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SYNTHESIZING MODELS OF NONLINEAR DYNAMIC OBJECTS IN CONCENTRATION ON THE BASIS OF VOLTERRA-LAGUERRE STRUCTURES

Purpose. Enhancing energy efficiency and quality of automated control of the technological concentration line, increasing extraction of the useful component into concentrate while processing iron-bearing ores of various mineralogical and technological types through developing principles and approaches to distributed optimal control over interrelated processes in concentration production on the basis of the dynamic space-time model.

Methodology. Based on the assumption that final results of concentration plant operation depend on a set of input parameters and results of functioning of interrelated nonlinear dynamic objects, the authors suggest an improved approach to simulating concentration processes for iron ore materials on the basis of Volterra-Laguerre structures by using input signals of certain technological stages characterizing granulometric composition of processed ore.

Findings. It is found that while synthesizing models of nonlinear dynamic objects of concentration, it is expedient to apply Volterra structures with the simulation error not exceeding 0.039 under the mean square deviation of 0.0594. Volterra models projected onto orthonormal basis functions enable simplifying parameterization and reducing sensitivity of models to noises. Among other orthonormal functions, Laguerre functions are reasonable to use. All this allows minimizing the number of model parameters in the course of their identification.

Originality. The method of identifying nonlinear dynamic objects of concentration on the basis of the space-time Volterra model is improved. This model is different from available ones by its projection onto orthonormal Laguerre basis functions to increase its robustness to noises.

Practical value. Testing results enable deducing efficiency of the space-time Volterra model in the condition space by means of the Laguerre network, thus increasing accuracy of simulation under noises as compared to the Volterra model through reducing the simulation error by 18.11 % under 40 iterations of identification. The experimental check of identification accuracy by means of the Volterra-Laguerre model in the iron content control system in various points of the technological concentration line confirms efficiency of the given method.

Keywords: *nonlinear dynamics, Volterra-Laguerre model, concentration, identification*

Introduction. Although all basic industrial processes are nonlinear in dynamics, in practice, most control systems are based on linear control methods. For soft nonlinearity, linear approximation of process dynamics around the working point is satisfactory in general. If there are great deviations from this working point, both the model and the control system should be readjusted.

If such readjustments are required too often, adaptive control is expedient to be used. It is evident that awareness of nonlinearity of the process combined with application of the method for controlling nonlinear dynamic objects could eliminate the necessity for adaptive control.

In case of nonlinear systems, when their dependencies on previous input data reduce quickly in time, the dependency of output signals on input ones is described by the Volterra integro-power series which characterize the control object's properties (its condition) as a sequence of multi-dimensional weight functions (Volterra kernels) invariant to the type of the input signal.

Application of models with Volterra series to identifying and simulating nonlinear systems can be explained by the following essential benefits as is indicated in the works by Soni, A. S., Parker, R. S. and [1, 2]:

- invariance as to the type of the input influence (i.e. possible solution of a problem for determined and random input signals);
- consideration of nonlinear and inertia (dynamic) properties of an object;
- universality, i. e. potential investigation into nonlinear continuous and nonlinear impulse systems, stationary and non-stationary stochastic systems with centred and distributed parameters as well as multidimensional systems with multiple inputs and outputs;
- interpretation of linear systems as a subclass of nonlinear ones to apply time and spectral methods developed in the theory of linear systems to nonlinear systems;
- potential research from the analytical and calculation standpoints.

Literature review. Identification of an object in the form of the second-order Volterra model implies the following. There

are L pairs of normalized values of "input-output" ($\psi(k)$, $\beta^d(k)$) that determine parameters β_0^d, $\alpha_1(i)$, $\alpha_2(i,j)$ of the second-order model considering disturbances [3]

$$\beta^d(k)=\beta_0^d+\sum_{i=0}^{M}\alpha_1(i)\psi(k-i)+\sum_{i=0}^{M}\sum_{j=0}^{M}\alpha_2(i,j)\psi(k-j)+e(k), \quad (1)$$

where $\{e(k)\}$ is a sequence of random values of the zero mean value which is statistically independent of the input sequence $\{\psi(k)\}$.

The problem of identifying is reduced to formation of equations to assess parameters of model (1) to ensure the minimum mean square error admitting at the same time that the input sequence $\{\psi(k)\}$ is stationary as is indicated by Doyle F.J., Pearson R.K., Ogunnaike B.A. and [3, 4]. The advantage of this approach is the fact that it produces simple evaluation equations with certain correlations that can be assessed by means of available data. To assess parameters of the second-order Volterra model, it is necessary to calculate the autocorrelation function of the input signal

$$R_{\psi\psi}(m) = E\{\psi(k-m)\psi(k)\}, \quad (2)$$

the intercorrelation function of 'input-output' signals

$$r_{\psi\beta}(m) = E\{\psi(k-m)\beta^d(k)\}, \quad (3)$$

and the inter-bi-correlation function of 'input-output' signals

$$t_{\psi\beta}(m, n) = E(\psi(k-m)\psi(k-n)\beta^d(k)). \quad (4)$$

While calculating the above mentioned correlation functions (2–4), the mathematical expectation operator $E\{\cdot\}$ is replaced by the mean value formula of the available sequence. As a result of this, under m, $n = 0, 1, ..., M$ the correlation functions are written down as [3]

$$\hat{R}_{\psi\psi}(m)=\frac{1}{L-M}\sum_{k=M+1}^{L}\psi(k-m)\psi(k);$$

$$\hat{r}_{\psi\beta}(m)=\frac{1}{L-M}\sum_{k=M+1}^{L}\psi(k-m)\beta^d(k);$$

$$\hat{t}_{\psi\beta}(m,n)=\frac{\sum_{k=M+1}^{L}\psi(k-m)\psi(k-n)\beta^d(k)}{L-M}.$$

Equations of parameter evaluations are synthesized on the basis of the mean square error expression

$$MSE = E\left\{\left(\begin{array}{l}\beta^d(k)-\beta^d{}_0-\sum_{i=0}^{M}\alpha_1(i)\psi(k-i)-\\ -\sum_{i=0}^{M}\sum_{j=0}^{M}\alpha_2(i,j)\psi(k-i)\psi(k-j)\end{array}\right)^2\right\}. \quad (5)$$

The equations aimed at assessing parameters β_0^d, $\alpha_1(i)$, $\alpha_2(i,j)$ are obtained by differentiating the expression MSE (5) for each parameter and equaling the obtained result to zero.

According to the recommendations of Doyle F.J., Pearson R.K., Ogunnaike B.A., [3, 4] in particular, the following calculations are greatly simplified under the following conditions

$$\begin{aligned}&E\{\psi(k)\}=0;\\ &E\{\psi(k-i)\psi(k-j)\psi(k-m)\}=0.\end{aligned} \quad (6)$$

Conditions (6) impose requirements of the third-order symmetry of the input sequence and, as a result, the linear component of the second-order Volterra model ($\alpha_1(i)$) maintains this unpaired symmetry, while constant and quadratic terms reveal paired symmetry.

Differentiation of MSE with respect to the linear parameter ($\alpha_1(\ell)$) leads to the following system of linear equations [3]

$$\begin{aligned}&\alpha_1(\ell)-r_{\psi\beta}(\ell)+2\sum_{i=0}^{M}R_{\psi\psi}(i-\ell)\alpha_1(i)=0,\\ &\ell=0,1,\ldots,M.\end{aligned} \quad (7)$$

The given system (7) is the Dzig-Walker equation system applied to identifying linear systems and is more concise as $R_{\psi\psi}a = r_{uy}$ where $a = [\alpha_1(0), ..., \alpha_1(M)]^T$ is a vector of linear parameters of the model; $R_{\psi\psi}$ is the symmetric Toeplitz matrix of $(M1) \cdot (M + 1)$ size where the elements with indices i, j are intercorrelation functions of the input signal $R_{\psi\psi}(i - j)$; $r_{\psi y} = [r_{\psi y}(0), ..., r_{\psi y}(M)^T]$ is a vector of the intercorrelation input-output function.

Purpose. The research aims at enhancing energy efficiency and quality of automated control of the technological concentration line, increasing extraction of the useful component into concentrate while processing iron-bearing ores of various mineralogical and technological types through developing principles and approaches to distributed optimal control over interrelated processes in concentration on the basis of the dynamic space-time model.

The given research aim conditions the necessity to solve the problem of developing and investigating into the space-time mathematical model of nonlinear dynamic objects of concentration as structures with distributed parameters.

Methods. To simulate nonlinear systems with distributed parameters, the authors suggest the space-time Volterra model considered by Han-Xiong Li, Chenkun Qi, Stephen Boyd, in [5, 6] in particular, in the form of

$$y(x,t)=\sum_{r=1}^{r}\int_0^{\pi}\cdots\int_0^{\pi}\int_0^{t}\cdots\int_0^{t}g_r(x,\xi_1\ldots\xi_r,\tau_1\ldots\tau_r)\times$$
$$\times\prod_{v=1}^{r}u(\xi_v,t-\tau_v)d\tau_v d\xi_v.$$

For better visualization, we provide a simple example of the three-dimensional kernel model under $R = 2$

$$\beta^d(x,t)=\int_0^{\pi}\sum_{\tau_1=0}^{t}g_1(x,\xi_1,\tau_1)\psi(\xi_1,t-\tau_1)d\xi_1+$$
$$+\int_0^{\pi}\int_0^{\pi}\sum_{\tau_1=0}^{t}\sum_{\tau_2=0}^{t}g_2(x,\xi_1,\xi_2,\tau_1,\tau_2)\psi(\xi_1,t-\tau_1)\psi(\xi_2,t-\tau_2)d\xi_1 d\xi_2.$$

The object's dynamics should be first decomposed into the kernel series resulting in the space-time distribution of kernels.

$$g_r(\cdot)=\sum_{i=1}^{n}\sum_{j_1=1}^{m}\cdots\sum_{j_r=1}^{m}\sum_{k_1=1}^{l}\cdots\sum_{k_r=1}^{l}\theta^{(r)}_{i\,j_1\ldots j_r k_1\ldots k_r}\times$$
$$\times\prod_{s=1}^{r}\phi_i(x)\varphi_{js}(\xi_s)\psi_{ks}(\tau_s)\ldots,R\ldots.$$

After assessing unknown parameters, the kernels are subjected to transformation through space-time synthesis. The quality of simulation will be improved as a result of increasing the number of kernels. As a set of kernels is used, a wide range of nonlinear integrated systems with distributed parameters can be approximated.

The lumped system is generally described by

$$\beta^d(t) = N(\{\psi(\tau)\}) + d(t),$$

where $\{\psi(\tau)\} = \{\psi(\tau) | \tau = 1, ..., t\}$ is an input signal; t is the discrete time; y is an output; d is stochastic disturbances, N is the operator with the extinct memory function approximated by the discrete Volterra model [6]

$$\beta^d(t)=\sum_{r=1}^{\infty}\sum_{\tau_1=0}^{t}\cdots\sum_{\tau_\gamma=0}^{t}\gamma_r(t,\tau_1,\ldots,\tau_r)\prod_{v=1}^{r}(\tau_v). \quad (8)$$

Transformation (8) for the system with distributed parameters as is shown in the work by Chenkun Qi enables [5]

$$\beta^d(x, t) = N(\{\psi(\zeta, \tau)\} + d(x, t)),$$

where $\{\psi(\zeta,t)\} = \{\psi(\zeta,t) \mid \zeta \in \Omega, \tau = \overline{1,t}\}$ is an input.

The space-time Volterra model is created through adding spatial variables to the conventional Volterra model

$$\beta^d\left(x,t\right) = \\ = \sum_{r=1}^{\infty} \int_{\Omega} \dots \int_{\Omega} \sum_{\tau_1=0}^{t} \dots \sum_{\tau_\gamma=0}^{t} \gamma_r\left(x,\zeta_1,\dots,\zeta_r,t,\tau_1,\dots,\tau_r\right) \prod_{v=1}^{r} \psi\left(\zeta_v,\tau_v\right) d\zeta_v, \quad (9)$$

where γ_r is the r^{th} order function of the space-time Volterra kernel that denotes the change at the input in space $\zeta_1, \dots, \zeta_r$ and time $\tau_1, \dots, \tau_r$.

Model (9) can be applied both to time-variable and stationary systems. For the system, which is currently invariant, the kernel will be invariant and, as is shown in the work by Chenkun Qi, can be presented in the following way

$$\gamma_r\left(x,\zeta_1,\dots,\zeta_r,t,\tau_1,\dots,\tau_r\right) = \gamma_r\left(x,\zeta_1,\dots,\zeta_r,t-\tau_1,\dots,t-\tau_r\right). \quad (10)$$

Similarly to (9), according to Chenkun Qi, the model can also function in the space of the variable or space-invariant systems. If the model is homogenous in the spatial domain, there is

$$\gamma_r\left(x,\zeta_1,\dots,\zeta_r,t,\tau_1,\dots,\tau_r\right) = \gamma_r\left(x-\zeta_1,\dots,x-\zeta_r,t,\tau_1,\dots,\tau_r\right).$$

Substituting (10) into (9), we obtain

$$\beta^d\left(x,t\right) = \sum_{r=1}^{\infty} \int_{\Omega} \dots \int_{\Omega} \sum_{\tau_1=0}^{t} \dots \sum_{\tau_\gamma=0}^{t} \gamma_r\left(x,\zeta_1,\dots,\zeta_r,t,\tau_1,\dots,\tau_r\right) \times \\ \times \prod_{v=1}^{r} \psi\left(\zeta_v,t-\tau_v\right) d\zeta_v. \quad (11)$$

Model (11) is not used because of its infinite order. In practice, according to Han-Xiong Li, Chenkun Qi, Stephen Boyd, in [5, 6], terms of higher order can be neglected and only the first R of the kernel is considered

$$\beta^d\left(x,t\right) = \sum_{r=1}^{R} \int_{\Omega} \dots \int_{\Omega} \sum_{\tau_1=0}^{t} \dots \sum_{\tau_\gamma=0}^{t} \gamma_r\left(x,\zeta_1,\dots,\zeta_r,t,\tau_1,\dots,\tau_r\right) \times \\ \times \prod_{v=1}^{r} \psi\left(\zeta_v,t-\tau_v\right) d\zeta_v + \upsilon\left(x,t\right), \quad (12)$$

where the last term $\upsilon(x,t)$ contains dynamics that is not simulated and noises. Accuracy of simulation and complexity of the model can be controlled through the R order. If we assume that kernels in (12) are fully integrant in the time domain [0, ∞) at any point of space x and ζ, it means that the corresponding space-time Volterra model is stable and can be presented by orthonormal time basis functions. The kernels should be applied to the input base $\left\{\psi_i(x)\right\}_{i=1}^{m}$, the output base $\left\{\varphi_i(x)\right\}_{i=1}^{n}$ and the time base $\left\{\varphi_i(t)\right\}_{i=1}^{q}$

$$\gamma_r\left(\cdot\right) = \\ = \sum_{i=1}^{n} \sum_{j_1}^{m} \dots \sum_{j_r=1}^{m} \sum_{k_1=1}^{q} \dots \sum_{k_r=1}^{q} \theta^{(r)}_{ij_1\dots j_r k_1\dots k_r} \left(\phi_i\left(x\right) \prod_{v=1}^{r} \psi_{j_v}\left(\zeta_v\right) \phi_{k_v}\left(\tau_v\right) \right), \quad (13)$$

where $\theta^{(r)}_{ij_1\dots j_r k_1\dots k_r}$ is a corresponding constant ratio.

Parameters n and q (13) should be infinite for systems with distributed parameters. In fact, for most parabolic systems, the finite values of n and q are quite realistic assumptions, according to Han-Xiong Li, Chenkun Qi and Stephen Boyd in [5, 6]. It is evident that they depend on required accuracy of simulation. On the other hand, n also depends on the frequency mode and the type of spatial basis functions, while q is associated with complexity of system dynamics.

The need to calculate a great number of ratios of the model, according to Soni A. S., Parker R. S., and others [2], is the factor that greatly complicates application of Volterra models.

$$\sum_{i=1}^{N} \left(\frac{M+i-1}{i} \right), \quad (14)$$

where N is the order of the Volterra model under study; M is the memory depth of the system. The scheme of the technological line of a concentration plant considered in this paper contains $L = 15$ testing points of technological parameters. According to the approach suggested, at each of L points, the granulometric composition of ore materials according to D = 9 size classes $\gamma_\ell\left(d\right), d = \overline{1,D}, \ell = \overline{1,L}$ and the useful component content in size classes $\beta_\ell\left(d\right), d = \overline{1,D}, \ell = \overline{1,L}$ are controlled. After considering one controlling influence for each technological aggregate, the number of model signals will increase up to 270 + 15 = 285.

The results of the research on dynamic properties of concentration in [5] indicate that while forming mathematical models, one should take account of 3–4 delayed values of signals.

Assessment of the number of parameters of the Volterra model by methods suggested by Abhishek S. Soni with the above number of signals including the delayed ones reveals the number of parameters of the Volterra model being over 11.500. Besides, under given conditions, sensitivity of the Volterra model to measurement noises is more evident.

One of the ways of simplifying parameterization and attenuation of sensitivity in Volterra models to noises is their projection onto a set of orthonormal basis functions. Various basis functions are under consideration: Laguerre functions for approximation of the linear block [6], Taylor series – for nonlinear term [7], radial basis functions for estimation of the solution of Volterra integral equations [8], Laguerre functions basis for expansion of transfer function [9], Laguerre and Kautz's basis functions [10]. Unlike the Volterra model, Laguerre and Kautz's orthonormal functions are composed of a set of smooth exponential functions. These models are noted for noise filtration. Orthonormal basis functions differ in their complexity and quality characteristics. For example, only one parameter (pole) is required to determine dynamics of the Laguerre model, while there should be two poles for the Kautz model.

The Kautz model can simulate oscillatory behavior due to its complexity when two poles are selected as combined ones. Considering the specific character of iron ore material concentration and taking account of the fact that a single parameter simplifies identification of Laguerre models, this model should be studied to solve the set tasks.

Nonlinear functions can be presented by linear combination of Laguerre filters. The basis Laguerre function, according to Abhishek S. Soni, in the discrete time is determined by

$$\varphi_j\left(i\right) = \sqrt{1-\alpha^2} \left\{ \sum_{k=0}^{j-1} \left[\begin{array}{l} \left(-\alpha\right)^k \binom{j-1}{k} \binom{i+k-1}{k-1} \times \\ \times \alpha^{i-j+k} U\left(i-j+k\right) \end{array} \right] \right\}, \quad (15)$$

where α is the Laguerre pole; $U(\cdot)$ is a unit-step function. The parameter α in (15) determines the exponential frequency of attenuation of Laguerre functions and dynamic behavior of the Laguerre model. It is within $0 \le \alpha < 1$.

The output of the Laguerre model is presented by the formula

$$\hat{\beta}_l^d\left(k\right) = c_0 + \sum_{i=1}^{\infty} c_i \ell_i\left(k\right) + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} c_{ij} \ell_i\left(k\right) \ell_j\left(k\right) + \\ + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} c_{ijn} \ell_i\left(k\right) \ell_j\left(k\right) \ell_n\left(k\right); \quad (16)$$

$$\ell_j(k) = \sum_{i=}^{\psi_l} \Phi_j(i)\psi(k-i), \quad (17)$$

where $\hat{\beta}_l^d(k)$ is the output of the Laguerre model; c is Laguerre ratios; ψ_l is the length of the input sequence ψ.

The benefit of the Laguerre basis is its efficiency as instead of the infinite sum used in (16, 17), it can be reduced to a relatively small number of elements [3]. Through increasing the number of filters included in the model, the simulation error caused by the final number of Laguerre filters decreases. Also, according to Dumont G.A., Fu Y., for the set accuracy of the Laguerre model, the simulation error can be reduced by selecting the parameter α. The total number of parameters N_p of the third-order Laguerre model with L filters is set through the ratio

$$N_p = (L+1)(L+2)(L+3)/6.$$

The Laguerre model can be also written down as a state-space. In case of the third-order discrete presentation of the model as is shown in the works by Dumont G.A., Fu Y., we obtain

$$\ell(k+1) = A(\alpha)\ell(k) + B(\alpha)\psi(k); \quad (18)$$

$$\hat{\beta}_l^d(k) = C^T\ell(k) + \ell^T(k)D\ell(k) + \sum_{i=1}^{L}\left(\ell(k)^T E_i\ell(k)\right)\ell_i(k), \quad (19)$$

where parameters $A(\alpha)$, $B(\alpha)$ depend on α, i.e. the dynamic behavior of the system in (18)–(19) is conditioned by the value of the parameter α

$$A(\alpha) = \begin{bmatrix} \alpha & 0 & 0 & \dots & 0 \\ (1-\alpha^2) & \alpha & 0 & \dots & 0 \\ (-\alpha)(1-\alpha^2) & (1-\alpha^2) & \alpha & \dots & 0 \\ \vdots & \vdots & \dots & \ddots & \vdots \\ (-\alpha)^{L-2}(1-\alpha^2) & (-\alpha)^{L-3}(1-\alpha^2) & \dots & \dots & \alpha \end{bmatrix};$$

$$B(\alpha) = \sqrt{(1-\alpha^2)}\begin{bmatrix} 1 \\ -\alpha \\ (-\alpha)^2 \\ \vdots \\ (-\alpha)^{L-1} \end{bmatrix}; \quad C^T = \begin{bmatrix} c_1 \cdots c_L \end{bmatrix};$$

$$D = \begin{bmatrix} c_{11} & \cdots & c_{1L} \\ \vdots & \ddots & \vdots \\ c_{L1} & \cdots & c_{LL} \end{bmatrix}; \quad E_i = \begin{bmatrix} c_{11i} & \cdots & c_{1Li} \\ \vdots & \ddots & \vdots \\ c_{L1i} & \cdots & c_{LLi} \end{bmatrix}.$$

Thus, the Laguerre model like the Volterra one belongs to Wiener models, i.e. consists of a linear dynamic element followed by the static nonlinear non-memory output.

Findings. As the Laguerre basis is orthonormal within the interval $[0, \infty)$ and complete in the space $L_2[0, \infty)$, the following statement is true. The kernel of the N order can be expanded to the Laguerre series in the following way, if it is stably separable and strictly proper, according to Zheng Q., Zafiriou E.

$$h_N(i_1,\dots,i_N) = \sum_{j_1=1}^{\infty}\cdots\sum_{j_N=1}^{\infty} c_N(j_1\dots j_N)\varphi_{j1}(i_1)\dots\varphi_{jN}(i_N), \quad (20)$$

where

$$c_N(j_1,\dots,j_N) = \sum_{i_1=1}^{\infty}\cdots\sum_{i_N=1}^{\infty} h_N(i_1,\dots,i_N)\varphi_{j1}(i_1)\dots\varphi_{jN}(i_N). \quad (21)$$

The separability condition of the kernel in (21) looks like

$$h_N(i_1,\dots,i_N) = \sum_{\hat{i}=1}^{q} v_{1j}(i_1)v_{2j}(i_2)\dots v_{Nj}(i_N), \quad (22)$$

where q is some finite number; $v_{gj}(i_g)$ is the actual function of a variable $g = \overline{1,N}$. The kernel is stably separable if $v_{gj}(i_g)$ satisfies the condition

$$\sum_{i_g}^{\infty}\left|v_{gj}(i_g)\right| < \infty.$$

The symmetric kernel is strictly proper if

$$h_N(i_1,\dots,i_N) = 0, \quad i_1,\dots,i_N = 0.$$

This implies that the Volterra model output is not a function of the current input value as it depends on previous inputs only. As Volterra kernels satisfy the above conditions, they can be projected onto the Laguerre basis.

As is shown above, instead of the infinite sum in (20), L elements can be used and the following expression is applied

$$h_N(i_1,\dots,i_N) = \sum_{j_1=1}^{L}\cdots\sum_{j_N=1}^{L} c_N(j_1,\dots,j_N)\varphi_{j1}(i_1)\dots\varphi_{jN}(i_N),$$

where $c_N(j_1,\dots,j_N)$ are Laguerre ratios obtained by means of Volterra kernels applying the following expression

$$c_N(j_1,\dots,j_N) = \sum_{i_1=1}^{L}\cdots\sum_{i_N=1}^{L} h_N(i_1,\dots,i_N)\varphi_{j1}(i_1)\dots\varphi_{jN}(i_N).$$

The main advantage of the Laguerre basis is adequate description of the system obtained at $L \ll M$. Thus, the number of parameters required to describe the Laguerre model is greatly reduced. To describe the Volterra-Laguerre model approximating the Volterra model, only two variables α and L are required. As parameters α and L are interrelated, the simulation error is the function of these parameters.

To determine the compromising size of the model (the number of Laguerre filters) and the model quality (the number of Laguerre filters and the Laguerre pole) the informational criterion Akaike is used as metrics. It is calculated as a quadratic error between inputs of the Volterra model and the Volterra-Laguerre model

$$AIC = K\log(SSE(\alpha, L)/K) + 2N_p. \quad (23)$$

The first element of (22) considers accuracy of the model, while K is the volume of data applied to identifying the model. The second element penalizes complexity of the obtained model. In the scope of the current research, K is the number of ratios of the Volterra models subjected to projection, SSE is a sum of error squares between the input Volterra model and outputs of the Volterra-Laguerre model projected, and Np is the number of parameters in the obtained Volterra-Laguerre model. Under the given set of data, SSE depends on both the number of Laguerre filters and the corresponding Laguerre pole. Each pair of α and L characterizes the Volterra-Laguerre model which has a unique value AIC associated with it. The model with the smallest AIC value is an optimal balance between the number of parameters and the model quality. In the given research, the optimal Laguerre pole for the given L is calculated to obtain the complete nonlinear Volterra model. The SSE term in the AIC expression is set as follows

$$SSE(\alpha, L) = \left(\hat{\beta}_v(k) - \hat{\beta}_l(k,\alpha,L)\right)^2,$$

where $\hat{\beta}$ and $\hat{\beta}_l(\alpha, L)$ are outputs of the Volterra and Volterra-Laguerre models respectively.

Dynamics of the Volterra-Laguerre model is set by the parameter α in (15), while L determines the size of matrices C, D, and E. Thus, the problem of optimization looks like

$$J(\alpha, L) = \min_{\alpha,L} K\log\left(\frac{\left(\hat{\beta}_v(k) - \hat{\beta}_l(k,\alpha,L)\right)^2}{K}\right) + 2N_p. \quad (24)$$

The given problem is the nonlinear optimization problem with constraints under nonlinear dependency of Laguerre filters on the parameter α.

According to the method described in [11], there is performed assessment of relative quadratic errors of four various approximations of the discrete Laguerre model within 8–32 with the sampling period of $T_s = 0.5$ sec. The method based on reduction of the quadratic error calls for inversion of the $R \times R$ matrix where R is the order of the desired concise model. Laguerre poles are calculated by the method developed by Moodi H. and Bustan D. These four Laguerre models are then used to calculate rational sixth-order models.

The research results reveal that for larger matrices, the minimal quadratic error can be obtained by applying the algorithm of singular value decomposition [12].

Fig. 1 presents comparison of the quadratic error areas obtained by different inversion methods. The obtained results indicate that the *SVD* algorithm is more accurate as compared to the pseudo-inverse matrix and the recursive least square method.

The efficient value of the error for the above methods (Fig. 1) makes: a) 3.65; b) 0.45; c) 2.37.

Relative quadratic errors calculated during the first 100 impulse responses are given in Table 1. The obtained results demonstrate that quality of the approximated model greatly depends on the quality of the initial model applied to its building.

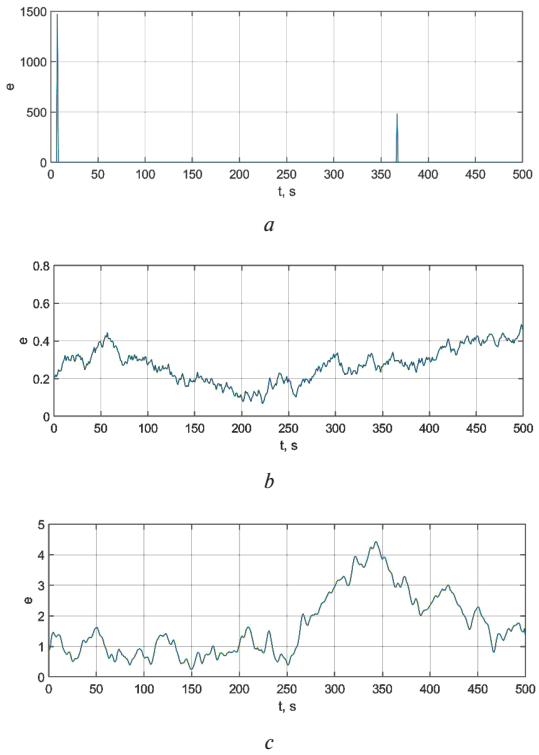

Fig. 1. Errors of the Volterra-Laguerre model identification:
a – pseudo-inverse matrix; b – SVD algorithm; c – recursive least-square method

Table 1

Relative quadratic error calculated for the 0–5 min interval

Laguerre model order	Laguerre model error	Error of the approximated sixth-order Laguerre model
8	$1.32 \cdot 10^{-2}$	$1.32 \cdot 10^{-2}$
16	$4.24 \cdot 10^{-3}$	$4.75 \cdot 10^{-3}$
24	$5.97 \cdot 10^{-4}$	$6.34 \cdot 10^{-4}$
32	$1.98 \cdot 10^{-4}$	$2.18 \cdot 10^{-4}$

The models obtained through applying this method are convincingly stable and maintain the first Laguerre R ratios of the input system.

Thus, the analysis of the results reveals potential considerable reduction of the order of the Volterra-Laguerre model accompanied by the retained practically identical response of the reduced model as compared to the input one.

Due to significant variability of the characteristics of the raw ore [13], nonstationarity and large spatial extent of concentrating production [14] and nonlinearity of its dynamic properties [15], to obtain model parameters, nonlinear optimization was used. To solve the problem of nonlinear optimization, the following methods are under study: the interior point method, iteration quadratic programming, and the active set method. The conducted analysis results in selecting the active set method. In particular, this method reveals better results of searching for the most efficient solution under various initial conditions, while other studied methods call for extra calculations to determine initial conditions.

Table 2 demonstrates the analysis of the results of comparing the Volterra and Volterra-Laguerre models in the absence of noises and under noises of 10 % amplitude of the useful signal.

With the increased number of identification iterations, the benefit of applying the Volterra-Laguerre linear identification reduces as compared to the standard Volterra model. One of the factors affecting the number of iterations is characteristics of the simulated process. It should be noted that under 30–40 iterations, the Volterra-Laguerre model reveals better results as compared to the Volterra model.

The identification result according to normallized "input-output" data is in Fig. 2.

Adequacy of the model is checked by the Cochren criterion according to the following conditions: the number of parallel experiments per series is 9, the number of series is 24, the significance level is 0.05. The results of experimental data procession reveal that the calculated Cochern criterion makes 0.4325, which is less than the table value of 0.4748. Thus, the conducted analysis confirms replicability of obtained results.

Thus, while using the Volterra-Laguerre model under noises, the simulation error reduces by 18.11 % as compared to the Volterra model under 40 identification iterations. Experimental testing of identification accuracy of the Volterra-Laguerre model applied to controlling the iron content at various points of the technological concentration line confirms efficiency of the method.

Conclusions. The approach to simulating iron ore materials concentration on the basis of Volterra-Laguerre is improved after considering granulometric composition of ore materials at the output of particular technological stages.

It is indicated that Volterra structures applied to simulating iron ore materials concentration enable reduction of the simulation error up to 0.039 under the mean square deviation of 0.0594.

Volterra models were projected onto a set of orthonormal functions to simplify their parameterization and attenuation of

Table 2

Increasing accuracy of the Volterra-Laguerre model as compared to the Volterra model

Iteration number	Error reduction (absence of noises), %	Error reduction (presence of noises), %
20	9.87	50.01
40	4.32	18.11
60	0.96	9.72
80	0.54	5.79
100	0.12	2.95

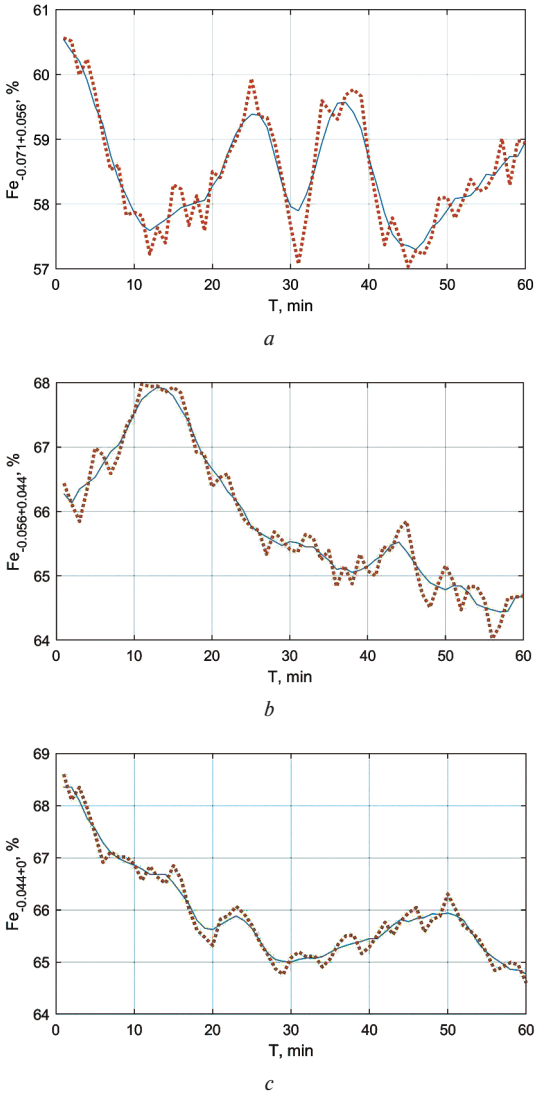

Fig. 2. Identification results of the Volterra-Laguerre model of the iron content in the second-stage crushing product:
• • • • • *– the experiment data;* —— *– the identification result*

sensitivity to noises. It is shown that among basis functions, Laguerre functions are the most expedient to be applied to minimizing the number of parameters for identifying models of ore concentration.

Testing results enable drawing the conclusion that the space-time Volterra model is efficient in the state-space through applying the Laguerre network, which allows increasing the accuracy of simulation under noises as compared to the Volterra model through reducing the simulation error by 18.11 under 40 % identification iterations. Experimental testing of identification accuracy of the Volterra-Laguerre model applied to controlling the iron content at various points of the technological concentration line confirms efficiency of the given method.

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Синтез моделей нелінійних динамічних об'єктів збагачувального виробництва на основі структур Вольтерра-Лагерра

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Мета. Підвищення енергоефективності та якості автоматизованого керування технологічною лінією збагачення, збільшення вилучення корисного компонента у концентрат при переробленні залізовмісних руд, представлених мінералого-технологічними різновидами, шляхом розроблення принципів і підходів до розподіленого оптимального керування взаємопов'язаними процесами збагачувального виробництва на основі динамічної просторово-часової моделі.

Методика. Ґрунтуючись на тому, що кінцеві результати роботи збагачувальної фабрики залежать від сукупності вхідних параметрів і результатів функціонування комплексу взаємопов'язаних нелінійних динамічних об'єктів, запропоновано удосконалений підхід до моделювання процесів збагачення залізорудної сировини на основі структур Вольтерра-Лагерра, із застосуванням вихідних сигналів окремих технологічних стадій, що характеризують гранулометричний склад руди, що переробляється.

Результати. Встановлено, що при синтезі моделей нелінійних динамічних об'єктів збагачувального виробництва доцільно використовувати структури Вольтерра, при цьому помилка моделювання не перевищує 0,039 при середньоквадратичному відхиленні 0,0594. Використання проеціювання моделей Вольтерра на ортонормовані базисні функції дозволило спростити процес параметризації та знизити чутливість моделей до шумів. Показано, що з ортонормованих функцій доцільно застосувати функції Лагерра. Зазначене дозволяє мінімізувати кількість параметрів моделей у процесі ідентифікації.

Наукова новизна. Удосконалено метод ідентифікації нелінійних динамічних об'єктів збагачувального виробництва на основі просторово-часової моделі Вольтерра, який відрізняється від наявних тим, що для підвищення робастності моделі Вольтерра до шумів здійснене її проеціювання на набір ортонормованих базисних функцій Лагерра.

Практична значимість. Результати апробації дозволяють зробити висновок щодо доцільності реалізації просторово-часової моделі Вольтерра у просторі станів за допомогою мережі Лагерра, що дозволяє підвищити точність моделювання в умовах дії шумів у порівнянні з моделлю Вольтерра зменшенням помилки моделювання на 18,11 % при 40 ітераціях ідентифікації. Експериментальна перевірка точності ідентифікації із застосуванням моделі Вольтерра-Лагерра в системі контролю вмісту заліза в різних точках технологічної лінії збагачення підтверджує доцільність даного методу.

Ключові слова: *нелінійна динаміка, модель Вольтерра-Лагерра, збагачувальне виробництво, ідентифікація*

Синтез моделей нелинейных динамических объектов обогатительного производства на основе структур Вольтерра-Лагерра

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Цель. Повышение энергоэффективности и качества автоматизированного управления технологической линией обогащения, увеличение извлечения полезного компонента в концентрат при переработке железосодержащих руд, представленных минералого-технологическими разновидностями, путем разработки принципов и подходов к распределенному оптимальному управлению взаимосвязанными процессами обогатительного производства на основе динамической пространственно-временной модели.

Методика. Основываясь на том, что конечные результаты работы обогатительной фабрики зависят от совокупности входных параметров и результатов функционирования комплекса взаимосвязанных нелинейных динамических объектов, предложен усовершенствованный подход к моделированию процессов обогащения железорудного сырья на основе структур Вольтерра-Лагерра с применением выходных сигналов отдельных технологических стадий, характеризующих гранулометрический состав перерабатываемой руды.

Результаты. Установлено, что при синтезе моделей нелинейных динамических объектов обогатительного производства целесообразно использовать структуры Вольтерра, при этом ошибка моделирования не превышает 0,039 при среднеквадратическом отклонении 0,0594. Использование проецирования моделей Вольтерра на ортонормированные базисные функции позволило упростить процесс параметризации и снизить чувствительность моделей к шумам. Показано, что в качестве ортонормированных функций целесообразно применять функции Лагерра. Это позволяет минимизировать количество параметров модели в процессе идентификации.

Научная новизна. Усовершенствован метод идентификации нелинейных динамических объектов обогатительного производства на основе пространственно-временной модели Вольтерра, который отличается от существующих тем, что для повышения робастности модели Вольтерра к шумам осуществлено ее проецирование на набор ортонормированных базисных функций Лагерра.

Практическая значимость. Результаты апробации позволяют сделать вывод о целесообразности реализации пространственно-временной модели Вольтерра в пространстве состояний с помощью сети Лагерра, что позволяет повысить точность моделирования в условиях действия шумов по сравнению с моделью Вольтерра, уменьшением ошибки моделирования на 18,11 % при 40 итерациях идентификации. Экспериментальная проверка точности идентификации с применением модели Вольтерра-Лагерра в системе контроля содержания железа в различных точках технологической линии обогащения подтверждает целесообразность данного метода.

Ключевые слова: *нелинейная динамика, модель Вольтерра-Лагерра, обогатительное производство, идентификация*

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