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METHOD OF IMAGE DENOISING BASED ON SPARSE REPRESENTATION AND ADAPTIVE DICTIONARY

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МЕТОД ЗМЕНШЕННЯ ШУМУ В ЗОБРАЖЕННІ НА ОСНОВІ РОЗРІДЖЕНОГО ПРЕДСТАВЛЕННЯ ТА АДАПТИВНОГО СЛОВНИКА

Purpose. Digital images are easy to be polluted in the communication. The research on image denoising is aimed to develop a new image denoising approach based on sparse representation, which will allow removing the noises in the digital images effectively and improve the image quality.

Methodology. By using K-SVD (K-means Singular Value Decomposition) algorithm, we trained the DCT (Discrete Cosine Transform) dictionary into a new dictionary, in which every atom was a linear combination of the atoms from the original DCT dictionary. The composition of these two dictionaries differs greatly, which proves that K-SVD algorithm is able to improve the dictionary structure effectively.

Findings. At first, we described and analyzed image denoising briefly and then discussed the relevant algorithms and techniques of sparse representation based on the initialization of DCT dictionary. Based on the above theories and techniques, a new image denoising method based on K-SVD and adaptive dictionary was developed.

Originality. By combining the construction and optimization of over-complete dictionary, we trained the atom dictionary with the samples of the images to be decomposed so that we managed to build the atom dictionary that can effectively reflect various image features. Through simulation analysis, this noise removal method allows denoising the images with profound details and increasing the peak signal to noise ratio of the image effectively.

Practical value. The image denoising method based on sparse representation has been developed. This approach makes contribution to the update steps of the dictionary and it solves the problem of matrix inversion by making iterative updates to every row of the matrix. Which is more important, this algorithm also updates the relevant coefficients while updating the atoms in the new dictionary and greatly reduces the computation complexity.

Keywords: *image denoising, sparse representation, adaptive dictionary, K-SVD algorithm, Matching Pursuit (MP) algorithm, Orthogonal Matching Pursuit (OMP) algorithm*

Introduction. Most of the information that people obtain from the outside world is visual. Image information has become an import source and means for people to obtain and utilize information because of its large amount of information, fast transmission speed, and long operating distance. In practice, images are usually digitalized. However, during transmission and processing, external and internal disturbance is inevitable and it raise noises. This kind of pollution affects the representation of the image scenes and degrades the image quality if the signal to noise ratio (SNR) is below a certain level. In addition, the noises may cover some significant image details, increase the entropy of the image and hinder the efficient compression of the image data to a certain extent. Denoising can reduce or remove the noises in the images to a certain degree, preserve details of the image, restore the image to a higher quality and reduce the errors with the original image. Therefore, the image denoising is critical in the image processing. Through the image denoising, we can obtain the filtered images with better quality and use it in the follow-up higher-level image processing [1].

Image denoising technique became a key point and a difficulty in the field of image processing. A great number of scholars researched and analyzed noise removal. They have developed many different denoising methods according to the image features, the rules and statistical characteristics of noise spectrum distribution, including spatial domain denoising, transform domain denoising and optimal linear denoising. However, there are still some defects in these methods. For example, existing filters are not competent in the preservation of image details and they cannot balance noise lowering and image details preservation. They do not do well in denoising effect or they cannot process the high-density noises. In the image denoising based on sparse representation, the useful image information usually has certain structural features, which coincide with the atom structure. However, the noises do not possess such feature so that they cannot be represented with few atoms. Therefore, the researchers used some mathematical transformations to obtain a fixed sparse dictionary, including discrete cosine transform (DCT), wavelet transform, Contourlet transform and Curvelet transform. Then they researched the sparsity of the image with the dictionary obtained by the mentioned transformations. Nevertheless, since these means of transformation consider only some of image features, they can insure that the image is sparse under a particular feature. After taking the method of machine learning into account, we conducted the dictionary learning to the image and got the necessary over-complete dictionary. Therefore, we can extract few fundamental atoms according to the image structure to rebuilt the structural information of the image and effectively separate the useful information from the noises to achieve the purpose of denoising [2].

Description and classification of image noises. Noises can be deemed as the factors that hamper people's sense organs to understand the source information they receive. The purpose of image denoising is to design corresponding methods to remove these noises and improve the given image by analyzing the characteristics of the noises in the image. In practice, there may be various noises in one image, which may be caused by transmission or requantization. The distribution characteristics of noises and the relationship between noises and image signal depend on the cause of noise. The common image noise filtering methods include mean filtering, median filtering, Wiener filtering and image wavelet filtering. In practice, the interference factors are uncertain, the sources of noises are numerous, and many different kinds of image noises exist. Therefore, the filtering of image noises aims to remove as many noises as possible, while preserving as many original details of the image as possible and improving the image quality. Since there is a need to receive accurate information from an image, the denoising pre-treatment is very critical for the follow-up processing. The common denoising methods include the mean filtering, median filtering, and frequency-domain filtering.

Mean filtering. Mean filtering is a typical linear filtering algorithm and its main idea is the neighborhood averaging method, namely to replace the mean value of several pixel grey-scales. To process the current pixel point, select a template, which is composed of several neighborhood pixels; obtain the mean value of all pixels in the template; give the mean value to current pixel point (x, y) and take it as the grey-scale value $g(x, y)$ at that point after the image is processed. I.e. $g(x, y) = 1/m \sum f(x, y)$ where m is the total number of pixels of the template (including the current pixel).

Although mean filtering can suppress noises properly, it processes the image edges and details unsatisfactory, and in spite of the excellent noise suppression effects, it makes the image fuzzier [3].

Median filtering. As a non-linear filtering, median filtering is a signal processing technology based on smooth filtering, which can suppress noises effectively. Under certain conditions, it can overcome image detail blurs brought by linear filtering. Mean filtering determines a neighborhood with a certain pixel as the center sorts the gray-scale value of every pixel in the neighborhood and takes the median value as a new gray-scale value of the center pixel. When the window moves, conduct smooth processing on the image with median filtering.

To conduct filtering processing on a digital signal series, define a long window L with an odd-numbered length. Assuming that at a certain moment, the window sample within the window is $s_{(i-n)}, \ldots, s_{(i)}, \ldots, s_{(i+n)}$ and S_i is the value of the signal sample in the window center. After sorting L signal sample values by size, define the sample value in i as the output value of median filtering according to the formula

$$S_i = Med\left\{s_{i-n}, \ldots, s_i, \ldots, s_{i+n}\right\}.$$

Median filtering is most effective in removing impulse interference and image scanning noises. However, for the image with many details, especially point, line and pinnacle details, median filtering is not applicable [4].

Frequency-domain filtering. In the image transmission, since the noises are mainly in the high-frequency part, the filter $F(u, v)$ is used to suppress high-frequency components, get through the low-frequency components and perform inverse Fourier transform to obtain the filtered image in order to remove the noises and improve the image quality. The widely used filtering functions are as follows.

The transfer function of two-dimensional ideal low-pass filter is

$$F(u,v) = \begin{cases} 1.\ S(u,v) \le S_0 \\ 0.\ S(u,v) > S_0 \end{cases}.$$

In this function, S_0 is a given non-negative value and $S(u, v)$ is the distance between Point (u, v) and the frequency center.

The transfer function of Butterworth high-pass filter is

$$F(u,v) = \frac{1}{1 + \left[\dfrac{S(u,v)}{S_0}\right]^{2n}}.$$

Where S_0 is the distance from the cut-off frequency and the origin. There is no ringing in first-order Butterworth filter.

The transfer function of Gaussian high-pass filter is

$$F(u,v) = e^{-S^2(u,v)/2S_0^2}.$$

$S(u, v)$ is the distance from the origin of the Fourier transform center and S_0 is the cut-off frequency.

In the frequency domain, smoothing can be realized by filtering the noises in the high-frequency part. In Fourier transform domain, the transform coefficients can reflect some image features. Noises usually correspond to the area with high frequency while the image entity is located in the low-frequency area. Therefore, the frequency domain is frequently used to eliminate image noises [5].

The principles of sparse decomposition. The typical algorithms of sparse representation theory include MP (Matching Pursuit) algorithm and OMP (Orthogonal Matching Pursuit) algorithm. As one of the greedy iterative algorithms, MP algorithm has low computation complexity and asymptotic convergence. Although it cannot restore the signal accurately, it can still obtain the approximate representation of the signal. What MP algorithm usually obtains is the sub-optimal solution. Compared with other sparse representation algorithms, MP has better approximation effect and lower computation complexity. MP analyzes the signal properties and selects the proper over-complete dictionary matrix according to the property to make the property of the atom in the dictionary matrix approximate to that of the signal. Perform matching operation between the original signal and every atom in the over-complete dictionary matrix, obtain the preserve the optimal matching atom. Represent the signal as the linear combination of the optimal atoms and get the linear representation of the signal in every optimal atom [6].

The flowchart of MP algorithm is shown in Fig. 1.

OMP algorithm uses the same atom selection criteria as MP algorithm and orthogonalizes the selected atom set with recursion method. It has the iterative optimality and a few iterations. Additionally, the reconstruction of OMP algorithm is based on the fact that the number of iterations is known. OMP algorithms can realize the purpose of orthogonalized processing on all the selected atoms through every step of iterations. It can guarantee the optimality of iteration and reduce the number of iterations [7]. The diagram of the image sparse representation is shown in Fig. 2.

The design of image denoising algorithm based on sparse representation and adaptive dictionary. We have combined the K-SVD algorithm with the construction and the optimization of over-complete dictionary matrix and trained the dictionary matrix with the sample of the image to be decomposed so that we can effectively construct the dictionary matrix, which can reflect various image features. We continuously updated and adjusted the atoms in the dictionary in order to achieve the maximum matching with the signal set to be used for training.

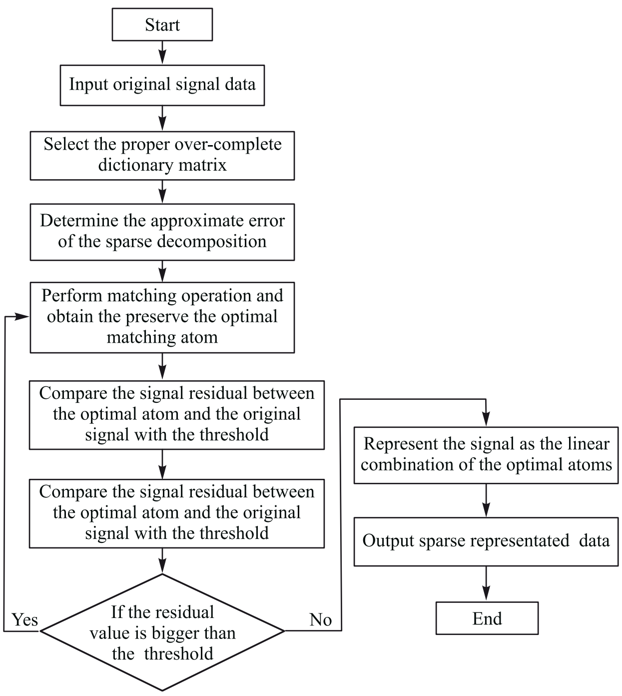


Fig. 1. The flowchart of MP algorithm

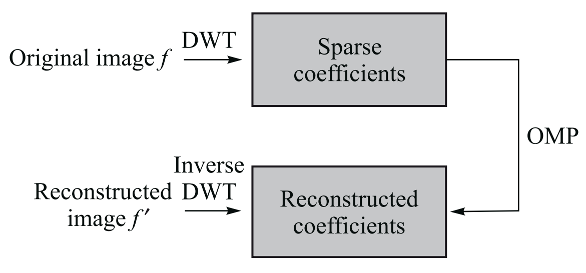


Fig. 2. Image sparse representation

The image denoising algorithm of sparse representation based on K-SVD and adaptive dictionary was proposed. The specific steps of the algorithm can be described as follows.

Step 1. Input the training sample set $S = [s_1, s_2, \ldots, s_n]$. The original signal is $Y = [y_1, y_2, \ldots, y_n]$.

Step 2. Initialize the over-complete DCT dictionary $D = D_0$. Perform sparse decomposition on the noisy image in the initial dictionary D with OMP algorithm. The objective function is expressed by the (1). Obtain the necessary sparse coefficient matrix X by solving the objective function.

$$\arg\min_{D,X}\{\|S - DX\|^2\} \quad \forall i, \|t_i\| \le t_{\max}. \tag{1}$$

Where D is the initial dictionary, X is the sparse coefficient matrix, S is the objective function value, $t_{\max}$ is the upper limit of the number of non-zero components in the sparse representation coefficients, namely the maximum diversity in the coefficient vector. Solving of the objective formula (1) is an iteration process [8].

Step 3. Extract pixel blocks to form the training sample set from the peak of the image matrix to the end point according to a certain step length. Decompose the matrix X to get the $X = U\Delta V^T$ with K-SVD method, update the dictionary, select d_k as the first column of U, update the sparse vector, select the product of the first column of V and $\Delta(1, 1)$, generate the optimized dictionary $\widetilde{D}$.

Step 4. Perform sparse decomposition denoising to the noisy image. The expression formula of the sparse decomposition is

$$\|Uy_i\|_2^2 = \|n_{\Delta\theta}\|_2^2 + \left\|U(\hat{y}_i + \hat{n}_{\Delta\theta})\right\|_2^2.$$

Where y_i is the column vector of the transformation of the noisy images, $\hat{y}_i$ is the useful information of the image, $\Delta\theta$ is the bandwidth of $\hat{y}_i$, $\hat{n}_{\Delta\theta}$ is the noise in the band within $\Delta\theta$, $n_{\Delta\theta}$ is the noise outside the band of $\Delta\theta$, U is decomposition matrix. Get the denoised image sparse representation coefficient matrix X'.

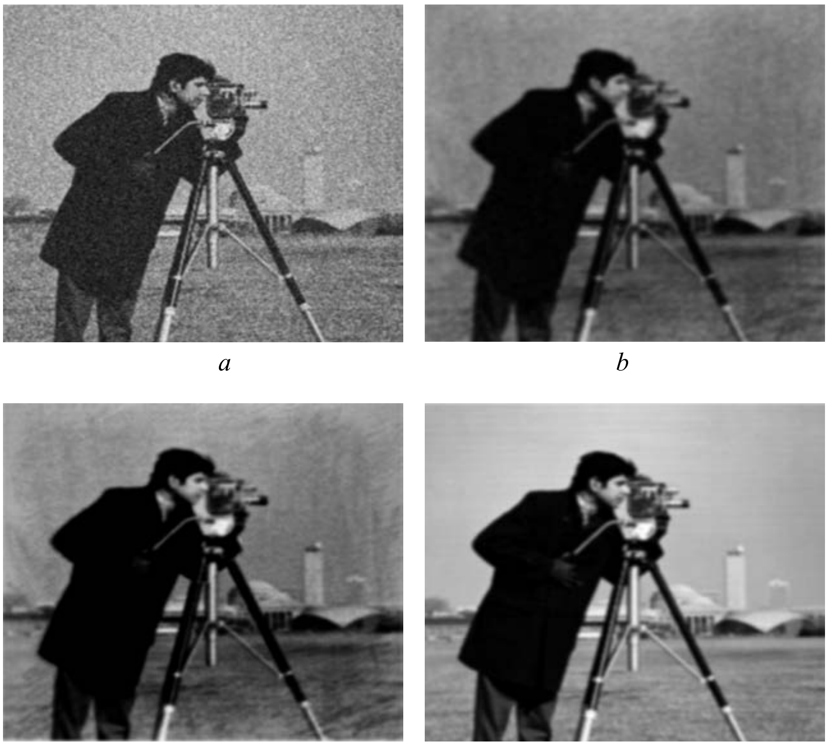
Step 5. Repeat *Step 2–4*, update the dictionary by column until all of the columns are updated, reach the termination conditions and output the dictionary $\widetilde{D}$, set the maximum number of iterations $t_{\max}$ as 100.

Step 6. After getting image sparse coefficient matrix X', perform product operation between this matrix and the previous learning dictionary according to the formula (2), get the denoised image matrix and restore the denoised image.

$$Y' = \widetilde{D}X'. \tag{2}$$

Where Y' is the denoised image matrix, X' is the sparse coefficient matrix, $\widetilde{D}$ is the learning type dictionary obtained on the last step.

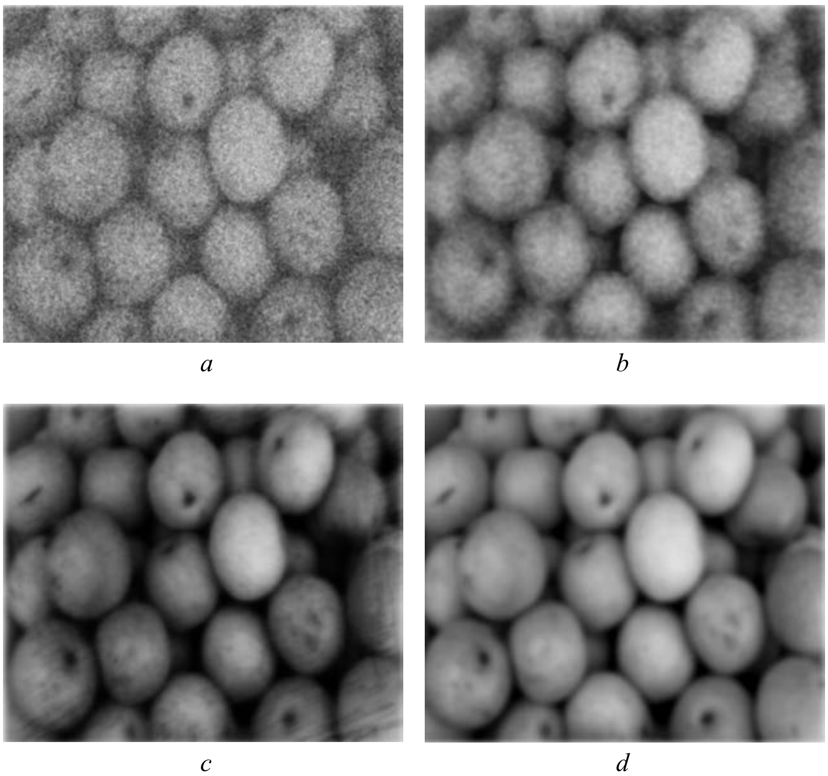
ISSN 2071-2227, Науковий вісник НГУ, 2016, № 2



a *b*

c *d*

Fig. 3. Camera operator images denoised by different denoising methods: a − noisy image; b − wavelet hard-threshold; c − wavelet soft-threshold; d − proposed method



a *b*

c *d*

Fig. 4. Pears images denoised by different denoising methods: a − noisy image; b − wavelet hard-threshold; c − wavelet sft-threshold; d − proposed method

Simulation experiment results. The CPU frequency of the computer used in this experiment is 3.3 GHz, the memory capacity is 4GB and the simulation platform is Matlab R2012a. The commonly used image set of the image library was utilized in the experiments. In the experiment, the K-SVD was used to train 5,000 clean sample image sets to get the global dictionary and obtain the adaptive dictionary by using K-SVD on the noisy image. After selecting the sparse dictionary, the sparse denoising was performed on the image with the same noise intensity with OMP sparse decomposition algorithms. The denoising effects were compared.

It can be seen from Fig.3, Fig.4, Table 1 and Table 2 that the denoising algorithms based on wavelet hard-threshold and wavelet soft-threshold did not allow receiving a denoised image with great vision. The commonly seen image noises belong to high-frequency part while the useful image information belongs to the low-frequency part. For the global dictionary after training the natural sample library with K-SVD, it can adapt itself to most natural images. Therefore, it can represent the general characters of the natural images, namely the effective low-frequency part in the image. According to the understanding of sparse decomposition on noisy signal, as the noise components increase, the signal noise ratio (SNR) will reduce and the useful signal components will reduce, namely, the structural components will reduce. Therefore, in the process of sparse decomposition, the fewer atoms match the signals, the sparser the signal representation is and the computation volume will fall sharply. That is why the K-SVD algorithm has a huge potential to optimize the dictionary structure and process the image with low SNR. In addition to removing blurs, it can preserve and strengthen the image edges and texture and improve the subjective effects and objective quality of the image. As for the pixel points which have been defined as noise points, it can filter them in a highly-efficient and accurate way and it can maintain the image details and edge information well.

Table 1

Comparison of PSNR (dB) of camera operator images

Noise Standard Deviation	Noisy Image	Wavelet Hard-Threshold	Wavelet Soft-Threshold	Proposed Method
10	26.2633	29.9827	28.8765	29.9716
20	21.2139	25.2802	25.8864	26.1069
30	17.2271	22.8017	23.6115	24.8293
40	16.0133	21.5802	23.0157	25.0386
50	14.0615	20.3768	22.9341	24.0138

Table 2

Comparison of PSNR (dB) of pears images

Noise Standard Deviation	Noisy Image	Wavelet Hard-Threshold	Wavelet Soft-Threshold	Proposed Method
10	27.0736	27.6065	26.1258	27.8921
20	22.1062	24.6507	23.9231	26.0266
30	19.1068	23.0016	23.1822	24.3628
40	16.5387	20.9325	21.6273	22.5692
50	13.9628	18.3828	19.4835	20.3016

Conclusion. The research was focused on several typical image denoising filtering algorithms and compares their advantages and disadvantages in the denoising and image processing. We attempted to combine DCT over-complete atomic library and K-SVD algorithm and proposed the image denoising method based on the sparse representation. This method allows us to build the dictionaries matrix reflecting various characteristics of images, realize the matching with the signal set used for training to the maximum, which reduces the computational complexity greatly if compared to other methods. In addition, this method improves effectively the peak signal to noise ratio of an image and produces better denoising effect.

Acknowledgements. This work was supported by the Ministry of Education Research of Social Sciences Youth funded projects. (Grant No: 15YJCZH208).

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Мета. Цифрові зображення схильні до погіршення якості у процесі комунікації. Дане дослідження зниження шумів у зображеннях направлене на створення нового підходу, заснованого на розрідженому представленні, що дозволить ефективно видаляти шуми з цифрових зображень і покращувати їх якість.

Методика. За допомогою сингулярного розкладання методом *k*-середніх (K-SVD) проводиться навчання ДКП-словника (дискретного косинусного перетворення) з метою перетворення його в новий словник, кожен елемент якого являє собою лінійну комбінацію з елементів вихідного словника. З точки зору структури, ці два словника значно відрізняються один від одного, що доводить здатність K-SVD алгоритму ефективно покращувати структуру словника.

Результати. На початку був наданий короткий опис і аналіз зменшення шуму в зображеннях, потім розглянуті відповідні алгоритми та методи розрідженого уявлення, засновані на ініціалізації ДКП-словника. Виходячи з наведених теорій і методів, розроблено новий метод зниження шумів у зображеннях на основі K-SVD алгоритму та адаптивного словника.

Наукова новизна. Об'єднавши побудову та оптимізацію повного словника, при навчанні словника, елементи зі зразками зображень розкладалися таким чином, щоб побудувати словник з елементів, що може ефективно описувати різні особливості зображення. За допомогою імітаційного аналізу, цей спосіб усунення шуму може проводити видалення шуму із зображень зі складними деталями та ефективно збільшувати пікове співвідношення сигнал-шум у зображенні.

Практична значимість. Запропоновано метод зниження шумів у зображеннях, заснований на розрідженому представленні. Такий підхід сприяє оновленню словника, вирішує проблему інверсії матриці шляхом ітеративного поновлення кожного рядка матриці. Ще важливіше те, що цей алгоритм також оновлює відповідні коефіцієнти при оновленні елементів у новому словнику та значно знижує складність обчислень.

Ключові слова: *зниження шуму в зображенні, розріджене уявлення, адаптивний словник, K-SVD алгоритм, алгоритм апроксимації з переслідуванням* (*MP*), *прямокутний алгоритм апроксимації з переслідуванням* (*OMP*).

Цель. Цифровые изображения подвержены ухудшению качества в процессе коммуникации. Данное исследование понижения шумов в изображениях направлено на создание нового подхода, основанного на разреженном представлении, который позволит эффективно удалять шумы из цифровых изображений и улучшать их качество.

Методика. С помощью сингулярного разложения методом *k*-средних (K-SVD) проводится обучение ДКП-словаря (дискретного косинусного преобразования) с целью преобразования его в новый словарь, каждый элемент которого представляет собой линейную комбинацию из элементов исходного словаря. С точки зрения структуры эти два словаря значительно отличаются друг от друга, что доказывает способность K-SVD алгоритма эффективно улучшать структуру словаря.

Результаты. Вначале было дано краткое описание и анализ подавления шума в изображениях, затем рассмотрены соответствующие алгоритмы и методы разреженного представления, основанные на инициализации ДКП-словаря. Исходя из приведенных теорий и методов, разработан новый метод понижения шумов в изображениях на основе K-SVD алгоритма и адаптивного словаря.

Научная новизна. Объединив построение и оптимизацию полного словаря, при обучении словаря, элементы с образцами изображений разлагались таким образом, чтобы построить словарь из элементов, который может эффективно отражать различные особенности изображения. С помощью имитационного анализа, этот способ устранения шума может производить удаление шума в изображениях со сложными деталями и эффективно увеличивать пиковое соотношение сигнал-шум в изображении.

Практическая значимость. Предложен метод понижения шумов в изображениях, основанный на разреженном представлении. Такой подход способствует обновлению словаря и решает проблему инверсии матрицы путем итеративного обновления каждой строки матрицы. Что еще более важно, этот алгоритм также обновляет соответствующие коэффициенты при обновлении элементов в новом словаре и значительно снижает сложность вычислений.

Ключевые слова: *понижение шума в изображении, разреженное представление, адаптивный словарь, K-SVD алгоритм, алгоритм аппроксимации с преследованием* (*MP*), *ортогональный алгоритм аппроксимации с преследованием* (*OMP*).

Рекомендовано до публікації докт. техн наук В. В. Гнатушенком. Дата надходження рукопису 16.04.15.