gree and prognostication of next-generation wagon undercarriage evolution. Review of the examples of application of the idealistic strategy of improvement of the undercarriage of railway universal freight gondolas.

Methodology. The research employed the law of technical systems ideality degree increase and approaches of theory of inventive problem solving. Thus, the ideal freight carriage was considered an ideal construction or ideal mental/virtual object, which serves a base for the concept of freight carriage and determines its essence. It is characterized by the indices and parameters that may provide transportation of maximum load with minimum/zero conversion cost and running expenses with certain production base and plying itinerary. We carried out the technical evaluation of the existent and promising freight carriages, construction troubleshooting, and suggested possible approaches to ideal implementations.

Findings. A general formula for the evaluation of freight carriages ideality degree was formulated and worked out in detail for the module of basket. Ways of improvement of techno-economic and operating indices of the freight wagon undercarriage were determined. The approaches to design of freight carriages of new generation have been suggested.

Originality. We offered the new aspect of freight carriages design based on the idealistic strategy of development of technical systems. We gave a scientific rationale for the necessity of the development of new-generation freight carriages and made the prognosis of possibility of its implementation.

Practical value. The forecast of stages of evolution of the freight gondolas undercarriage may serve a basis for subsequent research and experimental works directed on the development of designs with improved techno-economic and operating indices (models of new generation), for example drafting of the proper technical design specifications.

Keywords: *freight wagon, degree of ideality, undercarriage evolution prognosis*

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T.S. Kagadiy, Dr. Sci. (Phys.–Math.), Assoc. Prof.,
A.H. Shporta

State Higher Educational Institution "National Mining University", Dnipropetrovsk, Ukraine, e-mail: kagadiy@i.ua; shportaanna@ukr.net

THE ASYMPTOTIC METHOD IN PROBLEMS OF THE LINEAR AND NONLINEAR ELASTICITY THEORY

Т.С. Кагадій, д-р фіз.-мат. наук, проф.,
А.Г. Шпорта

Державний вищий навчальний заклад „Національний гірничий університет“, м. Дніпропетровськ, Україна, e-mail: kagadiy@i.ua; shportaanna@ukr.net

АСИМПТОТИЧНИЙ МЕТОД У ЗАДАЧАХ ЛІНІЙНОЇ ТА НЕЛІНІЙНОЇ ТЕОРІЇ ПРУЖНОСТІ

Purpose. To develop analytical method for complex properties constructions calculations. To study viscoelasticity influence on construction stress-strain state and the possibility of considering final deformations.

Methodology. The mathematical model of tree-dimensional problem on load transfer from supporting element to the viscoelastic massif was constructed.

Findings. The circle of analytical solutions for nonlinear elasticity and linear viscoelasticity theory problems was extended by means of elaborating perturbation method.

Originality. The asymptotic method for three-dimensional linear viscoelasticity of orthotropic bodies problems solutions or for nonlinear problems was elaborated. Analytical solutions of new problems on load transfer through supporting element to the viscoelastic material massif in spatial statement were received.

Practical value. The method suggested allows passing from the solution of the complex mixed tasks of mechanics to the consecutive solution of potential theory problems, which is the most developed section of mathematical physics. The solutions of a range of new complex challenges received due to the offered approach provide an opportunity to analyse stress-strain state of bodies with supporting elements. These results can be used in engineering calculations of piles foundations and substructures.

Keywords: *loading transfer, perturbation method, elasticity theory, viscoelasticity, nonlinearity, spatial and flat problems*

Problem statement. Modern constructions and mechanisms, which are used in mining, have a complex of properties. The aggressive environment and use of new technologies require previous estimates of the construction stress-strain state. Analytical solutions of the corresponding problems can be used to do this. Models to find such solutions have important positive qualities (simplicity and clarity), but may lack quite clear range of applications.

Recent research analysis. The methods of small parameter (geometric or physical) are widely used in the elasticity theory. The effective approaches have been suggested by V.M. Alexandrov, I.I. Vorovych, A.L. Goldenveiser, O.M. Huz, A.S. Kosmodamianskii, S.G. Lehnytskii, Yu.M. Nemish, G.Y. Popov. The paper by L.I. Manevich and A.V. Pavlenko [1] describes the asymptotic analysis of equations of elasticity theory of orthotropic environments conducted using parameters that characterize the anisotropy that would resolve a number of problems.

Determining the unresolved aspects of the problem. The asymptotic analysis is effective mathematical apparatus, which enables us to construct reasonable approximation equation and evaluate the application of different hypotheses [2, 3]. Obviously, requirements of engineering practices can be satisfied through the development of numerical methods and the results using a computer, but the need of analytical solutions is very significant. That is explained by the fact that the account of the real material properties, such as anisotropy or viscoelasticity and, particularly, nonlinear, leads to significant mathematical difficulties.

Task statement. In the cases mentioned above, the approximate analytical solutions helpto find problems qualitative features, to get asymptotics, to analyse critical points, and are often the basis for numerical calculations.

In this regard, it is necessary to generalize the approach to the solution of plane and spatial problems of elasticity based on finite deformation or physically nonlinear elasticity theory. Using the developed method of research, we can expand the range of problems of nonlinear and linear elasticity theory (viscoelasticity) which can be solved by analytical methods.

The basic material statement. ***The proposed research method.*** Let us state the foundations of the asymptotic method for three-dimensional linear viscoelasticity theory problem.

It is assumed that the material is orthotropic regarding both elastic and viscoelastic properties. The main directions of rectilinear anisotropy coincide with Cartesian coordinate axes x*, y, z.* The viscoelastic material properties are described with creep incremental nuclei. The relations between strains and stresses in such a material are as follows

$$e_{11} = s_1 - \upsilon_{12}s_2 - \upsilon_{13}s_3,\ s_i = \frac{1}{E_i}\left[\sigma_{ii} + \int_0^t K_{1i}(t-\tau)\sigma_{ii}d\tau\right],\quad (i=1,2,3);$$

$$e_{ij} = \frac{1}{G_{ij}}\left[\sigma_{ij} + \int_0^t K_n(t-\tau)\sigma_{ij}d\tau\right],$$

$$(i=2, j=3, n=1; i=1, j=3, n=2; i=1, j=2, n=3);$$

$$\upsilon_{12}E_1 = \upsilon_{21}E_2,\ \upsilon_{23}E_2 = \upsilon_{32}E_3,\ \upsilon_{31}E_3 = \upsilon_{13}E_1,$$

$$K_{12} = K_{21},\ K_{23} = K_{32},\ K_{31} = K_{13}. \qquad (1)$$

To receive e_{22}, e_{33} it is necessary to make the circular replacement of indexes in e_{11}. For creep nuclei approximation, the following analytic expressions are used:

$$K_{ij}(t-\tau) = k_{ij}(t-\tau)^{\alpha_{ij}-1}\exp\left[-\beta_{ij}(t-\tau)\right];$$

$$K_i(t-\tau) = k_i(t-\tau)^{\alpha_i^*-1}\exp\left[-\beta_i^*(t-\tau)\right];$$

$$\left(0 < \alpha_{ij}, \alpha_i^* \le 1\right).$$

After applying the Laplace transform in time t with parameter p for expressions (1) and substituting the expressions for connecting stresses and deformations in the equilibrium equation, we come to the integration of the latter regarding the transformant movements

$$\begin{aligned} &\tilde{u}_{xx} + \varepsilon\tilde{u}_{yy} + \varepsilon l_1\tilde{u}_{zz} + \varepsilon m\tilde{v}_{xy} + \varepsilon m_1 l_1\tilde{w}_{xz} = 0;\\ &\varepsilon\tilde{v}_{xx} + q\tilde{v}_{yy} + \varepsilon l_2\tilde{v}_{zz} + \varepsilon m_2\tilde{u}_{xy} + \varepsilon m_3 l_2\tilde{w}_{yz} = 0;\\ &\varepsilon l_1\tilde{w}_{xx} + \varepsilon l_2\tilde{w}_{yy} + q_1\tilde{w}_{zz} + \varepsilon m_4 l_1\tilde{u}_{xz} + \varepsilon m_5 l_2\tilde{v}_{yz} = 0;\\ &\varepsilon = \varepsilon_* F_3(p)/F_{11}(p),\ \varepsilon_* = G_{12}/E_1, \end{aligned} \qquad (2)$$

where l, l_1, m, m_i, q, q_1 are known factors that are expressed through the functions $F_i(p), F_{ij}(p)$ and stiffness properties of the material. Indices x, y, z in equations (2) further denote differentiation according to the relevant variables.

Equations (2) are similar to equations of equilibrium in elastic orthotropic body movements; under the asymptotic analysis, the smaller parameter is to be chosen. Such parameter is ε_*, because it is really small for real orthotropic materials. However, if the value of the Laplace transformation parameter *p* does not match zeros and poles of the function $F_3(p)/F_{11}(p)$, then this ratio does not exceed a unit and ε can be selected as a small parameter. Suppose *p* is not a critical point and is a small parameter is equal to ε.

In order to consider possible relations between the components of the displacement vector and velocity of their change according to the coordinates, affine transformation variables that depend on ***ε*** are introduced

$$\xi_1 = \alpha\varepsilon^{1/2}x, \eta_1 = y, \zeta_1 = z, \tilde{u} = U^{(1)};$$
$$\tilde{v} = \varepsilon^{3/2}V^{(1)}, \tilde{w} = \varepsilon^{3/2}W^{(1)}; \qquad (3)$$

$$\xi_2 = x, \eta_2 = \beta\varepsilon^{1/2}y, \zeta_2 = z,$$
$$\tilde{u} = \varepsilon^{3/2}U^{(2)}, \tilde{v} = V^{(2)}, \tilde{w} = \varepsilon^{3/2}W^{(2)}; \qquad (4)$$

$$\xi_3 = x, \eta_3 = y, \zeta_3 = \gamma\varepsilon^{1/2}z;$$
$$\tilde{u} = \varepsilon^{3/2}U^{(3)}, \tilde{v} = \varepsilon^{3/2}V^{(3)}, \tilde{w} = W^{(3)}. \qquad (5)$$

According to these changes, it can be assumed that the solutions of corresponding equations have different properties, namely: the solution of the system obtained from (2) after introducing (3–4) or (5) changes relatively more slowly by coordinate x (*y* or *z*) than analogous system solutions obtained after the application of other transformations.

The solution of the initial boundary problem is found as a superposition of components that correspond to the specified stress-strain state types.

Functions $U^{(n)}, V^{(n)}, W^{(n)}$ (*n* = 1,2,3), and the coefficients α, β, γ are presented in the form of the series by parameter $\varepsilon^{1/2}$

$$U^{(n)}\left(V^{(n)}\right)\left(W^{(n)}\right) = \sum_{j=0}^{\infty}\varepsilon^{j/2}U^{n,j}\left(V^{n,j}\right)\left(W^{n,j}\right),\ (n=1,2,3); \qquad (6)$$

$$\alpha\ (\beta)\ (\gamma) = \sum_{j=0}^{\infty}\varepsilon^{1/2}\alpha_j\ (\beta_j)\ (\gamma_j). \qquad (7)$$

After applying transformations (3–5) to equation (2) with (6,7), we obtain three systems of equations for functions $U^{i,j}, V^{i,j}, W^{i,j} (i = 1,2,3; j = 0,...,n,...)$. In the zero approximation, basic functions $U^{1,0}$, $V^{2,0}$, $W^{3,0}$ are found from the Laplace equations, all others (auxiliary functions) are found with basic ones using simple integration.

A theorem is proved according to which if the unknown coefficients $\alpha_j = \beta_j = \gamma_j$ (at $q = q_1 = 1$) are found with the formulas

$$\alpha_0 = 1, \alpha_j = \frac{1}{2} m^2 \left(\omega_{j-4} + \sum_{k=0}^{j-4} \alpha_k \omega_{j-k-4} \right);$$

$$\omega_s = \alpha_s + (1-m) \sum_{n=0}^{s-2} (2\alpha_n - c_n) \omega_{s-n-2};$$

$$(s \ge 0; s < 0, \omega_s = 0),$$

then the basic functions $U^{1,j}, V^{2,j}, W^{3,j}$ of each approximation are found from the Laplace equations and auxiliary functions are expressed in terms of the basic integration

$$\Delta U^{1,j} = 0,\ V^{1,j}_{\eta\eta} = f_1 \left(U^{1,j}, V^{1,j-2}, W^{1,j-2} \right);$$

$$W^{1,j}_{\zeta\zeta} = f_2 \left(U^{1,j}, V^{1,j-2}, W^{1,j-2} \right);$$

$$\Delta V^{2,j} = 0,\ U^{2,j}_{\xi\xi} = f_3 \left(V^{2,j}, U^{2,j-2}, W^{2,j-2} \right);$$

$$W^{2,j}_{\zeta\zeta} = f_4 \left(V^{2,j}, U^{2,j-2}, W^{2,j-2} \right);$$

$$\Delta W^{3,j} = 0,\ U^{3,j}_{\xi\xi} = f_5 \left(W^{3,j}, U^{3,j-2}, V^{3,j-2} \right);$$

$$V^{3,j}_{\eta\eta} = f_6 \left(W^{3,j}, U^{3,j-2}, V^{3,j-2} \right).$$

The method effectiveness. Thus, the method effectiveness depends on the ability to formulate boundary problems to determine basic functions.

It is shown that this can be done in many cases. In particular, if the tension on the bounding planes ***x = const*** is known

$$\tilde{\sigma}_{11} = \phi_1(y,z,p),\ \tilde{\sigma}_{12} = \phi_2(y,z,p);$$

$$\tilde{\sigma}_{13} = \phi_3(y,z,p),$$

boundary conditions for the basic functions have the following expressions

$$U^{1,j}_{\xi} = \phi_{1,j} + f_7 \left(U^{1,j-1}, o(j-2) \right);$$

$$V^{2,j}_{\xi} = \phi_{2,j} - U^{1,j}_{\eta} + f_8 \left(o(j-3) \right);$$

$$W^{3,j}_{\xi} = \phi_{3,j} - U^{1,j}_{\zeta} + f_9 \left(o(j-3) \right).$$

In the zero approximation ***(j = 0)*** the boundary conditions for $\boldsymbol{U^{1,0}}$ do not depend on higher approximations and solutions of stress state equations of types two and three. Therefore, function $\boldsymbol{U^{1,0}}$ is found independently and $\boldsymbol{V^{1,0}}$, $\boldsymbol{W^{1,0}}$ are determined by simple integration through $\boldsymbol{U^{1,0}}$. After that boundary conditions are formulated for finding other basic functions $\boldsymbol{V^{2,0}}$, $\boldsymbol{W^{3,0}}$ with Laplace equations. Having solved the boundary value problems and defined all auxiliary functions, one can obtain boundary conditions for finding $\boldsymbol{U^{1,1}}$ and so on. Similar results occur for second basic and confounded boundary problems. Correlation between the three types of stress-strain state is performed through boundary conditions for tangent stresses.

Thus, boundary problems of orthotropic linear viscoelasticity solids are reduced to consistent solution of boundary problems of the potential theory.

The solution of new problems using the proposed approach. The space problem of viscoelastic orthotropic semi-infinite body was considered, in which half-infinite elastic rod of rectangular cross-sectional area is placed and constrained closely with it; its area is small enough (i.e., thickness and half-width are small enough). Lines of the body anisotropy coincide with coordinate axes ***x,y,z***. Midline inclusion is perpendicular to the plane which limits the half-space and coincides with the axis Ox.

It is necessary to determine the law of distribution contact stresses between the rod and half-space when the endpoint inclusion has concentrated force P_0 directed along the axis of the rod. The force is applied to starting point and then remains constant.

It is assumed that the half-width is so small that in the contact tangent stresses σ_{23} can be neglected, i.e. only tangent contact stresses $\sigma_{12}(x,z)$ are active in strip connections inclusion of half-space.

In this problem the model of one-dimensional elastic rod is combined with a model of contact for the area in half-space where the law of distribution of contact stresses is given with a formula

$$\sigma_{12}(x,z) = \tau(x) \Big/ (\pi\sqrt{b^2 - z^2}),$$

where $\tau(x)$ is stress per unit of the inclusion length which must be determined. With these assumptions, it follows that in any cross-section of the rod axial stresses $\tau(x)$ are concentrated along the midline plane of contact.

In this formulation, the problem reduces to the integration of the half-space equilibrium equations with the following boundary conditions

$$\sigma_{11} = \sigma_{12} = \sigma_{13} = 0 \ (x = 0);$$

$$u = u_1;\ w = v = 0 \ (y = 0, z = 0).$$

At the infinity, all functions revolve to zero. Here E_c is elastic modulus of the inclusion material, $\delta(x)$ is Dirac function, p is Laplace transform parameter.

Tangent stress $\tilde{\sigma}_{12}(x,y)$ is determined only by function U_y, as $V = 0 \ (V_x = 0)$ with $y = o$, with

$$\tilde{\sigma}_{12}(x,y) = 2GF_1(p)U_y; \quad G = G_{12}.$$

The Cauchy problem solution can be obtained using the Fourier transforms. Returning to the original, we get

$$U_1(x) = -\frac{2}{\pi} \frac{P_0}{pE_cF_c} \int_0^\infty \frac{M(\theta)\cos xs}{\Delta} ds;$$

$$\tilde{N}(x) = \frac{2}{\pi} \frac{P_0}{p} \int_0^\infty \frac{sM(\theta)\sin xs}{\Delta} ds;$$

$$\tilde{\tau}(x) = \frac{2P_0\varphi(p)}{\pi p}\int_0^\infty \frac{\cos xs}{\Delta} ds \ ;$$

$$\Delta = s^2 M(\theta) + \varphi(p); \quad \varphi(p) = \varphi_0 F_1(p);$$

$$\varphi_0 = \frac{2\pi G}{E_c F_c}; \ M(\theta) = I_0(\theta)K_0(\theta); \ \theta = \frac{b\omega s}{2},$$

where $I_0(\theta), K_0(\theta)$ are modified Bessel functions, $\tilde{N}(x)$ is Laplace transform efforts rod. The inverse Laplace transform defines tension $\tilde{N}$ and τ based on the coordinates and time. To move to the original, let us present tension $\tilde{\tau}(x)$ in the form of series in a small parameter ε^*, which depends on p

$$\tilde{T}(x,p) = \left[T_0(x) + T_1(x)\varepsilon_* + T_2(x)\varepsilon_*^2 + ...\right]/p,$$

where $\tilde{T}$ presents either tension of $\tilde{N}$ or $\tilde{\tau}$.

The last formulas can be written for p when setting takes large values (corresponding to small values of time t), or when parameter p takes small values (large amount of time t).To obtain the originals of functions at arbitrary time values, two-point Padé approximant can be used.

The perturbation method for nonlinear elasticity problems. The analysis has been conducted regarding the relations between strains and movement of orthotropic body within the plane of the task elasticity theory considering finite deformations.

$$e_{11} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right];$$

$$e_{22} = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right];$$

$$e_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}.$$

It is shown that after the introduction of reforms

$$\xi_i = \varphi_i \varepsilon^{\alpha i} x, \ \eta_i = \omega_i \varepsilon^{\beta i} y \ ;$$
$$u = \varepsilon^{\gamma_i} U^{(i)}, \ v = \varepsilon^{\delta_i} V^{(i)}, \ (i = 1,2)$$

two types of strain state with different properties (as for a linear problem)can be detailed. Connection between these states is carried out through the tangential component strain that contains equal components of both types. Two types of stress state are distinguished, which correspond to specified types of strain, with tangents strain containing identical components of both types.

The equilibrium equations asymptotic integration is shown

$$\left[\left(1+\underline{u_x}\right)\sigma_{11} + \underline{u_y\sigma_{12}}\right]_x + \left[(1+u_x)\sigma_{12} + u_y\sigma_{22}\right]_y = 0 \ ;$$
$$\left[v_x\sigma_{11} + \left(1+v_y\right)\sigma_{12}\right]_x + \left[\underline{v_x\sigma_{12}} + \left(1+\underline{v_y}\right)\sigma_{22}\right]_y = 0 . \quad (8)$$

Considering the equation without underlined members and the value of ***G/E*** (as a linear formulation) for small parameter, one can split stress-strain state into two components with different properties after applying transformations.

The functions $U^{(i)}$, $V^{(i)} (i=1,2)$ and coefficients φ_1, ω_2, are defined in the form of series by parameter ε($\varphi_2 = \omega_1 = 1$). With the same degrees of ε coefficients, one can select $\varphi_{1j}, \omega_{2j}$ so that the basic functions $U^{1,j}$, $V^{1,j}$ in each approximation are defined with the Laplace equation. Auxiliary functions are found using basics of integration.

If general equations (8) are considered, in the zero and first approximations basic functions are found from the Laplace equations; to obtain higher approximations it is necessary to solve the Poisson equation, the right side of which contained the known functions of the previous approximations. In this case, we have a preference, because it is well developed boundary value general solving methods problems for such equations.

The analysis of boundary conditions was conducted; it shows the possibility of their formulating for basic functions.

The model problem solution. The model problem is solved on normal load action

$$\sigma_{11} = -\frac{P_0}{\pi}\frac{\alpha}{\alpha^2 + y^2}$$

for the boundary ***(x = 0)*** elastic orthotropic half-plane (***x≥0***, $|y| < \infty$) without tangent stress at the boundary. The obtained displacements and stresses values, such as normal stress σ_1^* on the line ***y = 0***, are

$$\sigma_1^* = \frac{1}{1+\varepsilon^{1/2}t_1} + C\frac{\varepsilon^{1/2}t_1}{\left(1+\varepsilon^{1/2}t_1\right)^2} +$$
$$+\varepsilon\left(\frac{1}{1+\varepsilon^{1/2}t_1} - \frac{1}{1+\varepsilon^{-1/2}t}\right) +$$
$$+\varepsilon^2\left(\frac{1}{1+\varepsilon^{1/2}t_1} - \frac{1}{1+\varepsilon^{-1/2}t_1}\right) + ... ;$$
$$\sigma_1^* = -\sigma_{11}\pi\alpha / P_0, C = P_0 / 4\pi E_1\alpha, t_1 = \frac{x}{\alpha}.$$

The item containing ***C*** describes the contribution of geometric nonlinearity.

Figure shows the normal stress at ***y=0*** if ε= 0.1 (solid line), ε = 0.35 (dashed line). Curves 1 correspond to the linear setting problems; curves 2 take into account the final strain.

Conclusions and prospects of further development. The perturbation method proposed to solve nonlinear differential equations in partial derivatives, has both theoretical and practical significance, is universal and can be applied for the analysis of various problems of mathematical physics.

The developed approach can be applied to solve the problems in which residual strain plays a significant role (bending of thin plates and shells). In the considered model problem, we succeeded in distinguishing the contribution of geometric non-linearity, but the class of problems considered above demonstrates the effectiveness of the method more clearly.

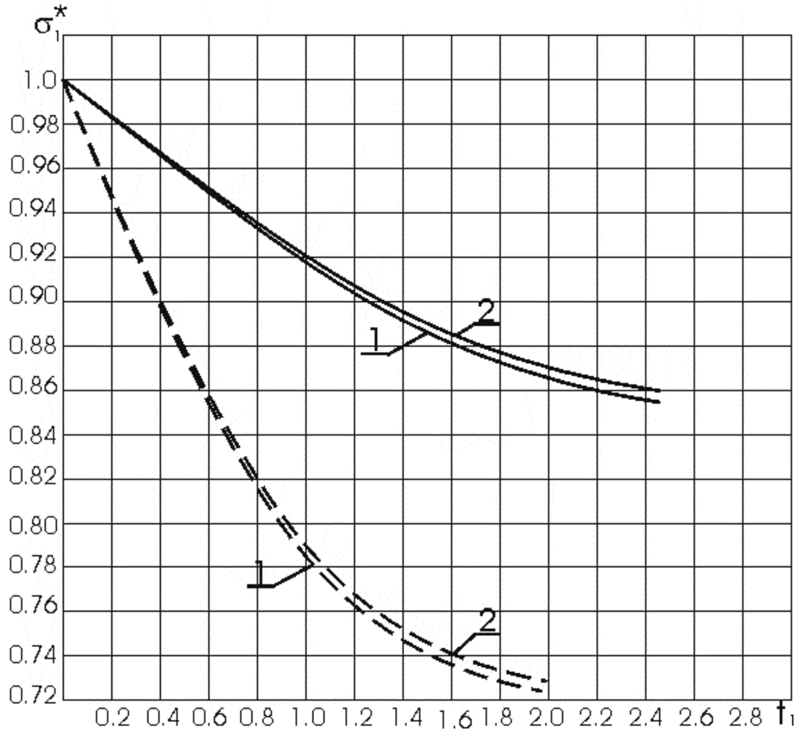

Fig. The normal stress change

The new solutions of complex problems obtained due to the proposed approach allow one to analyse the stress-strain state of multi-layered bodies ith straighten elements. These results can be used in engineering calculations of different constructions, foundations and layered bases.

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Мета. Розробка аналітичних методів для розрахун-ків конструкцій, матеріал яких має складний комплекс властивостей. Вивчення впливу в'язкопружності на напружено-деформований стан конструкції та можливості врахування кінцевих деформацій.

Методика. Побудована математична модель просторової задачі про передачу навантаження від підкріплюючого елементу до в'язкопружного масиву. Запропоновано метод для вирішення завдань теорії пружності з урахуванням геометричної нелінійності, перевірений на тестових завданнях.

Результати. Розширене коло завдань нелінійної теорії пружності та лінійної в'язкопружності, що можна розв'язати аналітичними методами, та розроблені самі методи збурення.

Наукова новизна. Розроблений асимптотичний метод вирішення тривимірних завдань лінійної в'язкопружності ортотропних середовищ, запропонований метод збурення для дослідження задач з урахуванням скінчених деформацій. Одержані аналітичні розв'язки ряду нових задач, а саме, про передачу навантаження через підкріплюючий елемент до масиву з в'язкопружного матеріалу у просторовій постановці.

Практична значимість. Запропонований метод дозволяє перейти від розв'язування складних мішаних задач механіки до послідовного розв'язування задач теорії потенціалу, що є найбільш розробленим розділом математичної фізики. Одержані завдяки запропонованому підходу розв'язки ряду нових складних задач дають можливість аналізувати напружено-деформований стан тіл з підкріплюючими елементами. Ці результати можуть використовуватися в інженерних розрахунках пальових фундаментів та підвалин.

Ключові слова: *передача навантаження, метод збурення, теорія пружності, в'язкопружність, нелінійність, просторові та плоскі задачі*

Цель. Разработка аналитических методов для расчетов конструкций, материал которых имеет сложный комплекс свойств. Изучение влияния вязкоупругости на напряженно-деформированное состояние конструкции и возможности учета конечных деформаций.

Методика. Построена математическая модель пространственной задачи о передаче нагрузки от подкрепляющего элемента к вязкоупругому массиву. Предложенный метод для решения задач теории упругости с учетом геометрической нелинейности проверен на тестовых задачах.

Результаты. Расширен круг задач нелинейной теории упругости и линейной вязкоупругости, которые можно решить аналитическими методами, разработаны сами методы возмущения.

Научная новизна. Разработан асимптотический метод решения трехмерных задач линейной вязкоупругости ортотропных сред, предложен метод возмущения для исследования задач с учетом конечных деформаций. Получены аналитические решения ряда новых задач, а именно, о передаче нагрузки через подкрепляющий элемент к массиву из вязкоупругого материала в пространственной постановке.

Практическая значимость. Предложенный метод позволяет перейти от решения сложных смешанных задач механики к последовательному решению задач теории потенциала, которая является наиболее разработанным разделом математической физики. Полученные благодаря предложенному подходу решения ряда новых сложных задач дают возможность анализировать напряженно-деформированное состояние тел с подкрепляющими элементами. Эти результаты могут использоваться в инженерных расчетах свайных фундаментов и оснований.

Ключевые слова: *передача нагрузки, метод возмущения, теория упругости, вязкоупругость, нелинейность, пространственные и плоские задачи*

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