. Principal components are linear combinations of the original data, which can be used to visualize similarities in an ensemble of signals. PCA or PCA-based methods are used to reduce the number of variables in a multivariate data set, while retaining as much variation as possible [3]. Ding, M., Tian, Z. and Xu, H. (2010) presented an adaptive kernel PCA method to improve computational speed and approximation. Wang, J., Barreto, A., Wang, L., Chen, Y., Rishe, N., Andrian, J. and Adjouadi, M., (2010) proposed a method based on PCA to determine the relationships between modal shapes and the principal component linear compound matrix, and those between modal responses and principal components.

Nevertheless, it is inevitable that samples contain noise, and this interference introduces uncertainty into the modal parameters. Traditional PCA is sensitive to Gaussian measurement noise, which means that noise in the samples may cause relatively large errors in the results, and even the loss of some important parameters. To distinguish real modes from computational, fictitious, or noise-generated modes, noise is often removed by preprocessing, allowing the extraction of useful data. The wavelet analysis method has excellent properties at different resolutions [4] and has been extensively applied to signal processing, pattern recognition, image processing, and many other fields. Akhtar et al. presented a framework, based on ICA and wavelet denoising, to improve the pre-processing of electroencephalographic (EEG) signals. This paper proposes a novel method that combines wavelet denoising with the PCA method. This method is applied to the model identification of dynamical systems, and the effectiveness of the technique using beams with different system boundaries, load types, and load positions is investigated.

**Modal parameter identification of a dynamic system using pca.** ***Modal parameter identification.*** In modal parameter identification, the modal shapes, eigenfrequencies and damping factors of a structure are identified. The modal shapes are a mathematical description of the deflection vibration patterns when the system vibrates at one of the eigenfrequencies. An alternative interpretation is that the modal shapes describe the participation of each independent oscillation in the output response.

In traditional modal analysis, the differential equations of motion for a linear vibration system with $n$-degrees of freedom can be written as

$$M\ddot{X}(t) + C\dot{X}(t) + kX(t) = F(t),$$

where $M$, $C$, and $K \in \mathbb{R}^{n\times n}$ are the mass, damping, and stiffness matrices, respectively, $F(t) \in \mathbb{R}^{n\times n}$ is the exciting force, and $\ddot{X}(t)$, $X(t)$, and $X(t) \in \mathbb{R}^{n}$ are the acceleration, velocity and displacement vectors, respectively. The free decay oscillations for a proportionally or lightly damped system are expressed as

$$X(t) = \Psi Q(t) = \sum_{i=1}^{m} \psi_j Q_j(t),$$

where $\Psi$ is the modal shape matrix comprising modal shapes $\psi_j$, and $Q(t)$ is the vector matrix formed by the modal response $Q_j(t)$ containing the modal coordinates.

According to vibration theory, if the eigenfrequencies of the system are not equal, the regularization modal shapes $\psi_j$ are normalized to be orthogonal to each other. Based on this theoretical model, independent modal parameter identification is a particular case of PCA decomposition.

***Mathematical description of PCA.*** Suppose that there are $m$ observed signals $X = [x_1, x_2, ..., x_m]^T$ in $\mathbb{R}^m$, and

$$X = PY; \quad (1)$$

$$P^T P = I_{n\times n}, \quad (2)$$

where $Y = [y_1, y_2, ..., y_n]^T$ are $n$ irrelevant unknown latent variables in $\mathbb{R}^n$, and $P \in \mathbb{R}^{m\times n}$ is a linear transformation matrix containing the principal components of $X$. In addition, the linear transformation matrix ($P$) and principal components ($Y$) satisfying equations (1) and (2) are principal component decompositions of $X$.

$C_{XX} = E[XX^T] \in \mathbb{R}^{m\times m}$ is the autocorrelation matrix of $X$. We assume that the rank of $C_{XX}$ is $n$ (where $n \le m$), and that eigenvalues $\lambda_i$, $i = 1, 2, ..., m$, satisfy $\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0$ and $\lambda_n + 1 = \lambda_n + 2 = \cdots = \lambda_m = 0$.

$$C_{XX} = V \Lambda V^T;$$

$$V \Lambda V^T = I_{n\times n},$$

where $\Lambda$ is a diagonal matrix formed by n nonzero eigenvalues $\lambda_i \ne 0$, $i = 1, 2, ..., n$ in $C_{XX}$, $V = [\upsilon_1\upsilon_2\cdots\upsilon_i\cdots\upsilon_n]$ is a transformation matrix composed of normalized eigenvectors, and $\upsilon_i \in \mathbb{R}^m$ corresponds to eigenvalue $\lambda_i$.

Therefore, when the rank of $C_{XX}$ is $n$ ($n \le m$), $X$ can be uniquely divided into

$$X = V(V^T X).$$

**Modal parameter identification using PCA.** In practice, signals observed by the devices always contain noise. If the observed signals contain measurement noise, the model in equation (1) becomes

$$\widehat{X} = PY + N,$$

where $N$ is Gaussian measurement noise. According to the information given in **Modal Parameter Identification of a Dynamic System Using PCA Section**, the largest $n$ eigenvalues of $C_{\hat{X}\hat{X}}$ are selected to form a diagonal matrix, and the transformational matrix consists of the corresponding eigenvectors. So, $C_{\hat{X}\hat{X}}$ can be decomposed into

$$C_{\hat{X}\hat{X}} \approx \hat{V}\hat{\Lambda}\hat{V}^T.$$

Meanwhile, the observed signal $\hat{X}_{(t)}$ can also be approximately decomposed into

$$\hat{X} \approx \hat{V}(\hat{V}^T \hat{X}).$$

Here, η is defined as the variance contribution accumulation of the first $n$ principal components, which is computed using the accumulated variance of the observed signal $\sigma_X^2$ and principal component $\sigma_Y^2$. That is,

$$\eta = \sigma_Y^2 \Big/ \sigma_X^2 = \sum_{i=1}^{n} \lambda_i \Big/ \sum_{i=1}^{m} \lambda_i . \qquad (3)$$

If we do not know the number of uncorrelated latent variables (i.e., $n$ is unknown), a threshold can be set in (3) to truncate the process.

From (3), the key idea behind modal parameter identification using PCA is the variance contribution accumulation, η. Note that η is easily affected by the observed signal. When the observed signal contains measurement noise, the contribution of some modal parameters to the variance is small, causing them not to be recognized by PCA. This is an inherent weakness of modal parameter identification using PCA when the observed signals contain measurement noise.

**Wavelet transform theory.** ***Fundamentals of the wavelet transform.*** In wavelet analysis, a variety of different wavelet basis functions can be constructed using translation and scaling. That is,

$$h_{a,b}(t) = \frac{1}{\sqrt{a}} h\left(\frac{t-b}{a}\right),$$

where $h(t)$ is the mother wavelet, $h_{a,b}(t)$ is called the wavelet basis function, $a \in \mathbb{R}^*$ is the scaling factor, and $b \in \mathbb{R}$ is the shift factor. If $a$ is large, the primary function is a stretched pre-image wavelet, which is a low-frequency function, whereas if $a$ is small, it is a contractible wavelet, which is a narrow, high-frequency function. The wavelet transform is defined as

$$WT_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} h^*\left(\frac{t-b}{a}\right) x(t)dt,$$

where $h^*(t)$ is the complex conjugate of $h(t)$, and the $WT_x(a,b)$ are known as wavelet transform coefficients.

The time-frequency resolution ratio of the wavelet transform is variable. The time range of the wavelet transform is shorter at higher frequencies, whereas the frequency width is narrower at low frequencies.

The wavelet decomposition of a signal is shown in Fig. 1. Because noise is typically in a high or low-frequency signal, the wavelet coefficient must be processed according to some threshold. We can then reconstruct the signal and reduce the noise.

In Fig. 1, the A processes return low-frequency coefficients while the D processes return high-frequency coefficients. The resolution is

$$S = A_n + D_n + \ldots + D_2 + D_1.$$

The low-frequency part of $A_n$ can be further split into low frequency $A_{n+1}$ and high frequency $D_{n+1}$.

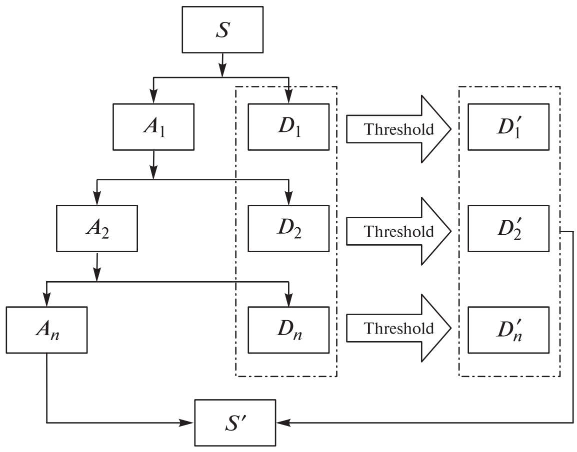


*Fig. 1. Wavelet decomposition of the signal*

After calculating the discrete wavelet transform of the sampled signal $f(t) = s(t) + n(t)$, the wavelet coefficients $W(a, b)$ are obtained. The wavelet transform still consists of two parts: $s(k)$, which corresponds to the wavelet coefficient $W_s(a,b)$, and $n(k)$, which corresponds to the wavelet coefficients, $W_n(a,b)$.

The signal and noise have different characteristics after the wavelet transformation. Therefore, the noise can be significantly suppressed using a suitable threshold, while preserving the main signal features. The noise reduction method in the wavelet domain is called wavelet shrinkage. The main step in wavelet denoising is to select an appropriate threshold value and rule. In hard-thresholding, important coefficients remain unchanged, whereas, in soft-thresholding, important coefficients are reduced by the absolute threshold value.

***Multivariate wavelet denoising.*** Multivariate wavelet denoising is an improvement on the wavelet denoising method that considers correlations among signals from different channels. Multivariate wavelet denoising regards multiple signals as a whole and transforms signals into the wavelet domain for denoising. The orthogonal basis of the transform domain can be determined from the estimated noise covariance matrix of the multivariate signals.

The following process is used to remove noise from signals. The process starts with a matrix $\hat{X} \in \mathbb{R}^{n \times m}$ containing $m$ signals presenting the columns of $\hat{X}$.

Step 1: Calculate the wavelet decomposition at level $J$ of each column of $\hat{X}$. Generate $J + 1$ matrices ($D_1, \ldots, D_J$) containing the detail coefficients at levels 1 to $J$ for the $m$ signals, and approximate coefficients ($A_J$) of the $m$ signals. $D_j$ is $(n/2j) \times m$ in size and $A_J$ is $(n/2^J) \times m$.

Step 2: Define an estimator of the noise covariance matrix $\hat{\Sigma\varepsilon}$, and then calculate the singular value decomposition of $\hat{\Sigma\varepsilon}$ to produce an orthogonal matrix $V$ such that $\hat{\Sigma\varepsilon} = V \Lambda V^T$, where $\Lambda = \mathrm{diag}(\lambda_i, 1 \le i \le m)$. After changing the basis $V$ (i.e., $D_j V$, $1 \le j \le J$), apply the $m$ univariate thresholding strategies to each detail, using the threshold $t_i$ = for the $i$-th column of $D_j V$.

Step 3: Reconstruct a denoised matrix $\hat{X}$ from the simplified detail and approximation matrices by changing the basis using $V^T$ and inverting the wavelet transform.

In the above process, the change in basis is first used to remove the correlation of the $m$ components of the noise. Then, $m$-univariate wavelet denoising is applied to the process. Thus, the proposed procedure can be regarded as a combination of multiple univariate denoising.

***Estimating the noise covariance.*** The most commonly used wavelet denoising method is threshold denoising, with the hard- and soft-thresholding methods currently the most widely used. In the one-dimensional case, a convenient estimate of the noise variance is needed to fine-tune the threshold. Generally, the finer detail coefficients are essentially Gaussian noise, which may be contaminated by a few large coefficients from the signal. A robust estimator based on the median absolute deviation is applied.

Similarly, as a natural extension to the multivariate case, a robust estimator of the covariance matrix must be defined. As in the one-dimensional case, $D_1$ is the matrix of details at level 1 and is essentially $m$-dimensional white noise from the covariance matrix $\Sigma_\varepsilon$, corrupted by various coefficients because of the robust estimator applied to $D_1$. Therefore, in this paper, we use the minimum covariance determinant (MCD) estimator and apply it to $D_1$.

**Improved pca for modal parameter identification using wavelet denoising preprocessing.** ***Wavelet denoising PCA framework.*** The proposed wavelet denoising PCA (WDPCA) method, combining PCA and wavelet denoising for modal parameter identification, is described in this section.

PCA is sensitive to noise, which means that noisy samples may result in relatively large errors, and some important parameters may be lost. This paper uses the wavelet transform to improve the capacity of PCA with regard to this issue. The wavelet transform has good time-frequency characteristics and low-entropy, can work at multiple resolutions, and can flexibly select the mother wavelets. These properties mean that it is an effective method for signal denoising and compression.

Wavelet denoising is based on how wavelets represent smooth signals using a sparse set of coefficients. Small coefficients can be suppressed to denoise the signals. The effect of wavelet denoising is closely determined by the mother wavelet and transformation parameters, amongst others.

A multiscale version of PCA (MSPCA) was proposed to fine-tune the limits in statistical process control. In this paper, MSPCA is applied to eliminate insignificant principal components, so that the proposed method reduces the dimension of the signal and removes any remaining noise.

***Implementation of WDPCA.*** The WDPCA method comprises the following steps:

Step 1: Let there be $m$ observed signals $\widehat{X} \in \mathbb{R}^{n\times m}$. Set the decomposition level $J$, and the iteration termination threshold value $\varepsilon$. Initialize counter variable $c = 1$, and the variance contribution accumulation $\eta = 0$.

Step 2: Define the estimator of the noise covariance matrix of the observed signals as $C_{\widehat{X}\widehat{X}} = MCD(D_1)$, where $MCD$ is described in Multivariate Wavelet Denoising Section. Then, compute $V$ such that $C_{\widehat{X}\widehat{X}} = \widehat{V}\widehat{\Sigma}\widehat{V}^T$, where $\Lambda = \mathrm{diag}(\widehat{\lambda}_i, 1 \le i \le m)$. Apply the $m$ univariate thresholding strategies to each detail after changing the base (i. e., $D_jV$, $1 \le j \le J$), using threshold $t_i = \sqrt{2\widehat{\lambda}_i \ln(n)}$ for the $i$-th column of $D_jV$.

Step 3: Matrix $A_J$, obtained in Step 2 (see Multivariate Wavelet Denoising Section) is computed by PCA, with the eigenvalues sorted in descending order.

Step 4: Use equation (3) to compute $\eta_c = \breve{\lambda}_c \Big/ \sum_{i=1}^{c} \breve{\lambda}_i$, update the variance contribution accumulation such that $\eta = \eta + \eta_c$.

Step 5: If $\eta > \varepsilon$, the first $c$ principal components satisfy the condition and the algorithm is terminated. Otherwise, let $c = c + 1$, and return to Step 4.

Step 6: Reconstruct a denoised matrix $\breve{X}$, from the simplified detail and approximation matrices, by changing the basis using $V^T$ and inverting the wavelet transform.

Step 7: Through a final PCA, extract the first $c$ eigenvectors from $\breve{X}$ and compute their latent variables by eigenvalues.

To reduce the influence of system measurement Gaussian distribution noise, a combined method is introduced to obtain a reconstructed signal. Then, the main modal parameters are identified from the reconstructed signals.

**Simulation results.** To confirm whether the proposed method can extract modal parameter information from the response signals, this paper uses a multiple-frequency sinusoidal superposition undamped simply supported beam under concentrated excitation. In addition, a limited-bandwidth damping cantilever beam subjected to a load with uniform Gaussian measurement noise is also introduced in this section. The basic parameters of the beams are as follows: length equal to 1.0 m; cross section is square; width and height both set to 0.005 m; the material is steel; modulus of elasticity is 205 GPa; Poisson's ratio of the material is 0.3; and density of material is 7850 kg/m$^3$. The beam can be treated as a one-dimensional continuous system lengthwise if the load is applied along the length.

LMS Virtual.lab is computer-aided engineering software for dynamic simulations. Using the finite element analysis (FEA) implemented by LMS Virtual.lab, the undamped modal shapes and eigenfrequencies can be obtained. The results are regarded as the real shapes and eigenfrequencies. In addition to a comparison of the modal shapes and eigenfrequencies, we investigated the modal assurance criterion (MAC), which is an important standard when considering the effectiveness of modal shape identification [3]. This is computed as

$$MAC_{\varphi_i,\phi_i} = (\varphi_i^T \phi_i)^2 \big/ (\varphi_i^T \varphi_i)(\phi_i^T \phi_i),$$

where $\phi_i$ is the $i$-th identified shape, $\varphi_i$ represents the real $i$-th shape, and $(\varphi_i^T \phi_i)$ represents the inner product of the two vectors. Note that $0 \le MAC_{\varphi_i,\phi_i} \le 1$, where a value closer to one represents a more accurate estimated shape.

***Multiple-frequency sinusoidal superposition undamped simply supported beam under concentrated excitation.*** Dividing the undamped, simply supported 1-m beam into 1000 parts at regular intervals generates 1001 measurement points. At frequencies of 205, 91.3, 366, 572, 824, 1121, and 22, observed signals can be generated from 0.2-m unit positions by loading multiple-frequency sinusoidal excitation with the corresponding power 60, 30, 30, 30, 30, 30, and 30 units, respectively. The sampling time and frequency interval are 1 s, and 4096 Hz, respectively, with 1 % Gauss measurement noise added.

In this experiment, response data of the undamped simply supported beam were selected at the first, 400th and 500th time steps, with 15 % additive Gaussian white noise. The AER and RER respectively represent absolute error reconstruction and relative error reconstruction. The performance of the wavelet denoising method is shown in Table 1. It is clear that the noise is reduced.

The first six principal components satisfy the condition. The PCA, WDPCA, and FEA results are compared in Table 2. Table 2 shows the Pareto charts for the PCA and WDPCA decompositions, for no and 1 % measurement noise. There are no significant changes in the accumulation contribution rate of the seventh principal component, and the fast Fourier transform results of the principal components identified by PCA and WDPCA, for no and 1 % measurement noise.

In Table 2, we can see that the variance contribution accumulation does not change significantly if there is noise in the response data. Because the modal contribution of the fifth principal component is small, no fifth modal shape is identified by PCA or WDPCA, additionally, the eighth and ninth modal shapes recognized by PCA are lost if the observed signals contain noise, so PCA is sensitive to measurement noise. On the contrary, WDPCA efficiently reduces the interference of noise. The eighth modal shape is identified by WDPCA with 1 % measurement noise.

*Table 1*

Error comparison of the reconstructed signals

| | PCA | | WDPCA | |
|---|---|---|---|---|
| | AER | RER | AER | RER |
| Signal 1 | 0.1243 | 54.62 % | 0.0229 | 9.16 % |
| Signal 2 | 0.1155 | 82.62 % | 0.0212 | 19.10 % |
| Signal 3 | 0.1151 | 78.70 % | 0.0256 | 14.85 % |

*Table 2*

Modal shape and fast Fourier transform for the PCA, WDPCA, and FEA methods applied to a simply supported beam

| | FEA modalnumber | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Eigenfrequencies identified by FEA (Hz) | 22.89 | 91.55 | 205.99 | 366.2 | 572.23 | 824 | 1121.6 | 1464.9 | 1854 |
| No noise | Modal shape number dentified by PCA | 1 | 3 | 2 | 5 | – | 4 | 6 | 7 | 8 |
| | MAC (%) | 100 | 100 | 100 | 100 | – | 100 | 100 | 98.34 | 98.34 |
| | Eigenfrequencies identified by PCA (Hz) | 23 | 92 | 207 | 367 | – | 825 | 1123 | 1466 | 1855 |
| | Relative percentage (%) | 0.481 | 0.492 | 0.005 | 0.210 | – | 0.121 | 0.125 | 0.075 | 0.054 |
| | Modal shape number identified by WDPCA | 1 | 3 | 2 | 5 | – | 4 | 6 | 7 | 8 |
| | MAC (%) | 100 | 100 | 100 | 100 | – | 100 | 100 | 98.34 | 98.34 |
| | Eigenfrequencies identified by WDPCA (Hz) | 23 | 92 | 207 | 367 | – | 825 | 1123 | 1466 | 1855 |
| | Relative percentage (%) | 0.481 | 0.492 | 0.005 | 0.210 | – | 0.121 | 0.125 | 0.075 | 0.054 |
| 1% addictive Gaussian measurement noise | Modal shape number dentified by PCA | 1 | 3 | 2 | 5 | – | 4 | 6 | – | – |
| | MAC (%) | 100 | 100 | 100 | 100 | – | 100 | 100 | – | – |
| | Eigenfrequencies identified by PCA (Hz) | 23 | 92 | 207 | 367 | – | 825 | 1123 | – | – |
| | Relative percentage (%) | 0.481 | 0.492 | 0.005 | 0.210 | – | 0.121 | 0.125 | – | – |
| | Modal shape number identified by WDPCA | 1 | 3 | 2 | 5 | – | 4 | 6 | 7 | – |
| | MAC (%) | 100 | 100 | 100 | 100 | – | 100 | 100 | 100 | 76.32 |
| | Eigenfrequencies identified by WDPCA (Hz) | 23 | 92 | 207 | 367 | – | 825 | 1123 | 1466 | – |
| | Relative percentage (%) | 0.481 | 0.492 | 0.005 | 0.210 | – | 0.121 | 0.125 | 0.075 | – |

A number of simulation processing results for the simply supported beam show that WDPCA is more effective than PCA, because the former method recognizes more modal parameters than the latter when the observed data contain noise. Additionally, because the resolution ratio of the fast Fourier transform is not sufficiently large, the frequencies identified by both PCA and WDPCA contain errors.

The variance contribution accumulation does not change significantly if there is noise in the response data. Because the modal contribution of the fifth principal component is small, no fifth modal shape is identified by PCA or WDPCA. The eighth and ninth modal shapes recognized by PCA are lost if the observed signals contain noise, so PCA is sensitive to measurement noise. On the contrary, WDPCA efficiently reduces the interference of noise. The eighth modal shape is identified by WDPCA with 1 % measurement noise.

A number of simulation processing results for the simply supported beam show that WDPCA is more effective than PCA, because the former method recognizes more modal parameters than the latter when the observed data contain noise. Additionally, because the resolution ratio of the fast Fourier transform is not sufficiently large, the frequencies identified by both PCA and WDPCA contain errors.

***Cantilever beam with additive white noise.*** Dividing a 1-m cantilever beam into 1000 parts at regular intervals generates 1001 measurement points with a 0.01 modal damping. This experiment applied the same white noise to each point to obtain the observed signals. The sampling time and frequency interval were set to 1 s and 4096 Hz, respectively, and 10 % Gaussian measurement noise was added.

In this experiment, the response data were selected from the cantilever beam at the 20th, 1000th, and 4000th time intervals with 10 % additive Gaussian white noise. The wavelet denoising results, shown in Table 3, demonstrate that the proposed method is effective.

The PCA, WDPCA, and FEA results are shown in Table 4. The variance contribution accumulation does not significantly change if the observed signals contain

*Table 3*

Errors in reconstructed signals

| | PCA | | WDPCA | |
|---|---|---|---|---|
| | AER | RER | AER | RER |
| Signal 1 | 0.0781 | 22.09 % | 0.0135 | 3.42 % |
| Signal 2 | 0.0799 | 42.32 % | 0.0139 | 6.13 % |
| Signal 3 | 0.0802 | 53.78 % | 0.0131 | 7.24 % |

*Table 4*

Modal shapes and fast Fourier transforms for PCA, WDPCA, and FEA applied to the cantilever beam

| | FEA modal number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Eigenfrequencies identified by FEA (Hz) | 801533 | 51.078 | 142.94 | 279.86 | 462.12 | 689.4 | 961.34 | 1277.5 | 1637.5 |
| No noise | Modal shape number dentified by PCA | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | MAC (%) | 100 | 100 | 100 | 100 | 100 | 100 | 99.99 | 99.96 | 98.67 |
| | Eigenfrequencies identified by PCA (Hz) | 8.2 | 51 | 143.2 | 279.6 | 461.6 | 689.6 | 959.2 | 1280.2 | 1651.6 |
| | Relative percentage (%) | 0.57 | 0.15 | 0.18 | 0.09 | 0.03 | 0.22 | 0.21 | 0.86 | 0.03 |
| | Modal shape number identified by WDPCA | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | MAC (%) | 100 | 100 | 100 | 100 | 100 | 100 | 99.99 | 99.96 | 98.67 |
| | Eigenfrequencies identified by WDPCA (Hz) | 8.2 | 51 | 143.2 | 279.6 | 461.6 | 689.6 | 959.2 | 1280.2 | 1651.6 |
| | Relative percentage (%) | 0.57 | 0.15 | 0.18 | 0.09 | 0.03 | 0.22 | 0.21 | 0.86 | 0.03 |
| 10 % addictive Gaussian measurement noise | Modal shape number dentified by PCA | 1 | 2 | 3 | 4 | – | – | – | – | – |
| | MAC (%) | 100 | 100 | 100 | 99.64 | – | – | – | – | – |
| | Eigenfrequencies identified by PCA (Hz) | 8.2 | 51 | 143.2 | 280 | – | – | – | – | – |
| | Relative percentage (%) | 0.57 | 0.15 | 0.18 | 0.05 | – | – | – | – | – |
| | Modal shape number identified by WDPCA | 1 | 2 | 3 | 4 | 5 | – | – | – | – |
| | MAC (%) | 100 | 100 | 99.99 | 99.98 | 93.15 | – | – | – | – |
| | Eigenfrequencies identified by WDPCA (Hz) | 8.2 | 51 | 143.2 | 280 | 462.2 | – | – | – | – |
| | Relative percentage (%) | 0.57 | 0.15 | 0.18 | 0.05 | 0.02 | – | – | – | – |

noise. Some modal shapes recognized by PCA are missing when the observed data contain noise, for example, the sixth and seventh modal shapes in Table 4. Therefore, noise can create problems when using PCA to identify modal parameters. WDPCA, however, deals more effectively with noise. The fifth modal shape was effectively extracted by WDPCA when the observed signals contained 10 % Gaussian measurement noise. Therefore, WDPCA is insensitive to Gaussian measurement noise, and can identify more modal parameters (such as modal shape and eigenfrequency) from the observed signals compared with PCA. Table 4 shows that PCA is sensitive to Gaussian measurement noise, but WDPCA reduces the interference. Especially for the fifth modal shape, the identified shape is very different to that recognized by FEA. Because there is a larger eigenfrequency error, the fifth modal shape extracted by PCA is missing. The fifth modal shape is identified by WDPCA, with a corresponding MAC of 93.15 %. Additionally, most of the eigenfrequencies of the modal shapes extracted by these two methods are similar; thus, the performance of WDPCA is better than that of PCA.

Numerical simulations of the simply supported and cantilever beams demonstrated that noise introduces uncertainties into the modal parameter calculations. This leads to low accuracy when identifying modes. The proposed method is insensitive to Gaussian measurement noise and succeeded in extracting the main contributory modal shapes and eigenfrequencies from the response signals with a relatively high accuracy.

**Conclusions.** By comparing the results obtained with and without noise, we have shown that the modal parameter identification method based on PCA is sensitive to measurement noise. This highlights the necessity for denoising the observed signals.

The proposed WDPCA method is a signal processing technique that can be applied to practical engineering problems. Wavelet transforms are used to determine time-frequency information and remove noise. Then, principal components and modal shapes can be extracted from the processed signal using PCA. The result of simulation experiments on beams demonstrates that the response signals processed by wavelet denoising result in more modal parameters being extracted. Additionally, these experiments indicate that the proposed method has some robustness to noise and is suitable for modal parameter identification.

However, modal identification based on WDPCA needs further verification. A strict proof of the mathematical theory of wavelet denoising is also required.

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**Мета.** Точна ідентифікація модальних параметрів є важливою передумовою для моніторингу цілісності конструкцій і виявлення пошкоджень.

**Методика**. Вейвлет-аналіз є одним з найбільш вигідних методів, оскільки він володіє здатністю представляти локальні особливості сигналу у часовій і частотній областях. Ідентифікацію модальних параметрів, що ефективно досягається за допомогою аналізу головних компонент (PCA), можна розглядати як тип системного розпізнавання.

**Результати.** Оскільки метод аналізу головних компонент чутливий до Гаусового шуму при вимірах, у роботі пропонується новий метод комбінування подавлення шуму на основі вейвлет-перетворення з методом аналізу головних компонент, і застосовується ця техніка в ідентифікації модальних параметрів.

**Наукова новизна**. Сигнали піддаються розкладанню на вейвлети з декількома шарами, а отримані вейвлет-коефіцієнти заздалегідь обробляються у відповідності до порогового значення. Потім вони реконструюються зі зменшенням впливу шуму. Дослідження цього аспекту раніше не проводилися.

**Практична значимість.** Результати моделювання на прикладі балок показують, що запропонований метод здатний розпізнавати основні модальні форми та власні частоти. Крім того, він дозволяє покращувати точність виявлених модальних параметрів і вилучати деякі раніше упущені коливання.

**Ключові слова:** *ідентифікація модальних параметрів, аналіз головних компонент (PCA), Вейвлет-шумоподавлення, системи розпізнавання, вейвлет-аналіз, Гаусів шум при вимірах*

**Цель**. Точная идентификация модальных параметров является важной предпосылкой для мониторинга целостности конструкций и выявления повреждений.

**Методика**. Вейвлет-анализ является одним из наиболее выгодных методов, поскольку он обладает способностью представлять локальные особенности сигнала во временной и частотной областях. Идентификацию модальных параметров, которая эффективно достигается с помощью анализа главных компонент (PCA), можно рассматривать как тип системного распознавания.

**Результаты.** Поскольку метод анализа главных компонент чувствителен к Гауссову шуму при измерениях, в работе предлагается новый метод комбинирования подавления шума на основе вейвлет-преобразования с методом анализа главных компонент, и применяется эта техника в идентификации модальных параметров.

**Научная новизна**. Сигналы поддаются разложению на вейвлеты с несколькими слоями, а полученные вейвлет-коэффициенты предварительно обрабатываются в соответствии с пороговым значением. Затем они реконструируются с уменьшением влияния шума. Исследования этого аспекта ранее не проводились.

**Практическая значимость**. Результаты моделирования на примере балок показывают, что предложенный метод способен распознавать основные модальные формы и собственные частоты. Кроме того, он позволяет улучшать точность выявленных модальных параметров и извлекать некоторые ранее упущенные колебания.

**Ключевые слова**: *идентификация модальных параметров, анализ главных компонент (PCA), Вейвлет-шумоподавление, системы распознавания, вейвлет-анализ, Гауссов шум при измерениях*

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# WAVELET IMAGE DENOISING BASED ON FUSION THRESHOLD FUNCTIONS

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# ВЕЙВЛЕТ-ФІЛЬТРАЦІЯ ШУМУ В ЗОБРАЖЕННЯХ, ЩО ЗАСНОВАНА НА ЗЛИТТІ ПОРОГОВИХ ФУНКЦІЙ

**Purpose**. Specific to the existing discontinuity and constant deviation in denoising of wavelet threshold function, this paper analysed the integrated denoising function of traditional wavelet threshold function.

**Methodology**. Through analysis of the defects of wavelet soft threshold function and hard threshold function and according to the characteristics of the traditional threshold function as well as the design idea and procedure, the paper establishes an integrated threshold function on the basis of the traditional threshold function and offers a simulation diagram extracted from the corresponding threshold function. Through the simulation diagram of the threshold function, it analyses the advantages of integrated threshold function.

**Findings**. According to the result, the integrated threshold function established on the basis of the characteristics and design idea of wavelet soft threshold function and hard threshold function integrates the advantages of traditional threshold functions, effectively overcoming the discontinuity of the hard threshold function and constant deviation of the soft threshold function.

**Originality**. Based on the structure and characteristics of the traditional wavelet threshold function, the article puts forward an idea how to combine the traditional wavelet threshold function and the fusion function which is not only used to transform the traditional threshold function, but also adds the fusion coefficient to modify it, which makes the fusion function adaptive.

**Practical value**. The results of the paper can effectively improve the denoising ability of an image, which apart from effective removal of image noise, reserves detailed information of images, laying solid foundation for in-depth processing of a high-quality image.

**Keywords:** *wavelet transformation, threshold function, fusion threshold function, image denoising*

**Introduction.** With the development of information technology, people are more and more dependent on information transmitted by digital images. However, images often have certain noise in the process of transmission and acquisition [1]. Meanwhile, when noise reaches a certain degree, it will blur the characteristics of images and greatly affect the further analysis and application of images [2–3]. Therefore, to make the subsequent image processing go on smoothly, people keep developing all kinds of denoising methods to preprocess images, to obtain better recovered images and satisfy demands of various image applications.

Traditional image denoising methods include two types: spatial domain and frequency domain [4–5]. Typical spatial-domain filters include mean filters and wiener filters. A mean filter signifies each pixel value in